Pressure- and q-profile effects on ideal infernal modes in tokamaks with an extended region of low magnetic shear

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Introduction An extended region of low magnetic shear is a common feature of the plasma in many tokamak experiments. Such a region can involve a substantial part of the entire plasma volume, for instance in “hybrid scenario” plasmas characterized by a nearly constant safety factor $q \gtrsim 1$ in the plasma core, and also in equilibria with similar profiles of q that are often developing in spherical tokamaks. Regions around rational surfaces where q is locally flattened could occur in the presence of an island, or as a result of partial reconnection. Such a flattening of q deteriorates the ideal stability of the plasma due to the reduced stabilising effect of field-line bending. A class of global instabilities that can grow in such regions are the “infernal” modes first discussed by Manickam *et al.* [1]. In the present work we study both q-profile and pressure profile effects on these kind of modes using an analytical formulation of the ideal MHD stability equations for a toroidal plasma with large aspect ratio (LAR). In the next section we present a system of equations useful for analyzing the ideal stability of LAR tokamaks including a region of low magnetic shear between an inner radius r_{IN} and an outer radius r_{OUT} (and finite magnetic shear outside this region). This system is thereafter used to investigate combined q- and pressure profile effects on the infernal modes in low-shear equilibria with $q \gtrsim 1$ in the core region, in particular the $m = n > 1$ modes. We consider first a family of profiles of similar nature as those used in Ref. [1], and thereafter look at the stability of a somewhat similar plasma, having low shear in the vicinity of the $q = 1$ rational surface and $q_0 < 1$ at the magnetic axis. It will be shown that, under certain circumstances, a broad spectrum of infernal $n > 1$ instabilities can exist in such equilibria, and a possible connection between such modes and the sawtooth instability is pointed out.

MHD stability equation for a LAR tokamak with a low-shear region By means of an asymptotic expansion in $\varepsilon = r/R$, with r and R denoting the minor and major radii of the plasma, respectively, and using the ordering $\beta \sim \varepsilon^2$, where $\beta = 2\mu_0\langle p\rangle/B_0^2$, the following equations for a perturbation $\xi_{m,n} \sim e^{i(m\theta - n\varphi)+\gamma t}$ within the low-shear region $r_{\mathrm{IN}} \le r \le r_{\mathrm{OUT}}$ of a tokamak plasma with circular cross section is obtained to leading order in ε:

$$\frac{d}{dr}\left(r^3 Q \frac{d\xi_{m,n}}{dr}\right) - r(m^2-1)Q\xi_{m,n} + m^2\frac{d\beta_0}{dr}\left[r^2\left(1-\frac{1}{q_r^2}\right)\xi_{m,n} - C_+ r^{1+m} - D_- r^{1-m}\right] = 0, \tag{1}$$

$$C_+ = -\frac{\Lambda_{(m+1)} m^4 R_0^2}{n^4 r_{\mathrm{IN}}^{2+2m}} \int_{r_{\mathrm{IN}}}^{r_{\mathrm{OUT}}} r^{1+m} \frac{d\beta_0}{dr}\, \xi_{m,n} dr,\ D_- = -\frac{\Lambda_{(m-1)} m^4 R_0^2}{n^4 r_{\mathrm{IN}}^{2-2m}} \int_{r_{\mathrm{IN}}}^{r_{\mathrm{OUT}}} r^{1-m} \frac{d\beta_0}{dr}\, \xi_{m,n} dr. \tag{2}$$

Here, $Q = n^2(\Delta q/q_r)^2 + (\gamma/\omega_A)^2(1+2q_r^2) \sim \varepsilon^2$, $\Delta q = q(r) - q_r \sim \varepsilon$ and $q_r = m/n$. Furthermore, $\beta_0(r) = 2\mu_0 p(r)/B_0^2 \sim \varepsilon^2$ and the quantities $\Lambda_{(m\pm1)}$ take into account the presence of the side-band harmonics $m \pm 1$ outside the low-shear region. Expressions for $\Lambda_{(m\pm1)}$ can be found in Ref. [2], where a variational formulation of the same equations is given. Boundary conditions for $\xi_{m,n}$ are $\xi_{m,n}(r_{\mathrm{IN}}) = \xi_{m,n}(r_{\mathrm{OUT}}) = 0$. With the exception of $m = 1$ modes in low-shear equilibria with an inner boundary ($r_{\mathrm{IN}} > 0$), Eqs. (1) and (2) are valid for general m and n (an analysis of the $m = 1$ case with $r_{\mathrm{IN}} > 0$, which needs a special treatment, will be presented elsewhere). The equations (1) and (2) modify similar equations [3-7] to account for $r_{\mathrm{IN}} \neq 0$.

