Job turnover, expectations, and the Phillips Curve

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Abstract

Job turnover makes a wage Phillips Curve less forward-looking, with a smaller coefficient for inflation expectations. Workers discount future wage income with a low discount factor if there is a strong flow of job turnover; this implies that future inflation is discounted more heavily with job turnover. The Phillips Curve flattens both in the short and long run, due to the correlation between output and inflation expectations. The paper then derives the optimal monetary policy: in particular, the price targeting result of the Ramsey policy is violated when there is turnover.

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1 Introduction

The Phillips Curve is central to macroeconomics but its shape has been questioned recently. The strong short run relationship between inflation and output (or unemployment) seems to have vanished in the aftermath of the 2008 financial crisis: unemployment increased and then fell sharply, while inflation remained low and positive. This would suggest that the short-run Phillips Curve has become flatter (see Blanchard et al., 2015 ; or Ball and Mazumder, 2019).

The idea of a vertical, or near-vertical long-run Phillips Curve, has also been questioned. In a recent Peterson policy brief (2016), Blanchard argues that the long run Phillips curve has become flatter, largely due to inflation expectations *anchored* at zero or low levels. The inflation expectation used in the Phillips Curve is a long-run expectation which is anchored around a reference point, and only adjusts partially to changes in short-run expectations. As such, the effect of short-run inflation expectations is largely *dampened*, and the Phillips Curve is no longer accelerationist or near accelerationist. This would imply a real trade-off between output and inflation in the long run.

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Dampened expectations

The Phillips Curve is often assumed to be accelerationist or near accelerationist: if current inflation increases one-to-one with short-run inflation expectations, this implies that the output gap is related to the acceleration of inflation.[1] But if the pass-through of short-run inflation expectations to current inflation is incomplete – or if the relevant inflation expectation is long run expectations which does not adjust fully to the short-run expectation – then the Phillips Curve is no longer accelerationist. The rate of inflation – and not only its acceleration – matters for the output gap. Short-run inflation expectations are dampened, or play a dampened role, in the sense that they matter less for agents (and hence current inflation) than what is usually assumed.

This paper shows that a Phillips curve with a dampened role for inflation expectations is not only flatter in the long run but also in the short run. When estimating a short run Phillips Curve, ignoring this dampening leads to a reduced coefficient on output/unemployment. The reason for this flattening bias is simple: since current inflation is correlated with output (or unemployment), expectations of future inflation are correlated with future output (unemployment), which is itself correlated with current output (unemployment). Mismeasuring the role of expectations necessarily biases the slope coefficients.

Explanations for these dampened expectations have mostly been behavioral so far: inflation expectations are either anchored around a reference (Blanchard, 2016), or agents are myopic about the future (Gabaix, 2018). While these effects are probably important, I show that the dampening can also occur in a micro-founded model with job turnover; such a micro-foundation is useful to study the optimal monetary policy. But whatever the source of dampened inflation expectations in the Phillips Curve, I show that dampening leads to a flatter long run curve, which is no longer vertical or near vertical. In the short run, the curve looks flatter if a wrong model without dampening is imposed. The optimal monetary policy for inflation targeting and stabilisation, is then derived.

Sources of bias

Suppose that we are estimating a simple New Keynesian Phillips Curve with inflation π_t and the output gap y_t:[2]

$$\pi_t = \kappa y_t + \beta \mathrm{E}_t[\pi_{t+1}] \tag{1}$$

In this setup β is the risk-less discount factor and κ the output coefficient. Suppose, however,that the true model features dampened inflation expectations:

$$\pi_t = \kappa y_t + \beta \delta \mathrm{E}_t[\pi_{t+1}] \tag{2}$$

$\delta \in (0, 1)$ is the dampening factor. It can come from anchored expectations, or a behavioral bias, or as I will show later, job turnover.

[1] In New Keynesian models of sticky prices, inflation doesn't exactly increase one-to-one with expectations, but the pass-through is close to unity, implying a near accelerationist curve.

[2] but we could also use wage inflation and cyclical unemployment

Even though it is not the right model, estimating eq (1) will provide an unbiased estimate of κ if the output and inflation expectation coefficient are jointly estimated (of course the estimated coefficient for expectations will estimate $\beta\delta$ not β). But eq (1) is often estimated with a calibrated β, which doesn't account for any dampening. This leads to a biased estimation of κ if the true model is eq (2) and the output gap (or cyclical unemployment) is serially correlated.

For example, let us estimate a reduced form Phillips Curve featuring only the output gap and current inflation $\pi_t = \tilde{\kappa} y_t$, and then backs out the structural parameters by relying on the auto-regressive properties of the output gap. This approach has been followed, for example, in Gali (2011).[3] Assume that output is serially correlated:[4] $y_t = \rho_y y_{t-1} + u_t$, with u_t a mean-zero disturbance.

Then we can iterate eq (2) forward:

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t \left[\pi_{t+1}\right] = \kappa \sum_{k \geq 0} \left(\beta\delta\right)^k \mathrm{E}_t y_{t+k} = \frac{\kappa}{(1 - \rho_y \beta\delta)} y_t$$

Estimating this reduced-form equation provides an estimate of $\tilde{\kappa} = \frac{\kappa}{(1-\rho_y\beta\delta)}$ from which κ can be uncovered if β, δ and ρ_y are known. But if the dampening factor is not accounted for (ie, having $\delta = 1$), the estimate of κ will be biased.

Property 1 *If κ is the true output coefficient, the estimated κ^* is smaller*

$$\kappa^* = \frac{(1 - \beta\rho_y)}{(1 - \rho_y \beta\delta)} \kappa < \kappa$$

Hence one can see immediately that the dampening factor δ will affect the slope of a traditional, reduced-form, Phillips Curve displaying only current inflation and output, $\pi_t = \tilde{\kappa} y_t$. A non-linear estimation of the Phillips Curve using the reduced form equation above and the auto-regressive properties of the output gap (or cyclical unemployment) to back out the slope of the true Phillips Curve will lead to a biased estimate if the wrong model without δ is estimated.[5]

The same bias occurs if equation (1) is directly estimated and a wrong restriction is imposed for the coefficient β. This is the case in the empirical estimates of Gali and Gertler (1999), where they use marginal costs instead of the output gap. They estimate $\pi_t = \lambda mc_t + \beta E \pi_{t+1}$. The estimated coefficient of marginal costs, λ, depends on the assumption about the coefficient of future inflation, β. When this coefficient is restricted to $\beta = 1$, the estimated value of λ is smaller than when there is no restriction and β takes a lower value.[6]

[3] Gali (2011) estimates an hybrid wage Phillips Curve featuring unemployment. As unemployment is assumed to be AR(2), a reduced form Phillips Curve without inflation expectations is first estimated, before backing out the structural parameters of the true hybrid Phillips Curve.

[4] Assuming an AR(1) process allows for simple expressions for the source and amplitude of the bias. Assuming a more sophisticated process changes the expressions but not the logic.

