Measurement of the cyclostationary third moment in the noise of a tunnel junction

Pierre Février
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
pierre.fevrier@usherbrooke.ca

Christian Lupien
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
christian.lupien@usherbrooke.ca

Bertrand Reulet
Université de Sherbrooke
Dépt. de Physique, Institut Quantique
Sherbrooke, QC, Canada
bertrand.reulet@usherbrooke.ca

***Abstract*—We measure the cyclostationary third moment in the photo-assisted noise of a tunnel junction. We use an homodyne detection where the junction bias is modulated at 14.55GHz, and the noise is demodulated at 4.85GHz. We identify contributions due to the presence of the measurement setup.**

***Index Terms*—Cyclostationary noise, non-Gaussian noise, third moment, tunnel junction**

In electronics, the term "current noise" often refers to the variance of current fluctuations, defining an effective temperature for a system. However, since fluctuations are in general not Gaussian, a lot of information can be extracted from the measurement of higher order moments. For example, in mesoscopic conductors, the existence of a third moment stems from the binomial statistics of charge transfer [1], and correlations between current and emitted photons [2]. It can also probe internal timescales [3] or electronic interactions [4]. In this work we present a new tool to study high frequency fluctuations in conductors. We measure the cyclostationary third moment of voltage fluctuations, i.e. the dynamical response of the third moment to an external drive.

In stationary regime, the third moment of current fluctuations is given by the correlator:

$$M_3(\tau,\tau') = \langle i(t)i(t-\tau)i(t-\tau')\rangle \tag{1}$$

which is invariant by translation in time t, and depends on the two delays τ and τ'. The brackets designate an ensemble or time average. With the Fourier transform of Eq.(1), we can show that, for a given bandwidth $[f-\Delta f, f+\Delta f]$, the measurement of M_3 requires a minimum width of $\Delta f > f/3$. Smaller bandwidth leads to $M_3 = 0$. This stringent condition on bandwidth makes the measurement of the third moment difficult at high frequency. However, in a cyclostationary regime, i.e. when the third moment is periodically modulated at nf_0 with $n \in \mathbb{Z}$, we can measure the *cyclostationary third moment* defined by the bispectrum:

$$K_n(f,f') = \langle i(f)i(f')i(-f-f'+nf_0)\rangle \tag{2}$$

This correlator can be non-zero when integrated over a small bandwidth $2\Delta f \ll f$ by choosing $f_0 = f$ and $n = \pm 1$ or $n = \pm 3$:

$$K_{\pm 1}(f) = \langle i(\pm f)^2 i(\mp f)\rangle, \quad K_{\pm 3}(f) = \langle i(\pm f)^3\rangle \tag{3}$$

These two correlators are measured with the static third order moment K_0 of the signal after a phase sensitive demodulation at frequency f of current fluctuations. In this work, we demonstrate the measurement of K_3, i.e. the measurement of the third moment of current fluctuations, while the system (here a tunnel junction) is driven at $3f$. This communication is organized as follows: we first describe our experimental setup, and the method used to extract the non-Gaussian contribution to the noise attributed to the tunnel junction. In a second part, we show how the presence of the measuring circuit modifies the statistics of current fluctuations in the junction.

I. Measurement of the cyclostationary third moment

A. Non-Gaussian noise generation using a tunnel junction

To generate the cyclostationary non Gaussian-noise, we used a tunnel junction with metallic electrodes. The junction have been made using usual lithography technique and evaporation of aluminum electrodes on a silicon substrate. The insulating tunnel barrier is obtained by controlled oxidation of the first electrode under oxygen atmosphere. The dc resistance of the junction is R =130Ω at 3.7K, with a RC cutoff frequency estimated around 8GHz. Because electronic transport in a tunnel junction is a Poisson process, all moments of current fluctuations are linear with the average current: $\langle (I-\langle I\rangle)^2\rangle = e|\langle I\rangle|$ and $\langle (I-\langle I\rangle)^3\rangle = e^2\langle I\rangle$. By modulating the bias current of the junction at $3f$ with an amplitude I_{ac}, we expect to measure a linear modulation of the third moment: $K_3 \propto e^2 I_{ac}$. The junction is current biased through a bias-Tee (see Fig. 1), allowing the separation of high frequency signal from DC bias.

