EXPERIMENTAL STUDY OF THE VERTICAL STABILITY OF HIGH DECAY INDEX PLASMAS IN THE DIII-D TOKAMAK


printed in February 1990
EXPERIMENTAL STUDY OF THE VERTICAL STABILITY OF HIGH DECAY INDEX PLASMAS IN THE DIII-D TOKAMAK


General Atomics
P.O. Box 85608
San Diego, California 92138-5608, U.S.A.

ABSTRACT. Experiments on the stabilization of highly elongated, vertically unstable plasmas were carried out on the DIII-D tokamak. Identification of the closed-loop transfer function showed that vertical stability could be usefully modelled as a second order dynamical system. The effect of varying the controller gains and the vertical field decay index was studied and found to be qualitatively as predicted by a low order model previously proposed [1]. The implementation of a new hybrid inboard/outboard coil positional control with differing controller dynamics allowed operation of DIII-D up to 92% of the limiting equilibrium field decay index of the vacuum vessel for those plasmas used. This improved control has allowed operation at plasma elongation up to $\kappa = 2.5$.

---

*Centre de Recherches en Physique des Plasmas, Association Euratom – Confédération Suisse, Ecole Polytechnique Federale de Lausanne 21, Av. des Bains, 1007 Lausanne, Switzerland.

†Fusion Energy Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, U.S.A.
1. INTRODUCTION

One method currently under study for increasing the viability of the tokamak as a fusion energy device is to operate with more strongly non-circular plasma cross-sections. In this way the current-carrying capability can be significantly enhanced, leading hopefully to higher attainable values of the plasma $\beta$ which scales theoretically and experimentally as $\beta \approx I/aB \approx (1 + \kappa^2)/2Aq$ where $I_p$ is the plasma current, $B_t$ is the toroidal field, $a$ is the horizontal minor radius, $\kappa$ is the plasma elongation, $A$ is the aspect ratio and $q$ is the safety factor. Furthermore, higher elongation allows a higher value of $I_p/B_t$ at fixed $q$, leading to better energy confinement.

In order to achieve the desired elongations, a strong quadrupole field must be added to the radially stabilizing vertical field. The relative strengths of the quadrupole and vertical (dipole) fields determine the curvature of the field lines, which is conveniently expressed as the equilibrium field decay index

$$n \equiv -\frac{X}{B_s} \left. \frac{\partial B_z}{\partial X} \right|_{X=X_0},$$

and which must be negative for $\kappa > 1$. $X$ is the plasma major radius, the symbol $R$ being reserved to denote electrical resistance. The toroidal plasma current in such a field configuration is positionally unstable in the vertical direction, and we must dynamically keep it close to a given metastable position using active feedback control of the radial field. All tokamaks with an elongation greater than that obtained naturally in a purely vertical field ($n=0$) have had to provide positional stabilization of this naturally unstable system. The analysis and understanding of the results of this paper depend on the formalism and discussions developed in [1]. This introduction repeats the salient points of the earlier study.
In Ref. 1 we showed that the limiting value for the decay index is related to the critical index of the shell,

\[ n_c = \frac{2M_{vp}^2 X_0}{\mu_0 \Gamma L_v}, \quad \Gamma = \frac{L_{ext}}{\mu_0 X_0} + \frac{L_i}{2} + \beta_p + \frac{1}{2}. \]  

(2)

Note that \( n_c \) is defined as a positive quantity, and that plasmas experience vertical instability for negative values of \( n \). The ideal limit for rigid-body vertical stability is \( n/n_c > -1 \).

Throughout the paper subscripts \( p, v \), and \( a \) refer to plasma, vessel (or shell), and active feedback coil respectively. \( L \) is the symbol for self inductance, and \( M \) is mutual inductance. \( I \) and \( V \) are current and voltage. The prime denotes partial differentiation with respect to vertical position. The notation otherwise follows that of Ref. 1.

Two problems will arise in the future. Firstly, the size and current of the plasmas to be stabilized is increasing, leading to significantly greater power handling requirements. Secondly, in experiments such as the DIII-D tokamak [2] and the TCV tokamak [3] we must stabilize more elongated plasmas, moving to more intrinsically unstable configurations.

For both of these cases it is important to develop a reliable operational understanding of the vertical stabilization problem, upon which an optimal control strategy can be based, allowing a minimization of the power requirements. The increase in the forces on the vessel and the poloidal coil system resulting from the motion of a high current plasma with greater elongation presents a further motivation. A final motivation resides in the fact that the poloidal system of a large tokamak represents a significant part of the system cost. The design of this system must, therefore, be carefully optimized.

In Ref. 1, an approach has been evolved for modelling and improving the DIII-D tokamak vertical position feedback control. The starting point for the vertical stability dynamics is a rigid massless plasma. The plasma is characterized by its size, current, poloidal \( \beta \), and the internal and external inductances. These are incorporated so as to be consistent with radial force balance, assuming stationary fields.
In the inhomogeneous equilibrium field the plasma current experiences a vertical force given by

\[ F_z = \int_{\text{plasma volume}} j_\phi(\vec{r}) B_r(\vec{r}) \, d^3\vec{r} \, . \]  \[ (3) \]

In order to retain the dominant physics while remaining algebraically simple, this force was approximated as \( F_z \approx 2\pi X_0 I_p B_r(X_0) \). The vacuum vessel provides a restoring force via the radial field resulting from the image currents induced by the plasma motion. The poloidal distribution of the vessel current is usefully decomposed into orthogonal modes, (Ref. 1, Appendix A). The first antisymmetric mode dominates the vertical control, both due to its long decay constant and its large radial field on axis. Retaining only the dominant vessel modes important for plasma control again simplifies the algebra, while retaining the physics important for plasma control. Up-down symmetric pairs of active feedback coils fed in antiparallel were added to the model in terms of their mutual inductance to the vessel current modes and their radial field at the plasma axis.

This simplified model leads to three determining equations. The first equation

\[ \frac{\mu_0 I_p \Gamma}{2 X_0} n_z - \frac{d M_{up}}{dz} I_v - \frac{d M_{ap}}{dz} I_a = 0 \]  \[ (4) \]

is the equation for vertical force balance for a massless plasma; the plasma is located at the position where the radial field vanishes.

We add the two circuit equations for the first antisymmetric vessel eigenmode current \( I_v \), and the active antisymmetric coil current \( I_a \)

\[ L_v \frac{d I_v}{dt} + R_v I_v + M_{sv} \frac{d I_a}{dt} + \frac{d M_{up}}{dz} I_p + \frac{d M_{ap}}{dz} I_a = 0 \, , \]  \[ (5) \]
\[ L_a \frac{da}{dt} + R_a I_a + M_{av} \frac{dl}{dt} + \frac{dM_{ap}}{dz} \frac{dz}{dt} I_p = V_a \]  

(6)

The applied active voltage in Eq. (6) is defined by a PD (proportional-derivative) controller,

\[ \frac{V_a(t)}{L_a} = G_z [z(t) - z_{ref}(t)] + G_v \frac{dz}{dt} [z(t) - z_{ref}(t)] \]

(7)

where \( G_z \) and \( G_v \) are the positional proportional and derivative gains respectively. In what follows, the values of \( G_z \) and \( G_v \) will be given in MKSA units unless explicitly noted otherwise.