[5] The same drawback would obviously apply to any joint estimation of a larger model, if inflation expectations are dampened but the estimated model doesn't account for it.

[6] The same logic would apply if an hybrid Phillips curve as in Gali and Gertler (1999) was estimated with a restriction on the coefficient of inflation expectations: it would create a bias in the estimation of other parameters.

Job turnover

While anchored or myopic expectations are likely to play a role in the dampening of expectations, I show that the dampening can also occur in a micro-founded, rational-expectations model with job turnover. Providing a simple micro-foundation is useful for the study of the optimal monetary policy.

In the New Keynesian wage Phillips curve models, such as the one pioneered by Erceg, Henderson and Levin (2000), workers (or unions) set staggered wages optimally. Current (wage) inflation depends on future (wage) inflation expectations as well as the output gap. In the log linear approximation, the coefficient of future inflation is β, the risk-less discount factor. However, when there is a probability that a worker quits, or that he will be fired and replaced by someone else, the net present value of his job will be discounted with a lower factor than the risk-less discount factor.[7] This probability of turnover makes the wage setting decision, and hence the wage Phillips curve, less forward looking.

A crucial assumption is that when a worker quits (or is dismissed) and is replaced by an entrant worker, the wage stickiness will be (at least partially) transmitted to the entrant. The entrant does not renegotiate her wage immediately, and has to abide by the wage of the previous incumbent she has replaced.[8] This model of entry has some Blanchard-Weil perpetual youth flavour. As hinted by Weil (1989), the crucial feature in these models is as much the probability of death of the agent, as the stream of newborns, who do not have a say over decisions made before their birth.[9] Here, when a new worker starts a job, he is bound by the decisions of his predecessor.[10] The externality between existing and new agents creates the additional discounting.

Related literature

Snower and Tesfaselassie (2017) derive a positive optimal long run inflation target in the presence of job turnover, but they do not investigate the short run properties much. Bilbiie, Ghironi and Melitz (2007) as well as Bilbiie, Fujiwara and Ghironi (2014) look at the optimal long run monetary policy in a similar setup: sticky prices with firm entry and exit. In their model, the exit probability affects the Phillips curve and optimal Ramsey policy. While they use a Rotemberg instead of a Calvo framework, and inflation offsets different long run distortions, the intuition, as well as the assumption that new workers

[7]It is important to distinguish layoffs and dismissals because persons who quit or are dismissed are replaced and hence count as turnover, while layoffs diminish employment and are not replaced by new hires.

[8]Or equivalently, there is no difference between incumbents and entrants in their distribution of wages. Assuming wage rigidity for new hires is crucial in models such as Hall (2005) or Gertler and Trigari (2009), who combine wage and labour search frictions. Gertler, Huckfeldt and Trigari (2016) find no evidence that the wage of new hires is more cyclical than for existing workers. Galuscak et al. (2012) find similar results for 15 EU countries.

[9]In the positive sense, the death probability creates the lower discount factor, but in the normative sense, the externality is caused by the stream of new workers.

[10]Or if wages are set by a union, it only cares about the welfare of its *existing* members.

cannot reset their wage,is largely the same.[11] But my paper shows how turnover affects the slope of the Phillips Curve both in the short and long run, and also investigates optimal stabilisation policies in the short and long run.

Different explanations have been put forward for the recently flatter Phillips Curve. Ball and Mazumder (2011) suggest that with menu costs, price changes will be less frequent when inflation is low, and the resulting Phillips Curve will be flatter. Changes in the labour market (see Haldane, 2017 or chapter 2 of the October 2017 World Economic Outlook (IMF, 2017)) or globalization (eg. Carney, 2017) have been suggested as possible sources of the flatter Phillips Curve, but without a proper underlying model. Dampened inflation expectations appear in models of anchored expectations (Blanchard, 2016) or rational inattention (Gabaix, 2018), but my paper shows how this dampening plays a crucial role also with the slope of the Phillips Curve itself.

This paper also belongs to the stream of literature that reassesses the New Keynesian model in light of the Great Recession and the Zero Lower Bound. While this paper introduces an extra discount factor in the Phillips curve, other papers have introduced a discount factor in the Euler equation instead, to explain the *forward guidance puzzle*. In McKay, Nakamura and Steinsson (2016, 2017) this is due to incomplete financial markets, while in Del Negro, Giannoni and Patterson (2015), it comes from a Blanchard-Yaari model of perpetual youth for households which is similar to this paper (where it applies to workers). The interaction between a discounted Phillips curve and a discounted Euler equation has been partially studied by Gabaix (2018).

Last, this paper is related to the literature on the optimal level of inflation, which does not solely rely on the Phillips curve. In their handbook chapter (2011), Schmitt-Grohe and Uribe document such other motives for positive inflation. If the price stickiness exhibits a quality bias (Schmitt-Grohe and Uribe, 2012), then a positive inflation will simply ensure that the hedonic price level remains constant. If wages are more rigid downwards than upwards, positive inflation will make relative wage adjustments easier (Akerlof, Dickens and Perry, 1996; Kim and Ruge-Murcia, 2009). A positive amount of inflation might also be useful to increase the nominal interest rate safely above zero, in case the zero lower bound needs to be avoided.

The paper is organized as follows: Section 2 builds a New Keynesian model with sticky wages, as well as job turnover. The non linear Phillips curve is derived and linearly approximated. I examine in section 3 the consequences of my model for the non linear long-run Phillips Curve, and how turnover affects the response of output and inflation to supply and demand shocks. Section 4 solves the welfare maximization problem, both in the non linear (steady state inflation) and quadratic setups (optimal stabilisation). Section 5 concludes.

[11] In my Calvo framework, workers adopt the wage distribution of existing workers. In a Rotemberg setup, it is assumed that new workers (or firms) take the existing symmetric wage (or price), and are not free to choose their starting wage (price) optimally

2 The model

2.1 A microfounded model

The model of wage rigidities closely follows Gali's (2008) notations, with monopolistic competition in the labour market. There is a continuum of wage-setting worker types, indexed by $j \in [0, 1]$.

