Cosmic Nonstationarity of the Coherent Gravidynamic Quantum 1/f Effect

Peter H. Handel
Department of Physics and Astronomy and Center for Nanoscience
University of Missouri - St. Louis, USA
corresponding author, handel@umsl.edu

Erika Splet
Department of Physics and Astronomy Visiting International Scholar
University of Missouri - St. Louis, USA
erikasplet@yahoo.de

Abstract—The Coherent Gravidynamical Quantum 1/f Effect is derived both electrically from an asymptotic QED propagator and directly in an intuitive semiclassical way. Like all quantum 1/f noise it is both an infrared-divergence-and-decoherence phenomenon. The asymptotic QED propagator, valid at large times and distances or low frequencies, includes the Coulomb field in the notion of particle. It also predicts in a particular instability, similar to the effects of “dark energy.”

Keywords—1/f noise, quantum 1/f noise, dark energy, coherent quantum 1/f effect, quantum gravidynamics

I. INTRODUCTION

Starting first from the asymptotic propagator of the coherent picture of Quantum-Electrodynamics (QED), we derive the QED coherent quantum 1/f effect (Q1/fE) and study the nature of its nonstationarity. This, more realistic, "coherent states" picture of QED was introduced by Kibble, Zwanziger et al. in the 1960’s. It included for the first time the long-range Coulomb field in the unperturbed hamiltonian of the charged particle (electron), obtaining a QED propagator for the first time, but only in asymptotic form, for large times and distances, i.e. at low frequencies. This propagator was proven by us to reproduce our earlier semi-classical derivation of the universal 1/f spectrum of the coherent Q1/fE with an added feature: a non-integrable factor (c/P) in the 1/f spectral density. Here \( \alpha = e^2/hc \) is Sommerfeld's fine structure constant, \( n_{p,s} \) the number of electrons in the state of momentum \( p \) and spin \( s \), the rest mass of the fermions, \( \delta_{ss'} \) the Kronecker symbol, \( c \) the speed of light, \( x=(r,t) \) any space-time point and \( V \) the volume of a normalization box. T is the time-ordering operator which orders the operators in the order of decreasing times from left to right and multiplies the result by \((-1)^P\), where \( P \) is the parity of the permutation required to achieve this order. For equal times, T normal-orders the operators, i.e., for \( t=t' \) the left-hand side of Eq. (1) is \( i<s_{\rho}(x')|\psi_s(x')\phi_o> \). The state \( \phi_o \) of the N electrons is described by a Slater determinant of single-particle orbitals. To calculate the current autocorrelation function we need the density correlation function, which is also known as the two-particle correlation function. The two-particle correlation function is defined by

\[
\phi_o|T\psi_s(x)\psi_s(x')|\phi_o> = \phi_o|T\psi_s(x)\psi_s(x')|\phi_o>,
\]

\[
\phi_o|T\psi_s(x)\psi_s(x')|\phi_o> = \phi_o|T\psi_s(x)\psi_s(x')|\phi_o>.
\]

The equality is always satisfied when decoherence took place, scrambling the phases. The first term can be expressed in terms of the particle density of spin \( s \), \( n/2 = N/2V = \phi_o|\psi_s(x)|\phi_o> \), while the second term can be expressed in terms of the Green function, Eq. (1):

\[
\phi_o|T\psi_s(x)\psi_s(x')|\phi_o> = (n/2)^2 \delta_{ss'}G_s(x-x').
\]

The "relative" autocorrelation function \( A(x-x') \) describing the normalized pair correlation independent of spin is obtained by dividing by \( n^2 \) and summing over \( s \) and \( s' \)

\[
A(x-x') = 1 - (1/n^2)\sum_s G_s(x-x')G_s(x-x') = 1
\]

\[
(1/n^2)\sum_s \sum_{s'} \exp[i(p\cdot p'-(p\cdot p')^2(t-t')/(2m)]\phi_o|n_{p,s}n_{p',s}>.
\]

Here we have used Eq. (1). We now consider a beam of charged fermions, e.g., electrons, represented in momentum space by a vector \( p_o \) and occupying \( N/2 \) orbitals \( e^{ip}\), the asymptotic propagator derived by these authors [2]-[6] can be reduced for large time components of \( x' - x \), to the non-relativistic form [7]

\[
-i<\phi_o|T\psi_s(x')\psi_s(x)|\phi_o> = \delta_{ss'}G_s(x-x') =
\]

\[
(i/V)\sum_s \exp[ip\cdot (r-r')-p^2(t-t')/(2m)]\phi_o|n_{p,s}n_{p',s}>
\]

\[
\exp[-ip\cdot r'+i(m^2c^2+p^2)^{1/2}(t-t')(c/\hbar)]\phi_o|\psi_s(x')\phi_o> =\sum_{s'}<\phi_o|\psi_s(x)|\phi_o> = (n/2)^2 \delta_{ss'}G_s(x-x').
\]