B. Homodyne detection

The experiment has been performed in an helium-free cryostat with a base temperature of 3.7K. The microwave signal from the junction propagates through an isolator before being amplified by a low-noise cryogenic amplifier with a bandwidth of 4-8GHz and a nominal noise temperature of 2.5K. The role of the isolator is to make the temperature and the impedance of the environment seen by the junction well defined. A room-temperature amplifier provides extra-amplification, before demodulation. In order to measure K_3,

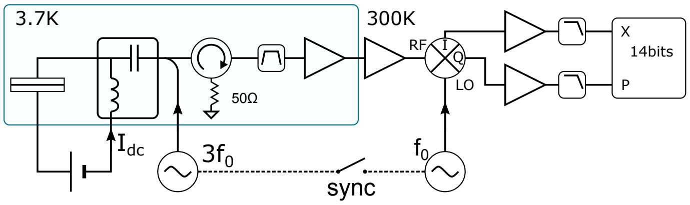

Fig. 1. Experimental setup for third order cyclic correlation measurement. Noise from the junction is preamplified by a cryogenic low noise amplifier, then amplified at room temperature before down-mixing by a high-IP3 IQ mixer. Both quadratures are low-pass filtered (Δf=225MHz) and amplified before digitization. The circulator fixes the environment seen by the junction to a 50Ω load at 3.7K.

we used an homodyne detection where the noise is demodulated at f=4.85GHz. The junction is photoassisted (i.e. ac driven) by a single tone at $3f$=14.55GHz using a directional coupler. Driving at $3f$ instead of f avoids dealing with the excitation reflected back in the amplification chain.

We use an IQ mixer with high linearity to separate the signal into two quadratures X and P. Each quadrature is amplified and low-pass filtered. The final measurement bandwidth Δf is $\pm$ 225MHz. The low frequency signal is digitized with a 2-channel fast acquisition card (14bits, 400MS/s), to compute the probability density $\mathcal{P}(X, P)$.

After demodulation the skewness on each quadrature $\langle X^3\rangle$ and $\langle P^3\rangle$ is given by the real and imaginary part of:

$$\langle X^3\rangle + i\langle P^3\rangle = \iint df_1 df_2 G(f_1)G(f_2)G(f_3)\times \left[\frac{1}{4}K_{v,3}(f_1,f_2)e^{i3\phi_0} + \frac{3}{4}K_{v,1}(f_1,f_2)e^{i\phi_0}\right] \quad (4)$$

$$f_3 = 3f - f_1 - f_2 \quad (5)$$

where $K_{v,3}$ and $K_{v,1}$ are the third order cyclic moments of voltage fluctuations at the input of the amplifier. G is the voltage gain of the amplification chain accounting for the finite bandwidth and losses. ϕ_0 is a global phase due to the delay between excitation and detection. For frequency independent gain, in a narrow bandwidth $[f-\Delta f, f+\Delta f]$ with $\Delta f \ll f$ Eq.(4) simply gives:

$$\langle X^3\rangle + i\langle P^3\rangle = \frac{1}{4}G^3\left[K_{v,3}(f)e^{i3\phi_0} + 3K_{v,1}(f)e^{i\phi_0}\right]\frac{3}{4}\Delta f^2 \quad (6)$$

The $3/4$ correction to the bandwidth comes from the convolution appearing in Eq.(4) due to the condition Eq.(5). The amplitude of the ac excitation I_{ac}, the gain and the noise of the amplification chain, are determined by measuring the variance of the photo-assisted noise $\langle X^2\rangle$ and $\langle P^2\rangle$ [5] (see Fig. 2). We find that the added noise temperature is around 3K which is close to the noise of the cryogenic amplifier. Signal to noise ratio is limited by the junction intrinsic noise at high bias. The estimated total power gain is $\sim$78dB.