Households and firms

Let me first look at the household. A worker of type j maximizes a utility

$$\mathrm{E}_0 \sum_{t \geq 0} \beta^t U(C_t(j), N_t(j)) \tag{3}$$

The period utility function U is separable in consumption and labour. The utility of consumption C, $u(C)$, is a concave function with inverse elasticity of intertemporal substitution σ, while the disutility of labour N, $v(N)$ is convex with an inverse Frisch elasticity ϕ. The utility from consumption and disutility from labour are scaled by a parameter λ:

$$U(C_t(j), N_t(j)) = u(C_t(j)) - v(N_t(j)) = \frac{C_t(j)^{1-\sigma}}{1-\sigma} - \lambda \frac{N_t(j)^{1+\phi}}{1+\phi} \tag{4}$$

Perfect competition is assumed in the goods market. The production function has diminishing returns to labour N_t, with a labour elasticity $(1-\alpha)$:

$$Y_t = N_t^{1-\alpha}$$

Labour is a CES aggregate of the labour of each type j, with a wage elasticity of substitution ϵ:

$$N_t = \left[\int_0^1 N_t(j)^{1-1/\epsilon} dj \right]^{\frac{\epsilon}{\epsilon-1}}$$

The aggregate wage index W_t is

$$W_t = \left[\int_0^1 W_t(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}}$$

The amount of labour of type j employed by firm i is

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon} N_t$$

Worker j maximizes the expected utility (3) subject to the budget constraint

$$P_t C_t(j) + Q_t B_t(j) = B_{t-1}(j) + (1-\tau_t) W_t(j) N_t(j) + D_t + T_t$$

where τ_t is a proportional labour tax (or subsidy) on his labour compensation $W_t(j)N_t(j)$, D_t is the dividend from owning a diversified portfolio of firms, and

T_t is a lump sum transfer (or tax) from the government. New bonds $B_t(j)$ can be bought or sold at price Q_t, the stochastic discount factor of the household. Balanced government budget in each period ($T_t = \tau_t W_t N_t$), as well as zero net supply of bonds, ensures that consumption and output are equal in each period:

$$P_t C_t = W_t N_t + D_t = P_t Y_t$$

With perfect competition for goods, prices are equal to marginal costs, or

$$P_t = MC_t = W_t \frac{N_t^\alpha}{1-\alpha}$$

Hence the real wage is linked to output as

$$\Omega_t = (1-\alpha) Y_t^{-\frac{\alpha}{1-\alpha}}$$

With decreasing returns to scale, firms make a profit $D_t = \alpha P_t Y_t$.

As in Erceg et al. (2000) or Gali (2008), let us assume markets with complete contingent claims for consumption but not leisure. This ensures full consumption smoothing across agents.

Lemma 1 *With complete markets, there is full consumption smoothing:*

$$\forall (t,j), \qquad C_t(j) = C_t = Y_t$$

The Euler equation of consumption pins down the risk-less discount factor

$$Q_t = \mathrm{E}_t \beta \frac{P_t}{P_{t+1}} \frac{u'(C_{t+1})}{u'(C_t)} = \beta \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \tag{5}$$

The labour supply decision for a worker j in problem (3) is equivalent to maximizing the following quantity in each period

$$u'(Y_t) \frac{(1-\tau) W_t(j) N_t(j)}{P_t} - \lambda \frac{N_t(j)^{1+\phi}}{1+\phi} \tag{6}$$

Distortions and dispersions

Let us define the first-best and flexible outcomes. Using the utility and production function, the first-best level of output is

$$\bar{Y} = \left(\frac{1-\alpha}{\lambda} \right)^{\frac{1}{\sigma + \frac{\phi+\alpha}{1-\alpha}}}$$

Lemma 2 *In the flexible outcome, the real wage $\Omega = \frac{W}{P}$ is a markup μ above the marginal rate of substitution of the worker:*

$$\mu = \left(\frac{\epsilon}{(\epsilon - 1)(1-\tau)} \right)$$

The flexible-wage output is

$$\tilde{Y} = \left(\frac{1-\alpha}{\lambda\mu}\right)^{\frac{1}{\sigma+\frac{\phi+\alpha}{1-\alpha}}} = \bar{Y}\left(\frac{1}{\mu}\right)^{\frac{1}{\sigma+\frac{\phi+\alpha}{1-\alpha}}}$$

The markups depend on the wage elasticity – with a high elasticity, the markup is close to 1. But it also depends on the wage tax τ. A positive tax creates an additional wedge, but a subsidy can offset the inefficiency caused by the finite wage elasticity. Unless the subsidies fully offset the wedges ($\mu = 1$), the flexible output will be inefficiently low as $\tilde{Y} < \bar{Y}$.

With staggered wages, the wage dispersion will be costly in terms of welfare. When wages are heterogeneous, the aggregate number of hours must increase to produce the same amount of goods.

Lemma 3 *The aggregate utility function can be written*

$$\int_0^1 U(C_t, N_t(j))dj = \tilde{Y}^{1-\sigma}\left[\frac{\left(\frac{Y_t}{\tilde{Y}}\right)^{1-\sigma}}{1-\sigma} - \frac{\frac{1-\alpha}{1+\phi}\Delta_t\left(\frac{Y_t}{\tilde{Y}}\right)^{\frac{1+\phi}{1-\alpha}}}{\mu}\right] \tag{7}$$

with the wage dispersions

$$\Delta_t = \int_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon(1+\phi)} dj \geq 1 \tag{8}$$

2.2 Sticky wages and the Phillips Curve

Worker discounting

A fraction θ of workers have sticky wages, and a fraction δ keeps their job from one period to another; the two are independent. The discount factor accounts for the wage and the firms survival probabilities θ and δ. Instead of maximizing the discounted sum of expression (6) with a discount factor β, the applicable rate of time preference will be $\beta\theta\delta$: the disutility of labour – attached to a wage and a worker – is discounted by $\beta\theta\delta$, while the labour compensation is discounted by $\theta\delta Q_t$.

It is assumed that when a worker is replaced, the new worker cannot automatically renegotiate his wage. Instead, he faces the same probability of sticky wages than existing workers. If they were completely free to choose new wages, the effect would die out; but as long as the new wage partly takes into account the wage of existing workers, the effect would be lessened but not die out. This gives a discrepancy between the joint survival probability $\theta\delta$ of the optimal wage setting decision, and the true wage stickiness θ that is featured in the dynamics of the aggregate wage and dispersion. This is the cause of the flatter wage Phillips curve.[12] As mentioned before, evidence in Gertler et al. (2016) or Galuscak et al. (2012) tends to support this assumption.

It is also possible to think about the case where it is the union which sets the wage of workers of type j, and the union insures workers against layoffs but not quits or dismissals. When a worker quits, or is dismissed, I assume that he leaves his labour type and finds a different occupation, where wages are set by a different union. As such, if the union maximizes the utility of its *existing* members, employed or not, it will have a short discounting horizon. And it will not represent *future* members, because they do not belong to this union *yet*.