The results presented here are mainly based on numerical solutions of Eqs. (1) and (2). Analytical solutions of these equations are available in special cases, for instance with $q = const$ in the low-shear region, and analytical results for such equilibria will also be reported elsewhere as an extension of the analysis in Ref. [6].

Infernal modes in low-shear equilibria with $r_{\mathrm{IN}} = 0$ We first consider low-shear equilibria of similar type as in Ref. [1], i.e. equilibria without an inner boundary of the low-shear region ($r_{\mathrm{IN}} = 0$), and where $q(r)$ and $p(r)$ are given by

$$q(r) = q_0 + q_1(r/a)^{\alpha_q} \qquad (0 \le r \le r_m) \tag{3a}$$

$$p(r) = p_0[1-(r/a)^{\alpha_2}]^{\alpha_1} \qquad (0 \le r \le a) \tag{3b}$$

Furthermore, we make the q-profile in the edge region $r_m \le r \le a$ reasonably experimentally relevant by setting $q(r) = q_0' + q_1'(r/a)^2$ there, with q_0' and q_1' chosen such that q is continuous at $r = r_m$ and $q(a) = 3.1$ [1]. The parameter values used here are, initially, $q_0 = 1.05$, $q_1 = 2.05$, $\alpha_1 = 4$, $\alpha_2 = 1.5$ and p_0 adjusted to give $\beta = 3\,\%$. Since $q \cong 1$ in the low-shear region $0 \le r \le r_m$, we only consider modes with $q_r = 1$, i.e. $m = n$, here. Fig. 1a illustrates the profiles in Eqs. (3) for $\alpha_q = 6$, $r_m/a = 0.4$ and $(\alpha_1, \alpha_2) = (4, 1.5)$ and $(2, 2)$.

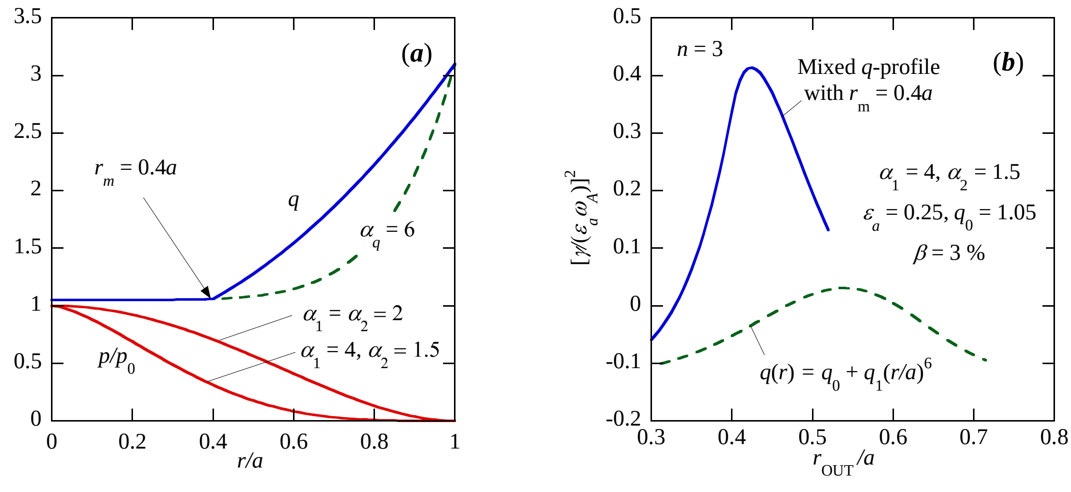

Fig. 1 a) Illustration of the q- and pressure profiles used in the calculations in this paper. b) Growth rate of the $n = 3$ mode vs r_{OUT} for $r_m/a = 0.4$ and 1. The relevant value of r_{OUT} is the radius where γ has its maximum.