These three determining equations lead to a characteristic equation corresponding to a dynamical system model of only second order. One aim of this paper is to determine whether such a simple dynamical system correctly represents the experimentally observed vertical movement of an elongated plasma.

The model as derived in Ref. 1 did not explicitly incorporate the poloidal shaping coils other than those fed with the vertical control signal. This point is developed later in the paper (Section 3.3).

The influence of different PD gain settings and different vertical field decay indices was studied systematically in Ref. 1. It is characteristic of such a second-order system that, once the system parameters are determined, the system stability can be characterized in terms of the derivatrive and proportional gains of the controller. The general behaviour of the plasma position control is reproduced in Fig. 1. The axes represent the proportional and derivative gains, Eq. (7). The stable operating regime is below the sloping line C–D and to the left of the vertical line B–B, i.e., Regions I and II. Within this stable region, the response can be oscillatory, Region I or overdamped, Region II. The solid contours give the oscillation frequency (\( \omega \)) and the dashed lines give the real growth rate (\( \gamma \)), negative
in the stable region. This diagram is characteristic of any second order system which is stabilized using a PD controller. It is determined for a particular value of $n$. As $n$ increases, the line C-D moves downwards. Increases in $-G_a$ increase the damping, and therefore the sluggishness of the system, whereas increases in $-G_x$ decrease the damping. The operating point must be chosen to be consistent with series of such stability diagrams which cover the range of $n/n_c$ over the desired tokamak operating range (0 to $-1$ in our case). We shall refer repeatedly to this diagram in describing the experimental results.

Several important features of the system were identified in Ref. 1

- There is a critical decay index, $n_c$, above which control is not possible. Beyond this limit, the plasma will in fact move with the Alfvén velocity. The vessel current restoring force is inadequate to stabilize the plasma.

- The vessel image current induced by a vertical movement is predominantly on the outboard side due to toroidicity.

- For $n > -n_c$ we can stabilize the plasma provided a suitable choice of active coil is made. Because the vessel shields the field from the outboard active coils very effectively, inboard coils are needed to reach the ideal limit.

- There is a vertical field decay index associated with the active coils, $n_a$, above which active feedback stabilization requires velocity gain.

- The power requirements are sensitive to the coil placement. The voltage requirement scales as $e^{-n/n_a}$ and the bandwidth requirement is the open-loop growth rate of the mode (Ref. 1, Section 4).

One result of this study is that there is no single choice of a control coil pair which provides both the stability to reach the ideal limit, and the uniformity of radial field desired for adjustment of the equilibrium vertical position. The suitability of a particular coil set for providing vertical stability is determined primarily by the strength of its interaction with the
plasma relative to the strength of its interaction with the vacuum vessel. As an alternative to a poor choice of a single coil pair, a multiple time scale controller using two coil pairs was developed. In this new hybrid control scheme, the outboard coils which produce a large and uniform radial field over most of the plasma volume are driven only on the vessel \(L/R\) timescale. In this way, they do not induce large vessel currents which destabilize the plasma. Control on a time scale faster than the instability growth time is provided by the inboard coils. The radial field produced by these currents is able to penetrate the vessel rapidly (\(\tau \approx 0.16 L_v/R_v\)), since they interact primarily with the higher order (spatial frequency) terms in the poloidal distribution of vessel current. \(L_v/R_v\) is the time constant of the slowest, antisymmetric vessel current eigenmode [Eq. (5)]; the higher order modes decay much faster. On the other hand, these inboard coils are less attractive for providing the equilibrium radial field establishing the plasma position on the longer timescale, requiring a far greater coil current for a given radial field. On DIII-D this hybrid control has allowed the control of plasmas at decay indices close to \(-n_{ci}\), the ideal MHD positional stability limit.

The motivation of this present paper is therefore fourfold. Firstly, to report on a series of experiments which demonstrate that the behaviour of the vertical control system is dominantly second order. Secondly, by examining a variety of plasmas with systematic variations in the decay index, as well as the derivative and proportional gains, we demonstrate the validity of the model developed in [1]. Thirdly, the improvement in vertical control using the hybrid system previously discussed is substantiated. Finally, we report results using this hybrid control to extend the range of elongation up to \(\kappa = 2.5\).

The remainder of this paper is organised as follows. In Section 2 we present the details of the experimental setup. In Section 3 we analyse the closed-loop response to a step input in \(z_{ref}\) and extract the order of the transfer function. In Section 4 we examine the effects of variations in decay index and controller gains and compare the experimental results with the model predictions of Ref. 1. In Section 5 we discuss the improvements observed with the hybrid control system and operation very close to the ideal stability limit. In Section 6 we report on higher elongations we have achieved, in particular plasmas with \(\kappa = 2.5\). Section 7 summarizes the work.
2. EXPERIMENTAL CONDITIONS

The experiments described were carried out on the DIII-D tokamak [2], with nominal parameters $X_0 = 1.67 \text{ m}$, $a = 0.67 \text{ m}$, $B_\phi = 2 \text{ T}$, $I_p = 1 \text{ MA}$. The poloidal field system is extremely flexible, with 18 close-fitting, independently controllable coils, labelled FnA, FnB in Fig. 2. The equilibrium field is programmed using all the coils, and the vertical position control is superimposed onto selected coil pairs. The vertical position is detected using a combination of flux-loop signals ($\psi$) and poloidal field pickup coils ($B$), shown on the figure, such that

$$I_p \cdot z = C_1(\psi_{7A} - \psi_{7B})$$

$$+ C_2(\psi_{2A} - \psi_{2B})$$

$$+ C_3(B_{67A} - B_{67B})$$

$$+ C_4(B_{2A} - B_{2B}) ,$$

where $z$ is the vertical displacement with respect to the vessel centre. The subscripts correspond to the different poloidal locations as shown in Fig. 2. This representation of the vertical position in terms of the available magnetic signals has been shown to be accurate for quiescent plasmas using a regression analysis of a large equilibrium database from which the coefficients $C_i$ were determined. With regard to the equilibrium aspect of vertical control, the flux loops alone are adequate to define $z$. However, the control of an unstable equilibrium requires the use of field measurements inside the vessel. Because of flux conservation of the plasma-vessel system when the plasma undergoes a position excursion, the flux loop signal will be delayed whereas the internally mounted poloidal field pickup coils react instantaneously. Without the internal probes, the phase shift in the $z$ measurement is not acceptable once the decay index is less than $-n_a$. 
Since it was unclear whether or not the stability of the control loop using this position estimate would still be susceptible to problems due to the image currents in the vessel, or the controlling currents in the active coils [Ref. 1, Section 7], a second uncalibrated measurement was also established using the soft X-ray emission from the plasma. We defined the soft X-ray vertical asymmetry factor, $z_X$, to be

$$z_a \sim \frac{\Phi_{XA} - \Phi_{XB}}{\Phi_{XA} + \Phi_{XB}},$$

(9)

where $\Phi_{XA}$ and $\Phi_{XB}$ are the emission signals from upper and lower soft X-ray chords, shown in Fig. 2. These chords were chosen to be on the steep part of the soft X-ray radial profile so as to provide the best sensitivity to a vertical displacement. The magnetically derived $z$-position always agreed well with $z_X$ at least down to a 1 ms time scale.