The non linear Phillips curve

When a worker is free to set a wage $w_t(j)$, he seeks to maximize the discounted sum of the wage compensation minus the disutility, defined in expression (6).

$$\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \left[u'(Y_T) \frac{(1-\tau_T) w_t(j) N_T(j)}{P_T} - \lambda \frac{N_T(j)^{1+\phi}}{1+\phi} \right]$$

Lemma 4 *The re-optimizing wage w_t^* is :*

$$\left(\frac{w_t^*}{W_t}\right)^{1+\phi\epsilon} = \frac{\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \mu_t \left(\frac{W_t}{W_T}\right)^{-\epsilon(1+\phi)} \lambda N_T^{1+\phi}}{\mathrm{E}_t \sum (\theta\beta\delta)^{T-t} \left(\frac{W_t}{W_T}\right)^{1-\epsilon} \Omega_T u'(Y_T) N_T} = \left(\frac{K_t}{F_t}\right) \tag{9}$$

with recursive terms F_t and K_t

$$F_t = (1-\alpha) Y_t^{1-\sigma} + \theta\beta\delta \mathrm{E}_t F_{t+1} \Pi_{t+1}^{\epsilon-1} \tag{10}$$

$$K_t = \mu_t \lambda Y_t^{\frac{1+\phi}{1-\alpha}} + \theta\beta\delta \mathrm{E}_t K_{t+1} {\Pi_{t+1}}^{\epsilon(1+\phi)} \tag{11}$$

This is where the job survival probability, δ plays a role, compared to the standard model. δ is an extra factor, appearing here in the worker's discounting, through the recursive F_t and K_t. In the recursive equation, F_t depends on the expected future value $\mathrm{E}_t F_{t+1}$, multiplied by the inflation and a discount factor $\theta\beta\delta$. The exact same phenomenon occurs for the recursive term K_t. The δ makes these two terms less forward looking than in the standard model, and it makes the wage Phillips curve flatter, as we will see with the linear approximation.

Each period, only a fraction $(1-\theta)$ of wages are re-optimized at the value w_t^*, while a fraction θ still follows the previous wage distribution, with an aggregate W_{t-1}. Using the definition of the aggregate wage, the wage level W_t is a weighted aggregate of the previous wage level W_{t-1} and the current optimal wage w_t^*:[13]

$$W_t^{1-\epsilon} = \theta W_{t-1}^{1-\epsilon} + (1-\theta)(w_t^*)^{1-\epsilon}$$

[12] In their Rotemberg setup, Snower and Tesfaselassie (2017) or Bilbiie et al. (2007; 2014) assume that new workers (or firms) start with the symmetric wage (price) of existing workers (firms). It is similar to here: entrants are bound by incumbents

[13] Importantly, the new hires follow existing wages, so that turnover δ doesn't play a role in this law of motion of the aggregate wage

This provides the dynamics for the wage inflation and dispersion

$$\frac{1-\theta {\Pi_t}^{\epsilon-1}}{1-\theta} = w(\Pi_t) = \left(\frac{w_t^*}{W_t}\right)^{1-\epsilon} = \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\phi\epsilon}} \tag{12}$$

$$\Delta_t = \theta\Delta_{t-1}{\Pi_t}^{\epsilon(1+\phi)} + (1-\theta)w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}} \tag{13}$$

Linear quadratic setup

Although I will look at the optimal steady state level of inflation that the non linear model yields, it is useful to derive a linear quadratic approximation around a zero inflation steady state. In the flexible wage steady state, there is no inflation ($\Pi = 1$), and no dispersion ($\Delta = 1$). The steady state values $\tilde{Y}$, $\bar{F}$ and $\bar{K}$ are easy to pin down. Let us define the percentage deviation of each variable: $\pi_t = \log \Pi_t$, and $d_t = \log \Delta_t$. Similarly y_t, f_t and k_t denote log deviations of the capital-letter variables from the steady state.

Property 2 *The linear wage Phillips curve is*

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t\left[\pi_{t+1}\right] \tag{14}$$

with $\kappa = \left(\frac{\phi+\alpha}{1-\alpha} + \sigma\right)\frac{(1-\theta)(1-\theta\beta\delta)}{\theta}\frac{1}{1+\phi\epsilon}$

This linear wage Phillips curve is similar with the standard wage Phillips curve in a model of wage stickiness. Current wage inflation positively depends on the output gap and future expected wage inflation, and negatively on the real wage. However, two differences stand out. The coefficient κ is slightly different as it features the parameter δ. But most importantly, future inflation is discounted by $\beta\delta$ instead of simply β. Intuitively, this is because $\beta\delta$ is now the discount factor that is applicable to the job tenure of the worker.

3 Implications

3.1 A flatter long run Phillips curve

The long run version of (14) implies a flatter long-run Phillips curves, and it is no longer vertical or nearly vertical as without turnover:

$$\bar{\pi} = \frac{\kappa}{1-\beta\delta}\tilde{Y}$$

When δ is smaller than 1, κ increases slightly. However the increasing effect on the denominator $(1-\beta\delta)$ largely dominates. This means that long run inflation will depend less strongly on the long run output gap, and the curve is not as vertical.

Property 3 *In the long run Phillips curve between inflation and output of the form* $\bar{\pi} = \chi\tilde{Y}$*, the coefficient* χ *decreases with turnover (*δ *falls):*

$$\chi = \left(\frac{\phi+\alpha}{1-\alpha} + \sigma\right)\frac{(1-\theta)}{\theta(1+\phi\epsilon)}\frac{1-\theta\beta\delta}{1-\beta\delta}$$

The non-linear long run Phillips curve

Because the linear equation is only an approximation of a highly non-linear model, it is useful to see the impact of turnover on the non linear long run Phillips Curve. Taking the steady state in equations (10), (11) and (12), output can be written in terms of inflation

Lemma 5 *The non linear long-run Phillips curve is*

$$\left(\frac{Y}{\tilde{Y}}\right)^{\frac{\phi+\alpha}{1-\alpha}+\sigma} = \left[\frac{1-\theta\beta\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\beta\delta\Pi^{\epsilon-1}} w(\Pi)^{-\frac{1+\phi\epsilon}{\epsilon-1}}\right] \tag{15}$$

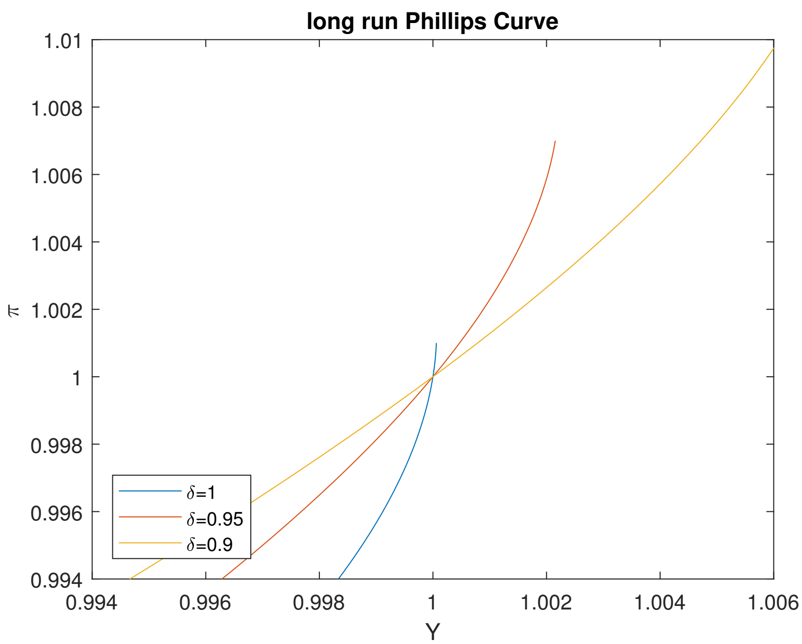

Figure 1: Non linear long run Phillips curve for different values of δ

Figure 1 displays the output level Y associated to a long run (annualized, wage) inflation Π. When $\Pi = 1$, $Y = 1$ (the flex price case). As Π increases, there is a limited output gain, at least to the first order. With turnover ($\delta < 1$), the long run trade-off is flatter than in the normal case without. This was true for the linear approximation of the curves around zero inflation, and it is also true for the non linear case.