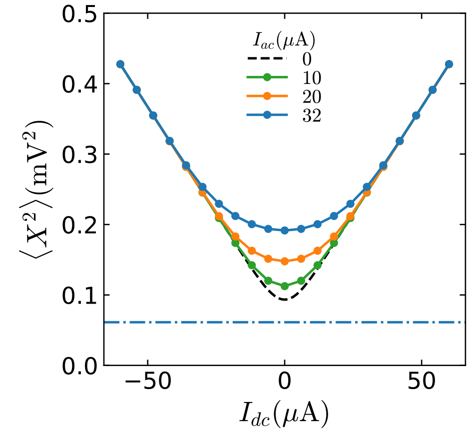

Fig. 2. Variance of voltage fluctuations on the X quadrature for different drive amplitude I_{ac}. The linear dependence at high bias is characteristic of shot-noise. The rounding appearing at low bias is a direct consequence of the noise modulation by the ac drive and can be used to extract I_{ac}. At I_{ac} =0, the variance is proportional to the voltage dependent current noise spectral density $S_i(V)$ in $\mathrm{A}^2\mathrm{Hz}^{-1}$ in addition to a constant contribution from the amplifier (dash-dotted line). The rounding is the cross-over with junction's thermal noise at 3.7K.

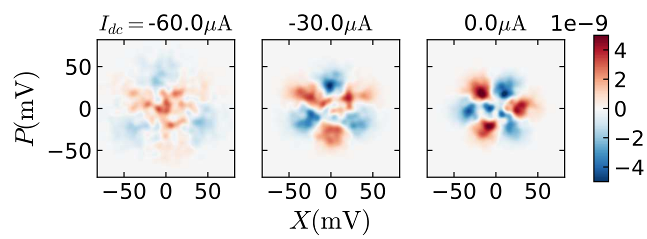

Fig. 3. Differential probability density $\Delta\mathcal{P}(X, P)$ of voltage fluctuations of a tunnel junction under cyclostationary excitation (after subtraction of the phase insensitive contributions). The existence of a third order moment is directly related to the rotational symmetry clearly visible in the data. Different plots correspond to different dc bias. The histogram rotates with dc bias, as a consequence of environmental feedback.

C. Third moment from the symmetry of the histograms

The phase coherence between the detection and excitation can be switched off by a slight detuning of one the sources. This averages to zero the contributions that depend on ϕ_0. Contributions to $\mathcal{P}(X, P)$ that are independent of the phase of the excitation are removed by measuring the difference in histograms obtained with and without phase coherence giving the differential probability $\Delta\mathcal{P}(X, P)$. These contributions include, non-Gaussian noise added by the non-linearity of the amplification chain, the non-linearity of the acquisition card and the almost Gaussian photoassisted noise showed in Fig. 2.

$\Delta\mathcal{P}(X, P)$ shows clear rotational symmetry (see Fig. 3). This is the direct consequence of the homodyne demodulation of the noise at a fraction of the noise modulation frequency. It is then natural to use polar coordinates: $X = r\cos(\theta)$ and $P = r\sin(\theta)$ and express $\mathcal{P}(r, \theta)$ as the Fourier series:

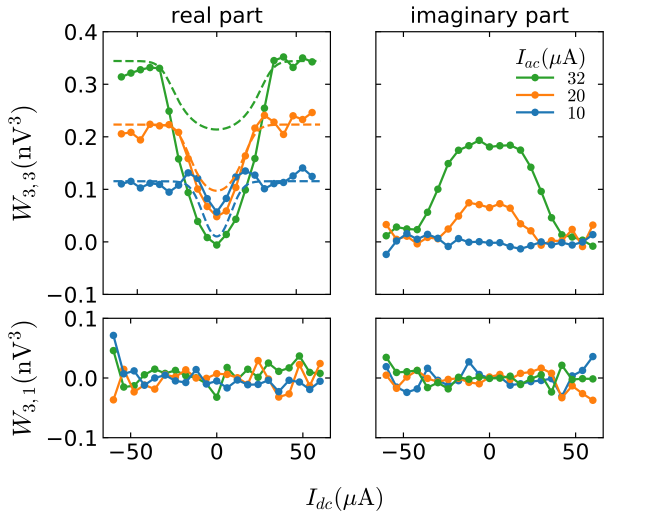

Fig. 4. Plain lines: Contributions to the third moment of current fluctuations extracted from the symmetries of the differential probability density $\Delta\mathcal{P}(X,P)$, as function of the dc current I_{dc} and the ac excitation $I_{ac}\cos 3\omega_0 t$. The distributions $\mathcal{P}(X,P)$ have been rotated by a global angle in order to maximize the signal on one quadrature. Dashed lines: expected signal with an environment at 3.7K (see text). The total averaging time is 50h.