The vertical feedback control was performed using two proportional plus derivative (PD) controllers with the magnetically derived vertical position $I_p \cdot Z$, as the input. The $Z$-position error signal was applied to either the outboard F7 coil pair or to both the F7 and the inboard F2 coils by different controllers, Fig. 3(a). The use of different controller dynamics for the inboard and outboard radial field generation is an essential part of the hybrid control of the vertical position. Simply applying the same corrector signal to many poloidal coils would not lead to the same improvement. In all cases, we refer to the antisymmetric component of the command signal to the FnA and FnB coils as the Fn coils demand signal, treating their currents in a similar way. No attempt was made to incorporate an integral term into the controller since the problems of precision were not the aim of these experiments.

The amplitude and phase curves of the two derivative terms in the controllers, as measured using a network analyzer, are shown in Fig. 3(b,c). The F7 coils controller was constructed to roll off at approximately 2 kHz as these outboard coils have a negligible effect at this frequency, due to the shielding by the dominant antisymmetric mode ($L_u/R_u \approx 5$ ms) of the vacuum vessel. It should be noted that the phase shifts for these controllers becomes significant at considerably lower frequencies than the 3db points. In such a controller
system noise is a problem. We cannot limit the bandwidth to that of interest because of the resultant phase shifts. At the same time we want to avoid driving the power supplies with this high frequency noise which can easily lead to saturation. This effectively limits the level of derivative gain permissible in the circuit. For our system, the "noise" will be a combination of plasma and electrical circuit noise, all MHD activity other than \( n = 0 \) contributes noise in this sense. The power supplies on the F7 and F2 coils are chopper amplifiers with chopping frequencies of typically 2 kHz [4].

In order to understand the dynamical system which we must stabilize, we performed a set of perturbation injection experiments. The perturbation chosen was a square-wave modulation (5 Hz) introduced into the reference signal of the control loop, Fig. 4. The vertical step was of the order 1–2 cm, 1% of the plasma height. In this figure, \( H_{OL}(s) \) is the unstable open loop transfer function between the command signal defined in Eq.(7) and the vertical plasma position. This transfer function includes the pair of power supplies fed in anti-parallel to provide the radial field, the reaction of the vessel currents and the plasma motion, plus all the other active circuits in DIII–D. The active power supplies, used in anti-parallel for the vertical control, are also used individually for the various plasma shaping control loops, not shown for simplicity. If we consider \( H_{OL}(s) \) together with the feedback loop as a closed loop system \( H_{CL}(s) \), shown by the dotted line of Fig. 4, then we can study the effect of the feedback gains on the closed loop stability. This transfer function relates the injected square wave to the plasma motion, and can be estimated from the experimental data, as will be described.
3. EXPERIMENTAL DETERMINATION OF THE SYSTEM STRUCTURE

In order to optimize the controller for the vertical control of DIII-D it was essential to determine the dynamics of the vertical movement. This section shows that a second order dynamical model reproduced the experimentally observed plasma motion satisfactorily, and that such a model was a good one on which the control improvements were to be based.

3.1. Qualitative behavior

Before attempting to identify $H_{CL}(s)$ formally, we inspect the system response for three characteristic discharges using the previous DIII-D vertical control on the F6 and F7 coils in parallel, with the F6 drive signal much smaller than that of the F7 coils. Fig. 5(a) shows a discharge in which the step response was overdamped, with a rise time of approximately 20 ms. The controller had both a large value of $G_v$ and a large value of $G_z$. Figure 5(b) shows an oscillating but damped response, still stable therefore, with much less velocity gain $G_v$. Finally, Fig. 5(c), we obtained an oscillatory but unstable discharge, with similar gains to Fig. 5(b), but at a slightly larger decay index. In this third case the positive pole, which defines the envelope of the unstable growth, remained small, around 10 s$^{-1}$, being vertically unstable in the closed-loop control sense rather than the ideal MHD sense. In this unstable case there is clearly no square wave excitation necessary to stimulate the system. In all these three figures, there is no evidence of any difference between the response of the magnetic and soft X-ray measurements of the vertical position. These responses are characteristic of a second order system.

Two points must be raised concerning the system linearity. Firstly, when larger amplitude square waves were injected in conditions which lead to an oscillatory response, the frequency of oscillation slowed down as the amplitude increased. The power supplies were being progressively saturated as the amplitude increased. This result can be interpreted as being equivalent to a progressive reduction of the controller proportional gain, shown to reduce the oscillation frequency, Fig. 1. Care was taken to remain unsaturated for the
quantitative response identification. Secondly, the chopper amplifier transfer function is itself non-linear. The two coil supplies may have different net currents for a given plasma boundary shape, resulting in differing bandwidths for the upper and lower coil responses. This asymmetry, which renders the vertical position modulation asymmetric, is visible in Fig. 5(b) in which the leading and trailing edge responses are noticeably different. In what follows the chopper transfer function is assumed to be linear.

3.2. System identification

Having qualitatively inspected the closed loop response, we can proceed with a formal identification of the dynamical system. The system identification procedure can be envisaged in Fig. 6. The goal is to estimate the modelled transfer function $H^*(s)$ which, when stimulated by the given experimental input $x(t)$, in our case the square wave modulation, outputs a waveform, $y(t)$, which is as similar as possible to the measured experimental one, $y(t)$. A system identification tool based on this diagram [5] had already been developed for dynamical studies on TCA [6]. The transfer function of the model is of the form

$$H^*(s) = g \prod_{p=1}^{P} \frac{(1 - \frac{s}{s_{0p}})}{\prod_{q=1}^{Q} (1 - \frac{s}{s_{00q}})} ,$$

(10)

Such a form represents any physical system described by ordinary differential equations. The DC gain $g$ and the set of zeroes $(s_{01}...s_{0p})$ and poles $(s_{001}...s_{00Q})$ completely parametrize such a system. The set of Eqs. (4) through (7) yielded a quadratic denominator ($Q = 2$) and a linear numerator ($P = 1$).

The experimental data available are not in the continuous time variable, but are sampled and stored digitally. Rather than continuous variables $x(t)$ and $y(t)$, we have discretized
data samples $x(t_i)$ and $y(t_i)$. In order to use these sampled data points, we use the bilinear transformation given by

$$s \rightarrow \frac{2}{\Delta T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right),$$  \hspace{1cm} (11)

where $\Delta T$ is the sampling period and $z^{-1}$ is the unit delay operator, such that $z^{-1} \cdot y(t_i) = y(t_{i-1})$.