3.2 Demand and supply shocks

I will now examine how this modified Phillips Curve changes the response of output and inflation to shocks in a simple New Keynesian framework.

Let me look at a standard Euler equation: in log-linear form,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1})$$

It can be combined with a Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_y y_t$$

ϕ_π and ϕ_y are the inflation and output coefficients. While McKay et al. (2016, 2017) or Del Negro et al. (2015) have a modified Euler equation, I do not look at this modification here because this equation becomes isomorphic to a standard Euler equation with modified parameters, once combined with a Taylor rule.[14]

With my Phillips Curve, the dynamics, incorporating simple shocks, write:

$$\begin{aligned} y_t =& E_t y_{t+1} - \frac{1}{\sigma}(\phi_\pi \pi_t + \phi_y y_t - E_t \pi_{t+1}) + u_t \\ \pi_t =& \beta\delta E_t \pi_{t+1} + \kappa y_t + v_t \end{aligned}$$

The fist equation is often thought as a demand equation, while the second is a supply equation. u_t is a demand shock – coming either from a shock to the natural rate of interest, or from a monetary policy shock – while v_t is a supply shock – generated either by a productivity or by a cost-push (markup) shock.

The role of persistence

It is only with persistent shocks that turnover will bring significantly different impulse responses to supply and demand shocks. It is easy to see why. If there is a transitory (white noise) supply or demand shock today, future expected variables are not affected, $E_t \pi_{t+1} = E_t y_{t+1} = 0$. Whether there is a coefficient β or $\beta\delta$ in the Phillips Curve is irrelevant, as the expected inflation is zero. Hence for purely transitory shocks, the model is not affected in any way.

On the other hand, the more persistent the shock, the bigger the difference between the standard model and this modified model. If the supply or demand shock is $AR(1)$ with $\rho \approx 1$, then $E_t \pi_{t+1} \approx \pi_t$. This means that the Phillips Curve becomes $(1 - \beta\delta)\pi_t \approx +\kappa y_t + v_t$ and is flatter with $\delta < 1$ than normally. Flattening the supply curve does make output more sensitive to demand shocks.

[14] In their modified Euler equation, $y_t = \gamma E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1})$
with $\gamma \in [0, 1]$ the dampening factor of the Euler equation. While it has strong implications at the Zero Lower Bound (where the Taylor rule does not apply), it is easy to see that when $\gamma \neq 1$, the modified equation can still be rearranged to *appear* as a standard Euler equation, without any dampening: $y_t = E_t y_{t+1} - \frac{1}{\sigma\gamma}(\phi_\pi \pi_t + (\phi_y + (1-\gamma)\sigma) y_t - E_t \pi_{t+1})$
Hence, when a modified Euler equation (with dampening factor $\gamma < 1$) is combined with a Taylor rule, the behavior is isomorphic to one with a lower elasticity of intertemporal substitution ($\gamma\sigma$ instead of σ) and a higher output coefficient in the Taylor rule ($\phi_y + (1-\gamma)\sigma$, instead of ϕ_y). Once this equation is combined with a standard Phillips Curve into the basic New Keynesian model, the dampening has the same effect as increasing the output coefficient: The model would show very similar responses to interest rate shocks or cost push shocks.

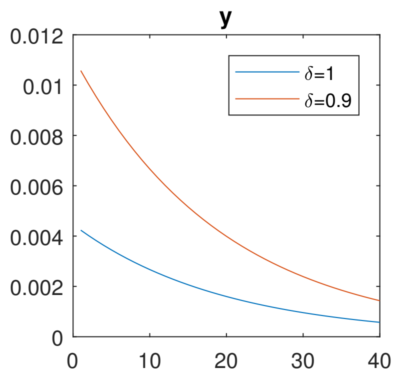

Figure 2: y and π in response to a persistent demand shock

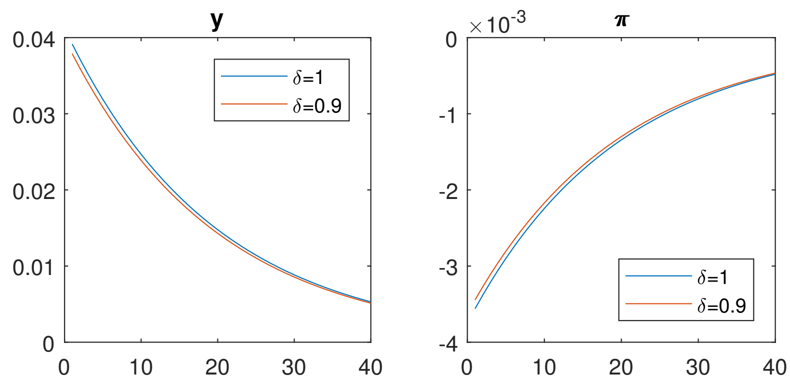

Figure 3: y and π in response to a persistent supply shock

Figures 2 and 3 display the impulse response functions of output and inflation to persistent demand and supply shocks respectively. The model is calibrated with $\sigma = 1$, $\kappa = 0.025$, $\phi_\pi = 1.5$ and $\phi_y = 0$. The shocks are very persistent ($\rho = 0.95$), and the turnover/dampening factor is set to 1 and 0.9 respectively.

One can see how turnover and dampened inflation expectations only have a significant impact on the response of output to persistent demand shocks. As hinted before, this is because turnover makes the supply curve flatter in the presence of persistent shocks. A demand shock has a bigger impact on output when the supply curve is flatter. This implies that for an observed fluctuation in output, the underlying fluctuation in the natural rate of interest is not as high as what the standard model would predict. Hence turnover and dampened inflation expectations significantly matter when the natural rate of interest is depressed for a while (a persistent negative demand shock) or when monetary policy persistently undershoots as compared to its baseline Taylor rule.[15]

[15]The case of the Zero Lower Bound is different as the Taylor rule no longer applies there.

4 Optimal Ramsey policy

4.1 Turnover and price (wage) targeting

As we will see, introducing turnover into a standard New Keynesian model has strong implications for the optimal Ramsey policy. Without turnover, price (wage) targeting is optimal for the Ramsey policy: even with steady state distortions, the long run optimal level of inflation is zero; while inflation reacts to cost push shocks in the short run, this is accompanied by deflation in the future, so that there is full mean reversion of the price level. In other words, there is long-run price targeting in response both to long term distortions and short term cost push shocks. But with turnover, price targeting is no longer optimal: long run inflation is non zero if there are steady state distortions; in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the price (wage) level.