(4)
space by a sphere of radius \( p_* \), centered on the momentum \( p_* \), which is the average momentum of the fermions, with \( p_{0i} \) parallel to \( r-r' \). The energy and momentum differences between terms of different \( p \) are large, leading to rapid oscillations in space and time which contain only high-frequency fluctuations. The low-frequency and low-wavenumber part \( A \), of this relative density autocorrelation function is given by the terms with \( p=p' \):

\[
A_i(x-x') = 1 - \left( \frac{1}{N^2} \right) \sum_{p} n_{p,i} (p_i-p_j) \delta\left( \frac{m^2 c^2 + p_{0i}^2}{\hbar^2} \right) \left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi} \left( r-r' \right)^{2\alpha/\pi} \left( - \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

\[
= 1 - \left( \frac{2}{N^2} \right) \left[ \frac{\gamma(2n)}{(2\pi)^2} \right] \int d^3 p (p_i-p_j) (r-r') \frac{1}{\hbar} \left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

\[
= 1 - \left( \frac{1}{N^2} \right) \left[ \frac{\gamma(2n)}{(2\pi)^2} \right] \int d^3 p (p_i-p_j) \delta\left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right) \left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi} \left( r-r' \right)^{2\alpha/\pi} \left( - \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

in which the fractional power could have been neglected in the integrand for all practical purposes except for the theoretical question of the integrability of the \( \alpha/\omega \) spectrum and stationarity. According to the Wiener-Khintchine theorem, the coefficient of the cos gives the spectral density. To get it for the fractional quantum fluctuations \( \delta j_i \) obtained by multiplying Eq. (4) on both sides with \( ep_i/m^2 \), and dividing by \( (en_p/m)^2 \) which is the square of the average current density \( j \), instead of just dividing by \( n^2 \). So it is the same as the fractional autocorrelation for quantum density fluctuations in the outgoing current. Indeed, for current density fluctuations \( \delta j_i \) we include a \( (\hbar/m)iV \) in front of each of the two \( \psi \) operators in Eq. (1), a factor \( pp_i/p_0^2 \) in Eq. (4) after the summation signs, a factor \( (p_i-p_j)^2 \) in the first form of Eq. (6), a factor \( (p_i-p_j)^2/p_0^2 \) in the second form, and no changes in Eqs. (7)-(8). Eq. (8) becomes

\[
S_{\delta j_i}(\omega) \approx \left[ 2\alpha/\pi(\alpha/\omega) \right] ^{2\alpha/\pi}
\]

This result coincides with our earlier theoretical result for coherent quantum 1/f noise if we replace \( N \) with \( N-2 \). Correlations are defined only for \( N \geq 2 \). The validity of this equation is restricted to low frequencies and wave-numbers. This equation is in excellent agreement with mobility and diffusion fluctuations 1/f noise in large electronic solid-state devices.

Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations. Most of the conventional Q1/f fluctuations are also in the mobility, but some of these are also found in the recombination speed or tunneling rate, being perceived and usually interpreted, as 1/f fluctuations in the concentration of carriers. This is how the quantum theory of fundamental 1/f noise solves the age-old controversy between those claiming 1/f noise in semiconductors was a carrier number fluctuation and those considering it a mobility fluctuation.

All integration and summations go up to infinity. Our result could be of cosmologic interest. Using identity [8],

\[
\gamma(2n) = \left[ \frac{2}{\pi} \right] \int_{0 \to \infty} 2\alpha/\pi \cos(\omega\alpha) d\omega/\alpha
\]

with arbitrarily small cutoff \( \omega_o \), with infinity as the upper limit, we obtain from Eq. (7) for the autocorrelation function of fractional current or density fluctuations the exact form

\[
A(x-x') = 1 + \left[ \frac{2}{\omega_{0}N} \right] \int d^3 p (p_i-p_j) \delta\left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right) \left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi} \left( r-r' \right)^{2\alpha/\pi} \left( - \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

This shows a \( \omega^{-1+2\alpha/\pi} \) spectrum and a 1/N dependence of the spectrum of fractional \( i \) and \( j \) fluctuations. We neglect the curly bracket in the denominator which is close to unity for very small \( \omega_o \). Eq. (9) for the coherent QED chaos process in electric currents can thus be written also in the form

\[
S_{\delta j_i} \approx \left[ 2\alpha/\pi(\alpha/\omega) \right] ^{2\alpha/\pi}
\]

This result derived directly earlier [9], [10], is in excellent agreement with the measurements [11]-[20], in large devices such as large \( n^+ \) Hg1-Cd3Te infrared detector diodes. It is also close to the empirical value of 0.0028N observed earlier by Hooge [20] in semiconductors and metals, after he understood the universal turbulence theory of 1/f noise. Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations.