Applied to the transfer function defined in Eq. (10), this transformation produces a transfer function expressed as a rational function in $z^{-1}$ whose numerator and denominator are $B'(z^{-1})$ and $A'(z^{-1})$ respectively: During the fitting procedure we evaluate the $z$-plane transfer function

$$H^*(z^{-1}) = \frac{B'(z^{-1})}{A'(z^{-1})}. \hspace{1cm} (12)$$

Recursive use of the delaying property of the $z^{-1}$ operator allows us to directly estimate the fitted output $\hat{y}(t_i)$ on the basis of the excitation signal $x(t_i)$. The coefficients of the two polynomials $A'$ and $B'$ are chosen so as to minimize the mean square error between the actual output $y(t_i)$ and the estimated output $\hat{y}(t_i)$ [7]. The $z \rightarrow s$ inverse mapping then allows us to reconstruct the $s$-plane transfer function $H(s)$ within the frequency band $f < (2\Delta T)^{-1}$, given by the Shannon Limit.

The experimental sampling interval of 1 msec therefore gives us an estimate of the transfer function $H^*(s)$ up to a frequency of 500 Hz. As we approach this limit, however, the accuracy of the modelled transfer function will degrade. On the other hand, for frequency components very much slower than the Shannon Limit, we become oversampled, and the data must be pre-treated, by a decimating filter for example [7].
The order of the numerator of $H(s)$ must be at least one fewer than that of the numerator if the determining equations of the system are physically meaningful. We have allowed the numerator to have this maximum order when performing the modelling for different denominator orders.

This fitting procedure was carried out on the data of Fig. 5(b), box-car averaging three square-wave cycles to improve the signal-to-noise ratio. The results in Fig. 7 show the measured response (dashed curve) and the modelled response (solid curve) to the square wave stimulus also shown. The agreement improves significantly as the order increases to second order, after which the general form of the fitted response is only slightly modified by the presence of more degrees of freedom. This saturation is quantitatively shown in Fig. 8, where the correlation coefficient, given by

$$R = 1 - \frac{\langle(y - \hat{y})^2\rangle}{\langle(y - \langle y \rangle)^2\rangle},$$

increases up to a second order denominator. The pronounced knee (Fig. 8) in this correlation coefficient when the model order is equal to and then exceeds the dominant order of the identified system is frequently used as a criterion to choose the optimum model structure [7].

The poles of the transfer function are common to all transfer functions of a given physical system. They therefore contain all the information on the dynamics of the system itself. The numerator of the transfer function is related to the observation of an action on the system, and depends on the choice of inputs and outputs, in our case the applied voltage and the plasma position. The transfer function between the applied voltage and the shell current will therefore have the same poles (the roots of the given characteristic equation) but will have its own numerator.

The zeroes and poles together define the structure of the amplitude and phase plots of $H^*(j\omega)$, the estimated transfer function. Table I summarizes the fitted poles and zeroes obtained when these different orders were fitted. From $(p, q) = (1, 2)$ onwards, the dominant
poles \((-130s^{-1} \pm 600s^{-1})\) hardly move. When the order increases from \((1,2)\) to \((2,3)\), the new pole and zero \((-3100s^{-1} \text{ and } -3441s^{-1})\) are very close, indicating that they almost factorize. This signifies that the fitting procedure was not able to use the extra degrees of freedom significantly, and that the new pole and zero taken together yielded a ratio of close to unity, in which case the actual value of this zero and pole pair has almost no effect on the function \(H^*(s)\). From \((p,q) = (2,3)\) onwards there is no identifiable pole which might be taken into consideration for the feedback optimisation. As the number of free coefficients increases, the modelled \(H^*(s)\) will be adjusted to fit the available data. Unless a pole appears and remains insensitive to the presence of subsequent poles and zeroes fitted, we cannot consider it to be significant, and certainly cannot usefully use it in a model in which the feedback controller could be optimized.

The correlation coefficient did not improve from second to fifth order and there were no new identifiable poles. Nonetheless the amplitude and phase plots were slightly modified towards the higher frequencies, Fig. 9 These plots were obtained by the substitution \(s \rightarrow j\omega\) in the inverse bilinear transformation and evaluating the amplitude and the phase of \(H^*(j\omega)\) for the frequency range plotted. The approximation of the physical system by the dominant second order model \(H^*(s)\) provides an excellent representation of the dominant feature of the closed-loop transfer function, which will determine the optimized corrector settings.

The difference at higher frequencies is due to the fact that the poles added to the second order transfer function are high frequency (Table 1). These poles are statistically badly estimated, however, and this difference is not significant.

3.3. Discussion of the required model properties

Having established that the experimentally observed behaviour of the vertical position control loop is that of a low order system, we reexamine the determining Eqs. (4) through (7) proposed in Ref. 1. Equation (4) which defines the instantaneous force balance can clearly not be simplified any further. The second equation, that of the first antisymmetric
vessel current mode, must obviously be retained since its time constant is a large part of the dynamical control problem. The third equation is of first order due to the term \( R_a \), the coil resistance, and the derivative gain \( G_o \) which we have added with the PD controller. The characteristic polynomial of these three determining equations is quadratic, i.e. the order found experimentally to be dominant.

This observation is extremely important. It implies that, in order to model the dominant vertical control features, we cannot require the presence of an additional dynamical equation, that is to say one which contains differing powers of \( s \). Put another way, if we had started with a set of four equations, including a new fourth dynamical equation, we would not even be able to identify the resulting model behaviour with the response observed on those particular tokamak discharges unless the resultant additional pole only had an effect at higher frequencies than those accessible in the experimental data.

This does not mean that the Eqs. (4) through (7) must be the exact equations defining the vertical movement. We are entitled to add a further equation defining another coil current, for example, provided that this coil has no significant resistance, in which case its equation is essentially factorizable by \( s \). This coil could also provide a radial field, modifying the first equation as well. Such an addition would not increase the order of the system or the possibility of providing a stable closed-loop. The model would have the correct dynamical structure, but would not have perfectly correct coefficients. In the case of DIII-D with its many poloidal coils, those coils which are not part of the vertical feedback loop are nonetheless controlled by shaping feedback loops with purely proportional gain. These coils will appear to satisfy the criterion of zero resistance, and their presence does not raise the order of the vertical control loop. If we were to add all these additional coils to Eqs. (4) through (7) we would find we could perform a model reduction on the resulting set of equations, and return to equations similar in structure to those of Eqs. (4) through (7). The coefficients would be modified due to the additional mutual inductances, and their derivatives with respect to the plasma vertical position.
4. VARYING THE CONTROLLER GAINS AND THE DECAY INDEX

In the previous section we have seen that the coupled system of the vertical movement plus vessel and active coil currents behaves predominantly as a second order dynamical system. We expect that the effect of varying the controller gains, Eq. (7), will be similar to that predicted in Ref. 1, summarized in Fig. 1. We also expect that the system will become less stable as the decay index is increased. In this section we will explore the results of experiments with such variations. Firstly we inspect the behaviour qualitatively; later we make use of the system identification method of the last section to extract the characteristic polynomial from the step response and compare the real and imaginary parts of the poles with the model predictions.