$$P(r,\theta) = \sum_n P_n(r) e^{-in\theta} \quad (7)$$

We define:

$$W_{\alpha,n} = \frac{\pi}{2}\int_0^\infty P_n(r) r^{\alpha+1} dr \quad (8)$$

Moments of the distribution $P(r,\theta)$ are related to the functions $W_{\alpha,n}$. Simple algebra shows that third order moments (skewnesses and coskewnesses) are given by the first and third order rotational symmetries:

$$\langle X^3\rangle = \mathrm{Re}(W_{3,3}+3W_{3,1}),\ \langle XP^2\rangle = \mathrm{Re}(W_{3,1}-W_{3,3})$$
$$\langle P^3\rangle = \mathrm{Im}(W_{3,3}-3W_{3,1}),\ \langle PX^2\rangle = \mathrm{Im}(W_{3,1}+W_{3,3}) \quad (9)$$

The measurement of the joint probability $\mathcal{P}(X,P)$ is necessary to separate the two contributions $W_{3,1}$ and $W_{3,3}$. In our experiment, it appears that $W_{3,1} \simeq 0$ (see Fig. 4). This corresponds to $K_{v,1}$ =0 as expected for a noise modulation at $3f$. We consider in the following $\langle X^3\rangle = \mathrm{Re}(W_{3,3})$ and $\langle P^3\rangle = \mathrm{Im}(W_{3,3})$, as it improves signal to noise ratio. We also choose a phase ϕ_0 that maximizes $\langle X^3\rangle$ and minimizes $\langle P^3\rangle$ at high bias.

$\langle X^3\rangle$ at high bias shows a plateau with an amplitude proportional to I_{ac}. This is qualitatively close to what is expected for a tunnel junction as discussed before. We however observe a deviation from this intrinsic contribution at low bias, a dip with a width of the order of I_{ac} on both quadrature. In the following, we show that this deviation can be attributed to contributions induced by the presence of the electromagnetic environment of the setup.

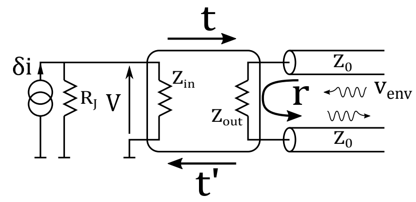

Fig. 5. Model for the impedance mismatch between the junction and the transmission line.

II. Effect of the measuring circuit

Ideally, currents fluctuations in a conductor are measured using a perfect ammeter with zero input impedance. This is usually not the case for radio-frequency measurements, since transmission lines have a 50Ω characteristic impedance. Additionally, the junction itself has a geometric capacitance in parallel with R_J. The wire bond between the junction and the 50Ω adapted microstrip launch pad also adds an inductance in series with some delay and dissipation. The circuit between R_J and the transmission line (TL) can be represented by a reciprocal two port network (see Fig. 5). This circuit shows an input impedance $Z_{in}(f)$,the impedance seen by the junction, and an output impedance $Z_{out}(f)$, causing an impedance mismatch with the TL. The voltage transmission coefficients $t(f)$ and $t'(f)$ characterize the transmission from the junction to the TL and from the TL to the junction. Because the circuit is reciprocal, we have $t' = t\frac{R_J}{Z_0}$. The *intrinsic* contribution to $K_{v,3}$ from the junction is:

$$K_{v,int}(f) = Z_{\text{eff}}(f)^3 t(f)^3 e^2 I_{ac} \quad (10)$$

where $Z_{\text{eff}}(f) = (R_J Z_{in}(f))/(R_J + Z_{in}(f))$.