Welfare function

Let me first define the aggregate welfare function. While workers discount future wages with the probability of job turnover, individuals do not die. Therefore, the aggregate utility function of the social planner is simply the aggregation of each household's utility given in equation (3). Using equation (7), this is

$$\mathrm{E}_0 \sum_{t\geq 0} \beta^t U(C_t, N_t(j)) = \mathrm{E}_0 \sum_{t\geq 0} \beta^t \tilde{Y}^{1-\sigma} \left[\frac{\left(\frac{Y_t}{\tilde{Y}}\right)^{1-\sigma}}{1-\sigma} - \frac{\frac{1-\alpha}{1+\phi} \Delta_t \left(\frac{Y_t}{\tilde{Y}}\right)^{\frac{1+\phi}{1-\alpha}}}{\mu} \right] \quad (16)$$

In terms of intuition, it is easier to look at the optimality of price targeting in a quadratic setup. Whens steady state distortions are small, the approximation of (16) and (13) bring a quadratic approximation that is not different from the case without turnover. This is because turnover plays no direct role in the utility function, or the dynamics of the dispersion.

Lemma 6 *The second order approximation of the aggregate utility is*

$$U = -\sum_{t\geq 0} \beta^t \left[\widetilde{\kappa} \frac{(y_t - \bar{y})^2}{2} + (1-\alpha)\epsilon \frac{\pi_t^2}{2} \right]$$

with $\bar{y} = \log \frac{\bar{Y}}{\tilde{Y}}$ *and* $\widetilde{\kappa} = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right) \frac{(1-\theta)(1-\theta\beta)}{\theta} \frac{1}{1+\phi\epsilon} \approx \kappa$

Contrary to κ, δ does not appear in $\widetilde{\kappa}$, which is exactly the same coefficient as in the case with no turnover. This is because the distortion is discounted with the discount factor of the household, where the death shocks play no role.

Let us also assume cost push shocks in the Phillips curve:

$$\pi_t = \kappa y_t + \beta\delta \mathrm{E}_t \pi_{t+1} + u_t$$

with u_t the cost push shock, an error term. I allow u_t to be an $AR(1)$ process with autocorrelation ρ_u ($\rho_u = 0$ denoting the white noise case).

Is it optimal to target the price (wage) level?

Denoting λ_t the Lagrange multiplier of the Phillips Curve at time t, the Lagrangian of the optimal Ramsey policy is

$$L = -\mathrm{E}_0 \sum_{t=0}^{+\infty} \beta^t \begin{pmatrix} \frac{1}{2}\left[(1-\alpha)\epsilon\pi_t^2 + \widetilde{\kappa}\left(y_t - \bar{y}\right)^2\right] \\ +\lambda_t(\pi_t - \beta\delta \mathrm{E}_t \pi_{t+1} - \kappa y_t - u_t) \end{pmatrix}$$

Taking first order conditions and simplifying λ_t, $(1-\alpha)\epsilon\kappa\pi_0 = \widetilde{\kappa}(\bar{y} - y_0)$ and $(1-\alpha)\epsilon\kappa\pi_t = \widetilde{\kappa}\left((\bar{y} - y_t) - \delta(\bar{y} - y_{t-1})\right)$. Used in the Phillips Curve, this shows that output y_t follows a second order difference equation:

$$(1 + \frac{\kappa^2}{\widetilde{\kappa}}(1-\alpha)\epsilon + \beta\delta^2)y_t = \beta\delta \mathrm{E}_t y_{t+1} + \delta y_{t-1} + (1-\delta)(1-\beta\delta)\bar{y} - \frac{\kappa}{\widetilde{\kappa}}(1-\alpha)\epsilon u_t$$

The steady state of output and inflation are zero only if $\delta = 1$:

$$y^* = \frac{(1-\delta)(1-\beta\delta)\widetilde{\kappa}}{(1-\delta)(1-\beta\delta)\widetilde{\kappa} + (1-\alpha)\epsilon\kappa^2}\bar{y} \qquad \pi^* = \frac{(1-\delta)\kappa\widetilde{\kappa}}{(1-\delta)(1-\beta\delta)\widetilde{\kappa} + (1-\alpha)\epsilon\kappa^2}\bar{y}$$

When $\delta = 1$, $\widetilde{\kappa} = \kappa$, $\epsilon\pi_0 = (\bar{y} - y_0)$ and $\epsilon\pi_t = -(y_t - y_{t-1})$, so we can integrate $\epsilon(w_t - w_{-1}) = (\bar{y} - y_t)$. Thus w_t also follows a stationary difference equation (level targeting). With $\delta < 1$, the wage level can no longer be integrated as a stationary variable. There is only partial mean reversion of the wage level.

Property 4 *Long run level targeting is the optimal Ramsey policy only when $\delta = 1$. When $\delta < 1$, targeting the nominal wage level is no longer optimal. Long run inflation is non zero if there are steady state distortions. And in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the wage level (see fig 4).*

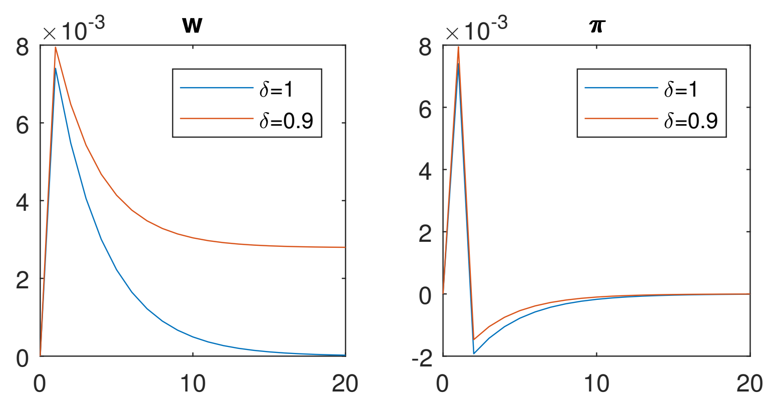

Figure 4: Ramsey policy in response to a wage cost-push shocks

The intuition is as follows: in the benchmark, by committing to give up some discretion in the future, the planner has some extra discretion in the present to offset cost push shocks, or an inefficient steady state. So that price (or wage) stability is optimal from today's perspective, but there is an incentive to renege tomorrow. With turnover, workers are less responsive to commitments, so that the current gain in terms of commitment no longer offsets the inefficiency in the future. Thus, even with a credible commitment, inflation will always be used to offset cost push shocks or steady state inefficiencies.

4.2 Long run optimal inflation

In this subsection, I derive the optimal steady state inflation implied by the non-linear model. While a closed form expression was available for the long run Phillips curve, the optimal level of steady-state inflation (for a given amount of steady state distortions) can only be defined implicitly. As such, it is useful to calibrate most of the parameters, to provide a graphical illustration. As in Gali, let us calibrate $\alpha = 0.25$, $\beta = 0.99$, $\epsilon = 5$, $\theta = 0.75$, $\phi = 5$ and $\sigma = 1$. Now I need to find values for δ. Let us consider a low turnover scenario ($\delta = 0.95$, or an average duration of 5 years) and an intermediate scenario with $\delta = 0.90$.