4.1. Varying the controller gains

Using only the F7 control coils in the feedback loop, we systematically varied $G_z$ and $G_v$. At a fixed value of $G_z = -0.15$ and varying values of the derivative gain, $G_v = -0.0008$, $-0.00025$, $-0.001$, we obtained the square-wave response results shown in Fig. 10, for a fixed value of the decay index, $n \approx 1.0$. As $-G_v$ is increased, the damping of the response is increased, as expected. The lowest damping, $G_v = -0.0001$, led to an oscillatory response at the leading and trailing edge, Fig. 10(a). With $G_v = -0.00025$, the response was close to critically damped, Fig. 10(b). Increasing the damping further, to $G_v = -0.0008$, gave an excessively sluggish response, Fig. 10(c).

The damping of the response, obtained by increasing $-G_v$, also leads to an increase in the RMS fluctuation level of the amplifier demand signals, but not in the size of the response to the steps, Fig. 10. As $-G_v$ increased from $0.0001$ to $0.00025$, to $0.0008$, the RMS value of the demand signal increased from 8.9 to 10.1 to 12.1. The leading and trailing edge response is seen in the demand signal of Fig. 10(a), but not Fig. 10(c). In the absence of noise on the position signal, we would expect a reduction in the demand signal as $-G_v$ is increased, as shown by the reduced presence of the response to the step.
As long as the amplifier is not saturated, the presence of an increased signal due to $G_v \cdot s \cdot Noise(s)$ is not detrimental to the control loop. However, the phase-amplitude relation of the controller output [Fig. 3(b)] requires that the bandwidth be extended beyond the characteristic frequency of the instability. Because of the noise in the position signal, the reduction of the demand signal with increased $-G_v$ is not seen. These data indicate how crucial the choice of the roll-off frequency will be when optimizing the controller.

Still using only the F7 coils, the proportional gain was then varied at a fixed value of the derivative gain, $G_v = -0.00004$. The decay index remained at $n = -1.0$. In Fig. 11(a), $G_x = -0.11$, the amplitude of the step is largest, and the oscillation frequency is lowest. The damping of the oscillation is also slowest. In Fig. 11(b), $G_x = -0.15$, the amplitude of the step is reduced, the oscillation frequency is higher and the damping is faster. Finally, in Fig. 11(c), $G_x = -0.22$, the oscillation is not visible in the noise, the response is rapid and the step amplitude is smallest. These observations agree with the predictions of Fig. 1, i.e., as $-G_x$ is increased both $\gamma$ and $\omega$ increase as well.

4.2. The effect of varying the decay index

As the vertical field decay index becomes more negative, the results of Ref. 1 predict an increase in the more dangerous root of the transfer function, and subsequent loss of control at a certain decay index. For $n < -n_a$ there is no stabilized solution without derivative gain, and for $n < -n_c$, there is no stabilized solution at all. Prior to this study, the loss of vertical control had always occurred in DIII-D at a decay index of $n \approx -0.95$, well above the value of $-n_c \approx -1.35$ which was calculated for the discharges studied. Using the control based on the F6 and F7 coils, we would calculate that $n_a$ should have a value of 0.65, whereas the decay index achieved without derivative gain was the $-0.95$ mentioned above. We believe that this discrepancy results from the action of the equilibrium shape control which tends to correct the vertical position through the change in flux at the controlling flux loops near the 18 coils when the plasma shifts position. In this manner the equilibrium control allows these 18 coils to act in a concerted effort which is similar to adding a lumped second shell.
Figure 12 shows the square wave response as the decay index is ramped from \( n \approx -0.8 \) to \( n \approx -1.1 \), with the controller gains set at \( G_s = -0.22, G_u = -0.0001 \). The response at the start of the ramp shows very little overshoot, being almost critically damped at \( t \approx 1.3 \text{ s} \). As the decay index decreases, the response starts to overshoot (\( t \approx 1.5 \text{ s} \)), and by \( t \approx 1.7 \text{ s} \) the response is clearly oscillatory. The oscillations increase until at \( t \approx 2.1 \text{ s} \) the oscillation becomes unstable and a disruption ensues. In this one discharge we have seen the characteristic behaviour predicted, namely that of the low order system whose stability boundaries are moving as the decay index varies (Ref. 1, Fig. 9).

The discharge in Fig. 12 ended in a vertical control disruption. Provided the proportional gain is adequate to keep the transfer function roots to the left of the line B–B of Fig. 1, a control disruption is easily identified by the control system having necessarily passed through an oscillatory phase before becoming unstable. If the decay index evolves slowly enough with respect to the period of the square-wave steps, this natural response must be detected experimentally at a leading or trailing edge. The signal remains oscillatory after the step at \( t = 1.996 \text{ s} \), and then the plasma disrupts immediately upon the next step at \( t = 2.096 \text{ s} \). This is seen more easily at \( t \approx 2.1 \text{ s} \) in Fig. 12(b) which is the same signal digitized at a faster rate (50 kHz). The higher frequency component distinguishable is the chopper frequency.

Disruptions which occur on a much faster time-scale, non-correlated with the square-wave excitation will not, in general, be avoidable by optimizing that particular control input which was stimulated by the square wave. This does not mean, however, that they are not controllable using other control loops, that is to say using either different detection coils or poloidal field coils or both.

Since the \( G_s, G_u \) settings determine the closed-loop poles, we expect the operational range, i.e. the decay index at which the real part of the most dangerous root goes to zero, to vary with the controller gains. In fact the discharges used in Fig. 10 show just such behaviour. In each case we measure the maximum value of the decay index achieved just before the disruption (Fig. 13). The decay index achieved increased significantly following
the addition of a significant derivative gain, from \( n = -0.96 \) to \(-1.18\). Between \( G_v = -0.00025 \) and \( G_v = -0.0008 \) little improvement was found, as predicted by Fig. 20 of Ref. 1. The value of \( n = -1.15 \) is the limiting value calculated in Ref. 1 for vertical control by the F7 coils, in remarkable agreement with the experimental result. In each case we measure the minimum value of the decay index for vertical control by the F7 coils, to be well above the critical value for these discharges (\( \approx -1.35 \)).