Due to the finite impedance Z_{in} at finite frequency, the voltage drop $V(t)$ at the junction shows fluctuations $\delta V(t)$. Indeed, $V(t)$ is influenced by the current noise $\delta i(t)$ generated by the sample as well as by an external noise source $v_{env}(t)$, the noise coming from the measurement setup:

$$\delta V(f) = -i(f)\frac{R_J Z_{in}(f)}{R_J + Z_{in}(f)} + t'(f) v_{env}(f) \quad (11)$$

Because photo-assisted noise variance depends on bias voltage, this leads to environmental corrections to the statistics of fluctuations. [6] [7] [8]. Similar corrections are expected in the measurement of the cyclostationary third moment [9].

Due to the impedance mismatch between the junction and the transmission line, part of the noise coming from the environment is reflected back to the measurement setup. In our case the environment is the 50Ω load on the circulator at 3.7K, and $\langle v_{env}^2\rangle = \frac{1}{2}k_B T_{env} Z_0$. This reflected signal is correlated with the fluctuations of junction noise variance through the fluctuation of bias (Eq.(11)). This adds a contribution to $K_{v,3}$:

$$K_{v,env}(f) = 3r(f)t(f)^2 Z_{\text{eff}}^2 \langle v_{env}(f) i(f)^2\rangle \quad (12)$$

If the modulation δV is small, it has been showed [9] that $i(f)^2$ can be rewritten as :

$$i(f)i(f') = \sum_{\alpha} S_\alpha \delta(\alpha f_0 - f - f') + D_\alpha \delta V(f + f' - \alpha f_0) \tag{13}$$

Where the S_α and D_α are the complex Fourier coefficients of the noise spectral density $S_i(V(t))$ (see Fig. 2), and its derivative $dS_i(V(t))/dV$ averaged over bias fluctuations δV.

From Eq.(11) and Eq.(13) we obtain a new expression for Eq.(12):

$$K_{v,env}(f) = Z_{\text{eff}}(f)^3 t(f)^3 \frac{3}{2} D_3 k_B T_{env} r(f) \frac{Z_{in} + R_J}{Z_{in}} e^{i2\phi(f)} \tag{14}$$

The reflection coefficient r is given by $(Z_{out} - Z_0)/(Z_{out} + Z_0)$. We also used the fact that $t^*(f) = t(-f) = |t|e^{-i\phi}$ where the phase ϕ accounts for the propagation between the junction and the transmission line. Only D_3, the modulation at $\alpha f_0 = 3f$ of dS_i/dV contributes to the environmental effect. D_3 is finite near zero bias where dS_i/dV is almost linear in I_{dc} and zero at high bias where dS_i/dV is constant, with a characteristic width given by I_{ac}. Because of the phase ϕ and the reflection that can be complex, we expect the contribution of the environment not to be in phase with the intrinsic contribution. To check the validity of Eq.(14), we increased the temperature of the environment by sending a Gaussian noise on the junction via the same coupler used for the ac excitation. Part of this noise is reflected on the junction and collected by the amplification chain. The effective noise temperature seen by the junction is estimated from the amplitude of the reflected signal, assuming $Z_{out} = R_J$. With a temperature much greater than 3.7K, the environmental contribution becomes the dominant one. We can see on Fig. 6 that this contribution is proportional to D_3 and T_{env} as expected. From this we can calculate the expected contribution with an environment at 3.7K (see Fig. 4). This is not in agreement with the measurement. Furthermore, around zero bias, $\langle X^3 \rangle$ have an almost parabolic dependence on I_{dc}, whereas $\langle P^3 \rangle$ is almost constant. This suggests the presence of another contribution. Indeed, Eq.(11) shows that fluctuations of the second moment $\langle i(f)^2 \rangle$ are correlated with current fluctuations $i(f)$. This adds a contribution to the correlation $\langle i(f)^3 \rangle$ known as feedback effect [9]. This effect needs further investigation and will be addressed in a future communication.

III. Conclusion

We report the measurement of the cyclostationary third moment in the photoassisted noise of a tunnel junction. The signal is consistent with the Poissonian statistics of current fluctuations in a tunnel junction. We also observe feedback and environmental contributions due to the presence of the measurement setup. This experiments opens the possibility to measure the frequency dependence of non-Gaussian noise due to the presence of an intrinsic dynamics. It could be for example, the classical dynamics of diffusion of electrons in a wire [10], or the quantum dynamics associated with the timescale h/eV [11]. We acknowledge fruitful discussions with Edouard Pinsolle and Charles Marseille, and the technical help of G. Laliberté. This work was supported by the Canada Excellence Research Chair program, the NSERC the MDEIE, the FRQMT via the INTRIQ, the Université de Sherbrooke via the Institut Quantique and the Canada Foundation for Innovation.