For the timeless Ramsey policy, I write the full dynamic Lagrangian (with Y_t, K_t and F_t re-normalized to flex wage values). The social planner maximizes the discounted sum of the per period utilities (7), subject, in each period, to the recursive expressions of F_t and K_t (equations 10 and 11), the ratio $\frac{K_t}{F_t}$ (equation 12), as well as the dynamics of Δ_t (equations 13). The Lagrangian writes

$$L = \sum \beta^t \left(\begin{array}{c} \left[\frac{1}{1-\sigma} Y_t^{1-\sigma} - \frac{1}{\mu_t} \frac{1-\alpha}{1+\phi} Y_t^{\frac{1+\phi}{1-\alpha}} \Delta_t\right] \\ +\phi_{1,t} \left[K_t w(\Pi_t)^{\frac{1+\phi\epsilon}{\epsilon-1}} - F_t\right] \\ +\phi_{2,t} \left[F_t - Y_t^{1-\sigma} - \theta\beta\delta \mathrm{E}_t F_{t+1} \Pi_{t+1}^{\epsilon-1}\right] \\ +\phi_{3,t} \left[K_t - Y_t^{\frac{1+\phi}{1-\alpha}} - \theta\beta\delta \mathrm{E}_t K_{t+1} {\Pi_{t+1}}^{\epsilon(1+\phi)}\right] \\ +\phi_{4,t} \left[\Delta_t - \theta \Delta_{t-1} {\Pi_t}^{\epsilon(1+\phi)} - (1-\theta) w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}}\right] \end{array} \right)$$

After taking the first order conditions, I look at the steady state value of each constraint and multiplier. As opposed to Benigno and Woodford (2005), because of turnover, the timeless optimal inflation is no longer zero. Figure 5 displays the optimal rate of inflation depending on the amount of steady state distortions (implied by $\bar{Y} > 1$), for different values of δ. When $\delta = 1$, I have the classic result of zero inflation in the long run, but it increases as this parameter decreases.

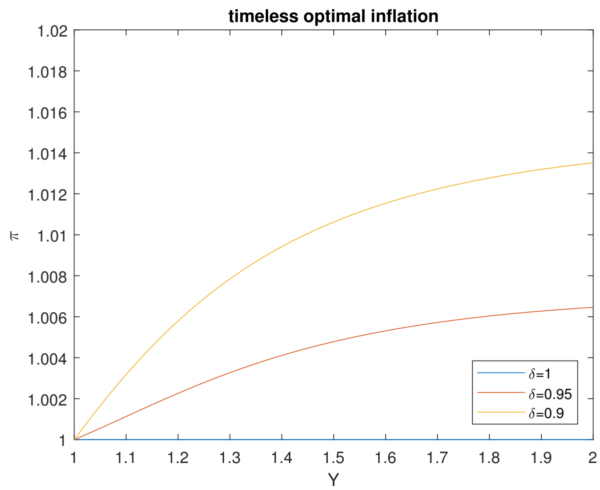

Figure 5: Steady state inflation without and with turnover

When $\delta = 1$, there is no inflation for the timeless Ramsey policy: this is the optimality of price stability. However, when turnover is introduced, the optimal level of inflation increases with the output gap, for both the constant and timeless cases. It is is in the order of $1 - 2\%$ annually with a high turnover and output gap ($\bar{Y} >> 1$), and would be higher if there was partial indexation.

5 Conclusion

This paper constructed a New Keynesian model with Calvo wage stickiness, as well as job turnover. I show how this leads to a Phillips Curve that is far less forward looking. When looking at a medium run Phillips Curve, with persistent output or unemployment disturbances, this can account for a flatter curve. If the coefficient of future inflation is restricted in a standard NK Phillips Curve, this creates a bias on the estimate of the slope of the Phillips Curve, and this bias increases with more turnover. In the long run, the Phillips Curve is also flatter, and no longer vertical or near-vertical.

I show how turnover breaks the optimality of price stability. Optimal Ramsey policy no longer targets the price level in response to cost push shocks. If turnover is large, and if the steady state distortions are high enough, the optimal level of inflation can reach $1 - 2\%$ annually. In fact, if there was partial price and wage indexation, the optimal inflation would be higher, or a same amount of inflation would be rationalized by a lower turnover or steady state distortion.

One fruitful avenue of future research would be to investigate the empirics. A cross section of different sectors, and different types of workers - eg, temporary vs. permanent employees - could provide strong empirical evidence. It would also be interesting to endogenise the turnover, as it does depend on institutions.

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Appendix

Linear quadratic approximation

Linear wage PC The log linear approximation of (10) and (11) yields,

$$\begin{aligned} f_t &= (1-\theta\beta\delta)\left[-\sigma y_t\right] + \theta\beta\delta \mathrm{E}_t\left[f_{t+1} + (\epsilon-1)\,\pi_{t+1}\right] \\ k_t &= (1-\theta\beta\delta)\left[\tfrac{\phi+\alpha}{1-\alpha} y_t\right] + \theta\beta\delta \mathrm{E}_t\left[k_{t+1} + \epsilon(1+\phi)\pi_{t+1}\right] \end{aligned}$$

While for equation (12) it is $\pi_t = -\frac{1-\theta}{\theta}\frac{1}{1+\phi\epsilon}\left(f_t - k_t\right)$, which brings

$$\pi_t = \frac{(1-\theta)\,(1-\theta\beta\delta)}{\theta}\frac{1}{1+\phi\epsilon}\left(\frac{\phi+\alpha}{1-\alpha}+\sigma\right) y_t + \beta\delta\mathrm{E}_t\left[\pi_{t+1}\right]$$

Dispersion Combining eqs (12) and (13), their 2nd order approximation is

$$d_t = \theta d_{t-1} + \frac{\theta}{1-\theta}\epsilon(1+\phi)(1+\epsilon\phi)\frac{\pi_t^2}{2}$$

Thus the discounted sum of the dispersions bring

$$\sum_{t\geq 0}\beta^t d_t = \frac{\theta\epsilon(1+\phi)(1+\phi\epsilon)}{(1-\theta)(1-\theta\beta)}\sum_{t\geq 0}\frac{(\pi_t)^2}{2} = \frac{\epsilon(1+\phi)}{\widetilde{\lambda}}\sum_{t\geq 0}\frac{\pi_t^2}{2}$$

with $\widetilde{\lambda} = \frac{(1-\theta)(1-\theta\beta)}{\theta}\frac{1}{1+\phi\epsilon} \neq \lambda$.