4.3. Tracking the closed loop poles

The qualitative agreement with the model is extremely encouraging, and we now look for a more detailed agreement. To do this, we took the three discharges of Fig. 10 which provide decay index ramps for three different values of velocity gain, \( G_v \). The data were analysed assuming a second order denominator, and the two poles of the transfer function were estimated for each edge of the square-wave stimulation. The signal-to-noise was poorer than before as we could not box-car average over several periods, as had been done for the stationary conditions of Fig. 7. The 1 ms sampling period was a little too long. The results shown are derived from the soft X-ray asymmetry factor, Eq. (9), although the magnetic measurement of \( z(t) \) gave similar results, but with a greater uncertainty due to its even longer sampling interval, 2 ms.

Figure 14 shows the imaginary part (\( \omega \)) and the real part (\( \gamma \)) of the fitted closed-loop response poles. The dotted line, solid circle data, \( G_v = -0.0001 \), shows relatively high frequency oscillations. The growth rate is \( \gamma \approx 500 \text{ s}^{-1} \) at the start of the ramp and tends towards \( \gamma \approx 0 \) at \( n = -1.08 \) at which point the plasma disrupted (open crossed circle). Increasing the velocity gain to \( G_v = -0.00025 \) (solid line, solid triangles) gave an overdamped response at the start of the ramp (\( \omega = 0 \)) and an oscillatory response above \( n \approx -0.92 \). The oscillation frequency remained lower than the previous case. The growth rate was more negative than before, and the \( \gamma \approx 0 \) disruption occurred at a higher value of \( n \approx -1.15 \). Increasing the velocity gain further to \( G_v = -0.0008 \) produced a damped
response for all decay indices \( \omega = 0 \) and a growth rate \( \gamma \) which was less negative than before and which increased negatively with the decay index ramp.

We derived the same curves from the model Eqs. (4) through (7), maintaining the same ratio between the velocity and proportional gains, and the same ratio between the different values of velocity gain. In order to obtain a reasonable quantitative match, we were forced to decrease the value of the shell time-constant, increasing \( R_\nu \) by 50\%. The absolute values of the gains were increased by a factor of 4. The resulting curves, Fig. 15, are in astonishingly good agreement with the experimental data of Fig. 14. Bearing in mind that the shell time constant was not estimated with the diagnostic ports cut out, the change to the time constant is not a cause for concern. The necessity of changing the gains is more disconcerting, although the fact that the presence of the remaining poloidal coils is not modelled correctly must have a significant effect here. We already have seen a change in \( n_\alpha \) from that predicted, which indicates that the effective value of \( M'_\alpha \) must differ from our calculated value. Additionally, there is uncertainty in the transfer function of the choppers [4] which contributes to the overall circuit gain.

An important point to note in Figs. 14 and 15 is that the disruption with the largest \( G_v \) appears to occur at \( n > -n_c \) while \( \gamma \) is negative. The F7 coils are incapable of controlling a discharge up to the critical index, as seen in Ref. 1, Fig. 20, and explained in Ref. 1, Section 5.2.1. For particular coils sets (F7, F8, and F9) the structure of the operating space is modified somewhat from Fig. 1 in that the line A–A is below C–D. In that case the stable operating space is further limited by an upper bound on \( -G_v \). As \( n \) approaches \( -n_c \) the line A–A moves upwards while C–D remains stationary. Eventually the stable space vanishes before reaching \( -n_c \). We now consider such a system in the experimental space of \( \gamma \) versus \( n/n_c \). The dependence of \( \gamma \) on \( n/n_c \) is shown in Fig. 16 for various values of derivative gain. Note that at high \( G_v \) the stability is increasing just prior to the instability. Further increases in \( G_v \) decrease the achievable range of \( n \). This behavior is a characteristic of using coils which interact too strongly with the stabilizing shell currents. The F2 coils do not exhibit this behavior and the system stability is always reduced as \( n \) approaches \( -n_c \).
The trend of the poles as the decay index, proportional gain, and derivative gain were varied is clearly of the form predicted by the model. The transfer function extracted from the data is in reasonable agreement with the calculated transfer function, provided we adjust the values of two of the eight parameters which enter the model Eqs. (4) through (7).
5. HYBRID CONTROL OF THE VERTICAL MOVEMENT

Our vertical control model derived in Ref. 1 led us to the conclusion that no one poloidal coil pair is optimal, and that the use of two selected coil pairs, one inboard and one outboard, would provide better vertical control. To a first approximation, the outboard coils provide good positional rigidity by providing a large radial field per unit current, whereas they are very effectively shielded by the vessel image currents making them ineffective for the production of the low amplitude variations in radial field at the high frequency necessary for vertical stabilization. The inboard coils provide a smaller radial field inadequate for positional rigidity, whereas they are shielded by a vessel current distribution which has a much faster decay rate. This concept is referred to as hybrid control of the vertical movement.

We do not simply add the vertical control signal to all the coil pairs, since for many of the coils such a signal is actually destabilizing at high \( n/n_e \) (see Ref. 1, Table 2). The coil currents will induce vessel currents in opposition to those produced by the plasma motion, counteracting the vessel effectiveness on the fast time scale \( (\omega > R_e/L_e) \). For a coil pair to improve control, its net field, including that produced by its interaction with the vessel, must oppose the plasma motion.

The hybrid control was tested on similar discharges for comparison all discharges were driven to an axisymmetric disruption. In Fig. 17 we plot the decay index achieved prior to disruption versus the derivative gain on the F7 coils. Our baseline for comparison with the simple control is the dashed line which describes the discharges previously discussed with only the F7 coils active, Fig. 13. Large values of feedback gains were used for the inboard coils, chosen to produce corrections of the order of the output range of the amplifiers. Adding only a large derivative feedback \( G_{uv} \) to the F2 coils, marked by a crossed circle, a value of \( n = -1.18 \) was achieved at low \( G_v = -0.003 \), already exceeding the F7 coil optimum of \( n = -1.16 \). Even adding only positional gain \( G_{zp} \) to the F2 coils, marked by a cross, increased the operational range. Since increasing \( G_z \) on the F7 coils alone did not change the maximum achieved, this simple observation already illustrates the difference in...
dynamics between inboard and outboard coil control. Our model simulations of this system also show that proportional gain in the F2 coils in conjunction with proportional gain on the F7 coils is stabilizing. These calculations indicated that this would not be an attractive option since the required power supply bandwidth would be higher than that for a system using derivative gain.

Increasing the outboard coil velocity gain $G_v$ to $-0.0008$, the additional velocity feedback on the F2 coils ($G_{vp}$) produced a further significant increase in the decay index achieved. These discharges are shown by the solid circles and solid square. The maximum decay index was not obtained with the maximum velocity gain, although the small sample of discharges available cannot confirm the presence of a general optimum value of $G_{vp}$, particularly since these high gains caused some saturation of the F2 coil drive signal. In Shot No. 60809 we reached 92% of the calculated maximum decay index for the measured plasma parameters.

The discharge marked by a triangle achieved a lower decay index, attributable to a decreasing plasma current following an accidental loss of control of the ohmic primary circuit. Such a reduction in the decay index can be attributed to the direct dependence of the critical decay index on the plasma current ramp-rate (Ref. 1, Section 2.3).