Fig. 1b shows the growth rate vs. $r_{\rm OUT}$ for an $m = n = 3$ mode in the equilibrium shown in Fig. 1a with $\alpha_1 = 4$ and $\alpha_2 = 1.5$. A comparison is made between the case $\alpha_q = 6$, $r_m/a = 0.4$, and an equilibrium where the q-profile with $\alpha_q = 6$ in Eq. (3a) is valid up to $r = a$. The outer boundary $r_{\rm OUT}$ of the low-shear region is a free parameter of the system (1)-(2), and the actual value of this parameter should be chosen as the radius that gives the largest growth rate [3]. For the case with $r_m/a = 0.4$ in Fig. 1b we see that $r_{\rm OUT} \cong r_m$, and this is often a good approximation of $r_{\rm OUT}$ if r_m is not too large. It is also seen that the growth rate in the case where the low-shear region is limited to $r \lesssim 0.4a$ is substantially larger than the growth rate in a plasma with a much wider low-shear region. The latter property is further illustrated in Fig. 2a, where $\gamma^2_{\rm max}$ is plotted for several modes (n = 1-4) as functions of r_m/a, with the other equilibrium parameters the same as in Fig. 1.

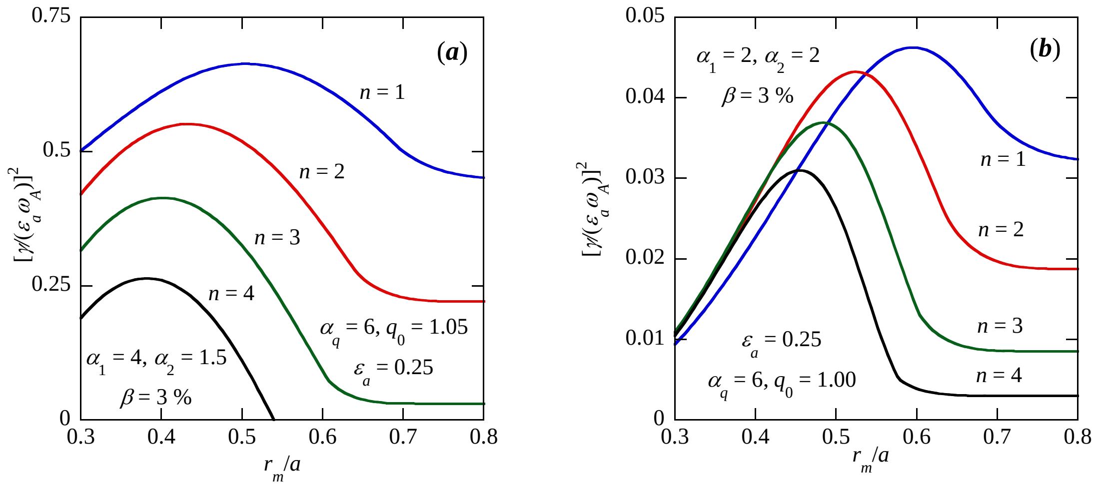

Fig. 2 a) γ^2 vs. r_m/a for the modes $n = 1, 2, 3, 4$ in an equilibrium with the parameters indicated. b) Growth rates for the same modes as in (a) but here $q_0 = 1$ and the pressure profile is broader.

In the case of a relatively large region of low magnetic shear ($r_m \gtrsim 0.6a$), $n = 1$ is obviously by far the most unstable mode, whereas restricting the low-shear region to approximately half of the plasma radius (or less), the growth rates of the higher n modes are seen to increase strongly. Two additional profile effects are, however, also of importance for the relative magnitudes of the growth rates of the different modes shown in Fig. 2. First, the fact that $q_0 = 1.05$ in Fig. 2a leads to a stabilising effect from the $n^2(\Delta q)^2$ term in the quantity Q in Eq. (1), a stabilising effect that increases strongly with n. Secondly, the profile with $\alpha_1 = 4$ and $\alpha_2 = 1.5$ used in Fig. 2a is rather strongly peaked at the axis (see Fig. 1a), and by making the pressure profile broader, the drive on the higher $m = n$ modes increases. The consequences of both reducing q_0 to 1 and, additionally, broadening the pressure profile (by using instead $\alpha_1 = \alpha_2 = 2$, see Fig. 1a) is shown in Fig. 2b. It is seen that, for $r_m \lesssim 0.5a$, this leads to a broad spectrum of high n infernal instabilities with practically the same growth rate. It is possible that the spectral property shown in Fig. 2b is related to the similar, broad spectrum of infernal instabilities seen in some of the results obtained with PEST in Ref. [1].