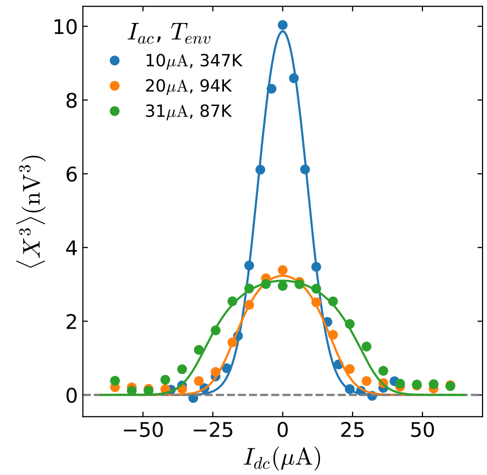

Fig. 6. : Effect of the temperature of the environment on the cyclic third moment. Dots represents data points and plain lines are given by Eq.(14) using the I_{ac} and an effective temperature extracted from the measurement of $\langle X^2 \rangle$. Global phase has been chosen to maximize signal on quadrature $\langle X^3 \rangle$.

References

[1] B. Reulet, J. Senzier, and D. E. Prober, "Environmental Effects in the Third Moment of Voltage Fluctuations in a Tunnel Junction," *Phys. Rev. Lett.*, vol. 91, no. 19, p. 196601, Nov. 2003.

[2] J. Gabelli, L. Spietz, J. Aumentado, and B. Reulet, "Electron–photon correlations and the third moment of quantum noise," *New Journal of Physics*, vol. 15, no. 11, p. 113045, 2013.

[3] N. Ubbelohde, C. Fricke, C. Flindt, F. Hohls, and R. J. Haug, "Measurement of finite-frequency current statistics in a single-electron transistor," *Nature Communications*, vol. 3, p. 612, Jan. 2012.

[4] E. Pinsolle, S. Houle, C. Lupien, and B. Reulet, "Non-Gaussian Current Fluctuations in a Short Diffusive Conductor," *Physical Review Letters*, vol. 121, no. 2, p. 027702, Jul. 2018.

[5] G. B. Lesovik and L. S. Levitov, "Noise in an ac biased junction: Nonstationary Aharonov-Bohm effect," *Physical Review Letters*, vol. 72, no. 4, pp. 538–541, Jan. 1994.

[6] C. W. J. Beenakker, M. Kindermann, and Y. V. Nazarov, "Temperature-Dependent Third Cumulant of Tunneling Noise," *Phys. Rev. Lett.*, vol. 90, no. 17, p. 176802, Apr. 2003.

[7] M. Kindermann, Y. V. Nazarov, and C. W. J. Beenakker, "Feedback of the electromagnetic environment on current and voltage fluctuations out of equilibrium," *Physical Review B*, vol. 69, no. 3, p. 035336, Jan. 2004.

[8] B. Reulet, J. Gabelli, L. Spietz, and D. E. Prober, "The third moment of current fluctuations in a tunnel junction: Experiments in the classical and quantum regimes," *arXiv:1001.3034 [cond-mat]*, Jan. 2010.

[9] T. T. Heikkilä and L. Roschier, "Cyclostationary measurement of low-frequency odd moments of current fluctuations," *Phys. Rev. B*, vol. 71, no. 8, p. 085316, Feb. 2005.

[10] S. Pilgram, K. E. Nagaev, and M. Büttiker, "Frequency-dependent third cumulant of current in diffusive conductors," *Physical Review B*, vol. 70, no. 4, p. 045304, Jul. 2004.

[11] K. Thibault, J. Gabelli, C. Lupien, and B. Reulet, "Pauli-Heisenberg Oscillations in Electron Quantum Transport," *Physical Review Letters*, vol. 114, no. 23, p. 236604, Jun. 2015.