These data show that the predictions of a simple vertical control model led us to significant improvements in the achieved vertical field decay index, by providing a simple clear picture of the controlled system. Such an improvement was obtained with a small amount of experimental data, contrary to the habitual trial and error optimization.

The time history of Shot No. 60809 is shown in Fig. 18. The shaping control system is programmed to provide a steadily increasing decay index once the plasma current has achieved a flattop. The response to the step input on $z_{ref}$ becomes increasingly rapid as $n$ becomes more negative, until finally after the step at 2.7 s the response becomes oscillatory, as discussed in the previous section. A hundred milliseconds later the plasma disrupts having exceeded $n/n_e = -0.92$. In the calculation of $n_e$ we have used the approximation...
$L_{\text{ext}} = \mu_0 X_0 [\ln (8X_0/\bar{a}) - 2]$, where $\bar{a} = \alpha_0 \sqrt{[1 + \kappa^2 (1 + \delta_{\text{upper}}^2 + \delta_{\text{lower}}^2)]/2}$. $\beta_p + \ell_i/2$ is obtained from magnetic analysis of the equilibrium field.

Subsequent analysis of the magnetic probe signals confirms that the disruption is indeed $m/n=1/0$ ($m$ is the poloidal mode number). The equilibrium just prior to disruption is shown in Fig. 19. Stability calculations using GATO [8] showed this plasma to be vertically stable. In order to see how close we were to ideal MHD instability, the wall was expanded in minor radius by 11% and GATO showed this plasma to be unstable to $n=0$ modes. This supports our filament calculation of $n/n_c = 0.92$. Calculations with a multiple filament plasma and multi-filament vessel using the PSTAB code [9], based on the same equilibrium calculation, yielded $n/n_c = -0.87$. The elongation for this discharge was not particularly high, $\kappa \approx 2.2$, for two reasons. Firstly, these experiments were run just after a vacuum leak, with the high oxygen content causing an abnormally high $\ell_i$, and secondly the shaping was controlled to produce a linear ramp in $n$, not an optimal ratio of elongation to applied quadrupole field.
6. HIGHER ELONGATED PLASMAS

Having established the improved performance of the hybrid control system compared with the previous control system, we attempted to shape higher elongation plasmas. One change, however, was made to the control circuit with respect to the preceding discussion. The control circuit used requires high gains, particularly the derivative gain. At the same time, the \( z \) measurement has a considerable noise component. This combination resulted in a large high frequency demand on the power supplies and a tendency to saturate some of the intermediate circuitry. In an attempt to reduce these saturation effects, we applied the higher frequency control signal to the F3 coils in addition to the F2 coils. (In Ref. 1 it was seen that the F3 coils are second only to the F2 coils in stabilizing the vertical plasma motion.) Similarly, the lower frequency drive is applied to the F6 coils as well as the F7 coils. This effectively increased the overall derivative gain.

Now our strategy was twofold. The first consideration was to optimize the plasma shape. Since to lowest order a hexapole field is not vertically destabilizing, we decided to make high triangularity (\( \delta \approx 0.75 \)) plasmas. Secondly, we tried to reduce the internal inductance \( L_i \). It is important to note that the relationship between the achievable elongation and the current profile is included in our simple filament model. Lowering \( L_i \) both increases the critical index through a change in \( \Gamma \) and reduces the quadrupole field (decay index) required to obtain any particular value of \( \kappa \).

Our best attempt to date has been an elongation of \( \kappa = 2.5 \). In this case an \( L_i \) value of 0.94 was reached with a combination of an \( L_i \) ramp and intense neutral beam heating (Fig. 20). This plasma disrupted vertically immediately after the equilibrium shown in Fig. 21. We were, however, able to sustain plasmas with \( \kappa \approx 2.45 \) for 500 ms during the plasma current flattop at \( L_i \approx 1.04 \). The \( \kappa = 2.5 \) plasma disrupted with a value of \( n/n_e = -0.85 \), although there is some uncertainty due to possible contributions to the field curvature from the additional vessel current induced by the significant \( L_i \) ramp.
7. DISCUSSION

The experimental behaviour of the DIII-D vertical control system is dominated by second order dynamics. This agrees with the predictions of a simple model of the vertical control problem [1]. Varying the controller proportional and derivative gains gave results which are in agreement with the model. When the vertical field decay index was varied, the experimentally measured closed-loop system response poles varied in a way well described by the model calculations. The qualitative and quantitative effects of the controller gain settings are in good agreement with the model. This simple model therefore provides us with a means of systematically optimizing the control loop.

The concept of hybrid control of the vertical movement has been implemented and tested, leading to significant improvements in the achievable decay index.

The observed improvement allowed by the use of inboard coils, at least indirectly, confirms the supposition in Ref. 1 that the poloidal distribution of vessel current is of prime importance in counteracting the problem of vessel shielding, and that such currents flow predominantly on the outboard side of the vessel. The correct prediction of the apparently benign increase in the stability followed by disruption before \( n = -n_c \) (Fig. 14 at high \( G_n \)) is solid confirmation of the importance of the vessel current distribution in the vertical control problem. It should be noted that with all the coils available in DIII-D, our model predicts that very few will provide adequate vertical stability (Ref. 1, Table 2).

We have shown that periodically monitoring the closed loop performance using a square-wave pulse train usefully allows us to identify the disruptions due to loss of control loop stability. The evolution of the control loop poles occurs on a timescale on which a crude form of closed loop adaptive control is envisageable. The plasma response to an external stimulus allows us to determine whether the particular stimulated coils are useful or not for improving the plasma control.
It is encouraging to find that the plasma behaviour is dominated by a small number of poles, so that a simple controller may be used on highly elongated plasmas. The quest for the optimal set of control coils and detection coils can be undertaken experimentally using these systematic techniques.

In subsequent experiments we have attempted to obtain even higher elongation in a double null configuration. This provides the highest triangularity and allows H-mode operation, which is also the easiest way to lower \( \xi \). In these plasmas we found that we could no longer sustain \( n/n_e \approx -1 \). We saw a systematic departure from this limit as the current profile was broadened and at \( \xi \approx 1 \) we were only able to operate at approximately 60% of this ideal MHD limit. That is, we were no longer able to stabilize plasmas having much lower rigid-body growth rates than in the higher \( \xi \) cases. Calculations with the PSTAB multi-filament model show the identical trend, thus we conclude this is not a problem of our single filament calculation of the rigid-body ideal limit. \( \kappa \) has been limited to 2.5 or below by this behaviour. These new vertical disruptions are not triggered by the response to a step, and we see no oscillatory phase just prior to the disruption, as observed in Shot No. 60809. Preliminary analysis using GATO indicates that the plasma is destabilized by the coupling of an \( m/n = 3/0 \) mode to the \( m/n = 1/0 \) mode. There is also some experimental evidence that this motion, which is not included in a rigid body model, is present. It is not yet clear whether improvements to the control system will solve this problem. However, these plasmas are seen to disrupt at values of \( n/n_e \) in the H-mode phase which were successfully sustained during the Ohmic (higher \( \xi \)) phase of the same discharge. New experiments are planned to investigate this in further detail, improving the control system and explicitly studying the effect of triangularity on \( n = 0 \) stability. Such behavior has been observed in the PBX tokamak for bean-shaped plasmas [10]. Similar behavior has been predicted for D-shaped plasmas with high triangularity [11]. The conclusion to be drawn from these latest findings is that the hybrid control described in this paper may have taken us to the useful limit of simple \( m/n = 1/0 \) vertical stability control. The more complex control of these new high performance plasmas will be an exciting challenge.
8. ACKNOWLEDGMENT