Note that the growth rates are reduced by a factor ~ 3-4 from Fig. 2a to Fig. 2b. This is due to the fact that, in order to get the same β, p_0 has to be smaller in Fig. 2b than in Fig. 2a.

Effect of an inner boundary of the low-shear region ($r_{IN} > 0$) We now consider a possible application of the result in Fig. 2b to a conventional tokamak scenario where $q_0 < 1$ and where the $q = 1$ radius is of order 0.4-0.5a or less. A q-profile of the sort shown in Fig. 3a may develop in a standard scenario plasma due to the presence of an island, partial reconnection, or another transport process in the vicinity of $q = 1$. An important difference relative to the q-profile shown in Fig. 1a is that there is now a radius r_{IN} marking the inner boundary of the low shear region.

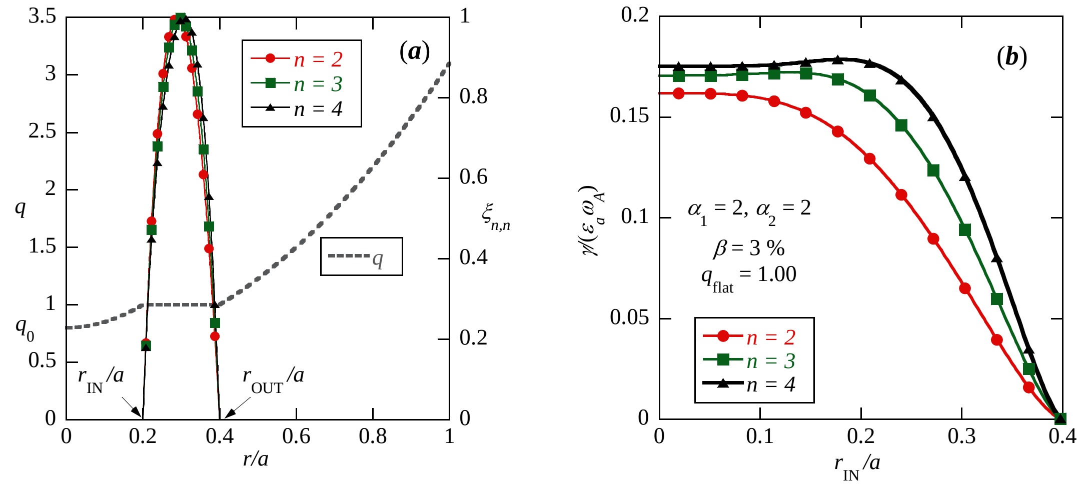

Fig. 3 a) q-profile with $q_0 = 0.8$ and a region of low shear between $r_{IN} = 0.2a$ and $r_{OUT} = 0.4a$, and the form of the $n = 2$-4 eigenfunctions. b) Growth rates for the same eigenmodes vs. a varying r_{IN} in the q-profile in (a).

Fig. 3b shows the growth rates for the modes $n = 2$-4 vs. r_{IN} with $r_{OUT} = 0.4$ fixed, calculated from Eqs. (1) and (2) for the q-profile in Fig. 3a, the pressure profile $\alpha_1 = \alpha_2 = 2$ in Eq. (3b), and $\beta = 3$ %. The form of the eigenfunctions, shown in Fig. 3a, are seen to be almost independent of $n > 1$. For simplicity, the growth rates and eigenfunctions are calculated with fixed r_{IN} and r_{OUT} here, so the growth rates are somewhat underestimated. It is seen that a very small region of low shear nevertheless is sufficient to destabilise a broad spectrum of fast growing infernal $n > 1$ instabilities within $r_{IN} \leq r \leq r_{OUT}$. Such instabilities could be present at, or soon after, the sawtooth crash event, and may be connected to the triggering of secondary instabilities (e.g. NTMs). In relation to this, the role of the ideal $m = n = 1$ mode in the type of scenario suggested here needs to be understood as well, and this is an area where results of relevance for this problem to some extent already exist in the literature [8-9].

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