III. NONSTATIONARITY

Consider, e.g., an infinite beam of particles of mass \( m \) in the cosmos, denoting \( \omega_o=2\pi/T_o \), where \( T_o \) is the 13.7\( 10^9 \) years age of the universe. Noticing that \( \alpha<1 \), and that the finite age \( T \) of the universe provides a natural cutoff \( \omega_o=1/T \), we re-write Eq. (11) in the form

\[
A(x-x') = \omega_o
\]

\[
= \left[ \frac{2}{\omega_{0}N} \right] \int d^3 p (p_i-p_j) \delta\left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right) \left( \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi} \left( r-r' \right)^{2\alpha/\pi} \left( - \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

\[
	imes \left[ 1 + \int_{0 \to \infty} \int_{0 \to \infty} 2\alpha/\pi \cos(\omega\alpha) d\omega/\alpha \right] ^{2\alpha/\pi} \left( r-r' \right)^{2\alpha/\pi} \left( - \frac{m^2 c^2 + p_{0j}^2}{\hbar^2} \right)^{2\alpha/\pi}
\]

\[
\approx \frac{2}{\omega_{0}N} = 0.06465 / \omega N
\]

This result derived directly earlier [9], [10], is in excellent agreement with the measurements [11]-[20], in large devices such as large \( n^+ \) Hg1-Cd3Te infrared detector diodes. It is also close to the empirical value of 0.0028N observed earlier by Hooge [20] in semiconductors and metals, after he understood the universal turbulence theory of 1/f noise. Being observed in the presence of a constant applied field, these fundamental quantum current fluctuations are usually interpreted as mobility fluctuations.
fluctuations. To find the spectral density of these inescapable fluctuations which are known to characterize any quantum state which is not an energy eigenstate, we use an elementary physical derivation based on Schrödinger’s definition of coherent states.

The coherent quantum 1/f effect will be derived in three steps: first we consider a hypothetical world with just a single mode of the gravitational field coupled to a beam of material particles. Considering the mode to be in a coherent state, we calculate the autocorrelation function of the quantum fluctuations in the particle-density (or concentration) which arise from the nonstationarity of the coherent state. Then we calculate the amplitude with which this one mode is represented in the field of an electron, according to electrodynamics. Finally, we take the product of the autocorrelation functions calculated for all modes with the amplitudes found in the previous step.

Let a mode of the gravitational field be characterized by the wave vector $q$, the angular frequency $\omega = cq$ and the polarization $\lambda$. Denoting the variables $q$ and $\lambda$ simply by $q$ in the labels of the states, we write the coherent state of amplitude $|z_q|$ and phase arg $z_q$ in the form

$$|z_q\rangle = \exp\left[-\frac{1}{2}|z_q|^2\right]\exp\left[-x^2/2\right]\exp\left[-z_q^2e^{-2i\omega t}\right] |0\rangle = \exp[-(1/2)|z_q|^2] \sum_{n=0}^{\infty} (z_q^n)/n! |n\rangle.$$  

Here $a_q^+$ is the creation operator which adds one energy quantum to the energy of the mode. Let us use a representation of the energy eigenstates in terms of Hermite polynomials $H_n(x)$

$$|n\rangle = (2^n n! \sqrt{\pi})^{1/2} \exp[-x^2/2] H_n(x) e^{i\frac{\omega}{\hbar} t}.$$  

This yields for the coherent state $|z_q\rangle$ the representation

$$\Psi_q(x) = \exp[-(1/2)|z_q|^2] \exp[-x^2/2] \sum_{n=0}^{\infty} \{|z_q e^{i\omega t}/\sqrt{n!(2^n \sqrt{\pi})}\}^{1/2} H_n(x).$$  

In the last form the generating function of the Hermite polynomials was used. The corresponding autocorrelation function of the probability density function, obtained by averaging over the time $t$ or the phase of $z_q$, is, for $|z_q|<1$,

$$P_q(t,x) = \Psi_{q,t}^* \Psi_{q,t}^2 = \Psi_{q,t}^* \Psi_{q,t}^2 >$$

$$= \{1 + 8x^2|z_q|^2[1 + \cos \omega t] - 2|z_q|^2\} \exp[-x^2/2].$$  

Integrating over $x$ from $-\infty$ to $\infty$, we find the autocorrelation function

$$A^1(\tau) = (2)^{-1/2}\{1 + 2|z_q|^2 \cos \omega \tau\}.  