Two of the authors (E.A. Lazarus and J.B. Lister) would like to thank Ron Stambaugh, Tony Taylor and the DIII-D physics staff for their hospitality during their detachment at General Atomics. The work described was partly funded by EURATOM, the Ecole Polytechnique Federale de Lausanne and the Fonds National Suisse de la Recherche Scientifique and the Office of Fusion Energy, United States Department of Energy, under Contract Nos. DE-AC03-89ER51114 and DE-AC05-84OR21400.
REFERENCES


<table>
<thead>
<tr>
<th>$(m, n)$</th>
<th>Poles (s$^{-1}$)</th>
<th>Zeroes (s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0, 1)</td>
<td>-391</td>
<td></td>
</tr>
<tr>
<td>(1, 2)</td>
<td>-129 ± 589</td>
<td>1839</td>
</tr>
<tr>
<td>(2, 3)</td>
<td>-130 ± 597</td>
<td>1757</td>
</tr>
<tr>
<td></td>
<td>-3100</td>
<td>-3441</td>
</tr>
<tr>
<td>(3, 4)</td>
<td>-154 ± 614</td>
<td>126 ± 1430</td>
</tr>
<tr>
<td></td>
<td>-403</td>
<td>-326</td>
</tr>
<tr>
<td></td>
<td>-5397</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF FIGURES

FIG. 1. Schematic of the operating regions in the $G_z$ $G_v$ plane using the F2 coils. The stable operating region is below $A-A$, below $C-D$, and to the left of $B-B$. The thick curve separating Regions I and II is the $\omega = 0$ contour. The dashed lines are contours of constant $\gamma$ (in s$^{-1}$) and the thin solid lines are contours of constant $\omega$ (rad/s). In Region I the solutions are stable and underdamped, in Region II they are overdamped, and in Region III the solutions are unstable and oscillatory.

FIG. 2. The layout of the DIII-D tokamak showing the poloidal field coils, the effective limiter outline, the vertical position detection coils and the soft X-ray chords used to monitor vertical position.

FIG. 3. (a) Schematic of the hybrid controller. The amplitude and phase curves of the derivative term for both the (b) F7 and (c) F2 controllers.

FIG. 4. A schematic of the vertical position control loop, shown isolated from the rest of the poloidal system for simplicity.

FIG. 5. Characteristic system response of the vertical position (a) overdamped, (b) oscillatory but stable and (c) oscillatory and unstable. The vertical position is indicated by both the magnetic and the soft X-ray asymmetry measurements.

FIG. 6. Schematic of the equivalent system identification problem. $x(t_i)$ and $y(t_i)$ are the input and output discrete-time data. $\hat{y}(t_i)$ is the modelled output data.

FIG. 7. Results from the system identification code for the oscillatory but stable case of Fig. 5(b). The denominator is tested up to 5th order. The measure fitted correlation coefficient $R$ is given.

FIG. 8. Improvement to the correlation coefficient as the denominator order is increased.

FIG. 9. Amplitude and phase plots of the modelled transfer function for 2nd and 5th order denominators.

FIG. 10. Variation of the square wave response ($z$) as the derivative gain is varied. $G_v =$ (a) $-0.0001$, (b) $-0.00025$, (c) $-0.0008$, $G_z = -0.15$. The amplifier demand signal ($V_{com}$) is also shown.

FIG. 11. Variation of the square wave response ($z$) as the proportional gain is varied. $G_z =$ (a) $-0.11$, (b) $-0.15$, (c) $-0.22$, $G_v = -0.00004$. The step response and voltage demand signals are shown.

FIG. 12. (a) Evolution of the square wave response raw data as the vertical field decay index (dotted curve) increases during one discharge; $G_z = -0.15$, $G_v = -0.0001$ (b) Expanded timescale for the last step. The disruption begins promptly at the step at 2.096 s.

FIG. 13. The decay index achieved prior to disruption as the F7 controller gains were varied.

FIG. 14. The variation of the poles of the closed-loop response $H_{CL}(s)$ as the decay index was ramped, measured for three different velocity gains ($G_v$). ($s = \gamma + j\omega$).
FIG. 15. The variation of the poles of the closed-loop response $H_{CL}(s)$, as a function of the decay index, calculated by the model ($s = \gamma + j\omega$).

FIG. 16. The least stable root versus $-n/n_c$ for various $G_v$ values using the F7 coils. $G_z$ is constant. Gains are in arbitrary units.

FIG. 17. Increase in the achievable decay index using the hybrid control. The data points of Fig. 13 are reproduced by the dashed line.

FIG. 18. The time evolution of shot No. 60809 which reached 92% of the ideal stability limit, (a) plasma current, (b) $n/n_c$ and $n$, (c) plasma vertical position in meters, (d) Amplitude of $n = 0$ oscillations in TESLA, (e) centre frequency of $n = 0$ oscillations in kilohertz.

FIG. 19. Shot No. 60809, equilibrium approximately 0.1 prior to disruption as calculated from the experimental data. The parameters are $I_p = 1.0$ MA, $B_t = 2.0$ T, $X = 1.68$ m, $a = 0.59$ m, $q_{95} = 5.6$, $\beta_p = 0.37$, $\ell_i = 1.5$, $\kappa = 2.19$, and $\delta = 0.37$.

FIG. 20. The time evolution of Shot No. 62458 which reached $\kappa = 2.5$ prior to a vertical disruption. (a) Plasma current (MA) and neutral beam power (MW), (b) decay index, and (c) plasma vertical position (m).

FIG. 21. Shot No. 63458, equilibrium just prior to disruption as calculated from the experimental data. The parameters are $I_p = 1.0$ MA, $B_t = 1.7$ T, $X = 1.69$, $a = 0.54$ m, $q_{95} = 7.7$, $\beta_p = 0.51$, $\ell_i = 0.94$, $\kappa = 2.50$, and $\delta = 0.55$. 

33
RELATIVE AMPLITUDE (db)

FREQUENCY (kHz)

PHASE (DEGREES)

LISTER, et al. Figure 3(b)
CORRELATION COEFFICIENT, $R$

DEGREE OF DENOMINATOR
LISTER, et al. Figure 12(a)
LISTER, et al. Figure 20