This result shows that the probability distribution contains a constant background with small superposed oscillations of frequency $\omega$. Physically, the small oscillations in the total probability describe self-organization or bunching of the particles in the beam. They are thus more likely to be found in a measurement at a certain time and place than at other times and places relative to each other along the beam. Note that for $z_q = 0$ the coherent state becomes the ground state of the oscillator which is also an energy eigenstate, and therefore stationary and free of oscillations. Note also the presence of four single-particle wave functions, because the
two-particle wave function without interactions is a product of two single-particle wave functions.

We now determine the amplitude $z_q$ with which the gravitational field mode $q$ is represented in the correct definition of the physical particle. The simple way to do this, like in the QED case that was treated first, is to let (in QGD) a bare particle also dress itself, this time through its interaction with the gravitational field, i.e. by performing first order perturbation theory with the non-relativistic interaction Hamiltonian

$$H' = \hbar \mu \phi .$$

(20)

where $\phi$ the scalar gravitational potential. This corresponds to a Fourier expansion $-4\pi G m^2 q^2$ of the gravitational potential $-Gm^2 r$ of a material particle in a box of volume $V$, and multiplication with a squared gravitonic "wave function" $(\hbar c q V)^{-1}$. This way we obtain

$$|z_q|^2 = \pi G (m/q)^2 (\hbar c q V)^{-1} .$$

(21)

Considering now all modes of the gravitational field, we obtain from the single-mode result of Eq. (5)

$$B(\tau) = C \Pi_q \{1 + 2|z_q|^2 \cos \omega_q \tau\} = C \{1 + \Sigma_q 2|z_q|^2 \cos \omega_q \tau\} = C \{1 + 4(V/2)^2 \} |z_q|^2 \cos \omega_q \tau .$$

(22)

Here we have again used the smallness of $z_q$, and we have introduced a constant $C$ proportional to the squared velocity of the particles in the beam. Using Eq. (21) we obtain

$$B(\tau) = C \{1 + 4(\pi V/2)^2\} (4\pi V)G(m^2/\hbar c) \frac{1}{2}(dq/q) \cos \omega_q \tau .$$

(23)

Here $B = G m^2 c^2 / \hbar = 10^8 m^2 / \text{gram}^2$ replaces in our case the fine structure constant. The first term in curly brackets is unity and represents the constant background, or the d.c. part of the mass current density defined by the motion of the beam of particles through vacuum. The autocorrelation function for the relative (fractional) density fluctuations, or for the fractional mass-current density fluctuations in the beam of material particles is obtained therefore by dividing the second term in curly brackets by the first term. The constant C drops out when the fractional fluctuations are considered. According to the Wiener-Khintchine theorem, the coefficient of cos $\omega t$ is the spectral density of the fluctuations, $S_{\omega t}$ for the particle concentration, or $S_j$ for the current density $j = e(k/m) \psi^2$

$$S_{\omega t} = 2(\beta/\pi N) = 2 G m^2/\pi N c h$$

$$= 4.4 \times 10^{-3} m^2/\pi N c h^2 .$$

(24)

Here we have included the total number $N$ of material particles of mass $m$ that are observed simultaneously in the denominator, because the noise contributions from each particle are independent. For example, for $m = 10^{-5} g = 1 \mu g$, we get about $10^{-3} / \text{N from Eq. (24)}$. This is similar to the coherent QED 1/f result calculated above.

V. DISCUSSION

The results obtained in the last section show that the new coherent Gravidynewt Quantum 1/f Effect (QGD 1/fE) is hard to be observed on atomic particles, but is easy to observe as a new form of 1/f noise in beams of mesoscopic aggregates (almost involving the squared Avogadro number in $\beta$) or even in macroscopic flows of matter. The new effect differs from the well known earlier conventional form, because it is present in any current of matter, and not only as a result of scattering, as we know, was the case for the conventional Gravidynewt Quantum 1/f Effect (QGD 1/fE). The latter was a property of the physical quantum mechanical cross sections that we had introduced as part of a new aspect of quantum mechanics, that we called quantum 1/f noise. We now understand that in the gravidynanical case, just as in the electrodynamical or lattice-dynamical (piezoelectric) cases discussed elsewhere, the observed quantum 1/f noise represents macroscopic quantum fluctuations including both coherent and conventional contributions. Simple formulas allow for calculating both the coherent and conventional quantum 1/f effect, combining them (see Gravidyn in these Proc.)