Welfare The 2nd order approximation of the utility function (7) is

$$\begin{aligned} & y_t - \sigma\tfrac{y_t^2}{2} - \tfrac{1}{\mu}\left(y_t + \tfrac{\phi+\alpha}{1-\alpha}\tfrac{y_t^2}{2}\right) - \tfrac{1}{\mu}\tfrac{1-\alpha}{1+\phi}\left(d_t\right) \\ &= \left(1-\tfrac{1}{\mu}\right) y_t - \left(\sigma + \tfrac{1}{\mu}\tfrac{\phi+\alpha}{1-\alpha}\right)\tfrac{y_t^2}{2} - \tfrac{1}{\mu}\tfrac{1-\alpha}{1+\phi}\left(d_t\right) \end{aligned}$$

Denoting $\bar{y}$ the log of the natural output, $\left(1-\frac{1}{\mu}\right) \approx \log\left[\mu\right] = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\bar{y}$

Up to a constant, the time-discounted objective function of the social planner can be written with output and the dispersions

$$U = -\sum_{t\geq 0}\beta^t\left[\left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\frac{(y_t-\bar{y})^2}{2} + \frac{1-\alpha}{1+\phi}d_t\right]$$

Replacing the discounted sum of dispersion with inflation, we get

$$U = -\sum_{t\geq 0}\beta^t\left[\left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\frac{(y_t-\bar{y})^2}{2} + \frac{(1-\alpha)\epsilon}{\widetilde{\lambda}}\frac{(\pi_t)^2}{2}\right]$$

Long run Ramsey Policy

This closely ressembles Benigno and Woodford (2005), with added turnover

Lagrangian (with Y_t, K_t, F_t renormalized to flex price values)

$$L = \sum \beta^t \left(\begin{array}{c} \left[\frac{1}{1-\sigma} Y_t^{1-\sigma} - \frac{1}{\mu_t} \frac{1-\alpha}{1+\phi} Y_t^{\frac{1+\phi}{1-\alpha}} \Delta_t \right] \\ +\phi_{1,t} \left[K_t w(\Pi_t)^{\frac{1+\phi\epsilon}{\epsilon-1}} - F_t \right] \\ +\phi_{2,t} \left[F_t - Y_t^{1-\sigma} - \theta\beta\delta \mathrm{E}_t F_{t+1} \Pi_{t+1}^{\epsilon-1} \right] \\ +\phi_{3,t} \left[K_t - Y_t^{\frac{1+\phi}{1-\alpha}} - \theta\beta\delta \mathrm{E}_t K_{t+1} {\Pi_{t+1}}^{\epsilon(1+\phi)} \right] \\ +\phi_{4,t} \left[\Delta_t - \theta \Delta_{t-1} {\Pi_t}^{\epsilon(1+\phi)} - (1-\theta) w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}} \right] \end{array} \right)$$

First order conditions

$$Y_t: \qquad \left[Y_t^{-\sigma} - \frac{1}{\mu} \Delta_t Y_t^{\frac{\phi+\alpha}{1-\alpha}} \right] = \phi_{2,t} \left[(1-\sigma) Y_t^{-\sigma} \right] + \phi_{3,t} \left[\left(\frac{1+\phi}{1-\alpha} \right) Y^{\frac{\phi+\alpha}{1-\alpha}} \right]$$

$$\begin{array}{lcl} \Delta_t & : & -\frac{1}{\mu} \frac{1-\alpha}{1+\phi} Y_t^{\frac{1+\phi}{1-\alpha}} + \phi_{4,t} - \phi_{4,t+1} \beta\theta \Pi_{t+1}^{\epsilon(1+\phi)} = 0 \\ K_t & : & \phi_{1,t} w(\Pi_t)^{\frac{1+\phi\epsilon}{\epsilon-1}} + \phi_{3,t} - \phi_{3,t-1} \theta\delta \Pi_t^{\epsilon(1+\phi)} = 0 \\ F_t & : & -\phi_{1,t} + \phi_{2,t} - \phi_{2,t-1} \theta\delta \left(\Pi_t \right)^{\epsilon-1} = 0 \end{array}$$

$$\begin{aligned} \Pi_t: \qquad & \phi_{1,t} \left[K_t \tfrac{1+\phi\epsilon}{\epsilon-1} w'(\Pi_t) w(\Pi_t)^{\frac{1+\phi\epsilon}{\epsilon-1}-1} \right] \\ & = \phi_{2,t-1} \left[\theta\delta F_t (\epsilon-1) \Pi_{t+1}^{\epsilon-2} \right] + \phi_{3,t-1} \left[\theta\delta K_t \epsilon (1+\phi) \Pi_t^{\epsilon(1+\phi)-1} \right] \\ & +\phi_{4,t} \left[\theta \Delta_{t-1} \epsilon (1+\phi) \Pi_t^{\epsilon(1+\phi)-1} + (1-\theta) \tfrac{\epsilon(1+\phi)}{\epsilon-1} w'(\Pi_t) w(\Pi_t)^{\frac{\epsilon(1+\phi)}{\epsilon-1}-1} \right] \end{aligned}$$

Steady state of constraints $\qquad K w(\Pi)^{\frac{1+\phi\epsilon}{\epsilon-1}} = F$

$$\left(1 - \theta\beta\delta \Pi^{\epsilon-1}\right) F = Y^{1-\sigma} \qquad \left(1 - \theta\beta\delta \Pi^{\epsilon(1+\phi)}\right) K = Y^{\frac{1+\phi}{1-\alpha}}$$

$$\left(1 - \theta \Pi^{\epsilon(1+\phi)}\right) \Delta = (1-\theta) w(\Pi)^{\frac{\epsilon(1+\phi)}{\epsilon-1}} \qquad w(\Pi) = \frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}$$

Steady state of FOCs

$$Y: \qquad \left[Y^{-\sigma} - \tfrac{1}{\mu} \Delta Y^{\frac{\phi+\alpha}{1-\alpha}} \right] = \phi_2 \left[(1-\sigma) Y^{-\sigma} \right] + \phi_3 \left[\left(\tfrac{1+\phi}{1-\alpha} \right) Y^{\frac{\phi+\alpha}{1-\alpha}} \right]$$

$$\begin{array}{lcl} \Delta & : & \frac{1}{\mu} \frac{1-\alpha}{1+\phi} Y^{\frac{1+\phi}{1-\alpha}} = \phi_4 \left(1 - \beta\theta \Pi_{t+1}^{\epsilon(1+\phi)} \right) \\ K & : & -\phi_1 w(\Pi)^{\frac{1+\phi\epsilon}{\epsilon-1}} = \phi_3 \left(1 - \theta\delta \Pi^{\epsilon(1+\phi)} \right) \\ F & : & \phi_1 = \phi_2 \left(1 - \theta\delta \Pi^{\epsilon-1} \right) \end{array}$$

$$\begin{array}{rl} \Pi: & \phi_1 \left[K \frac{1+\phi\epsilon}{\epsilon-1} w'(\Pi) w(\Pi)^{\frac{1+\phi\epsilon}{\epsilon-1}-1} \right] \\ = & \phi_2 \left[\theta\delta F (\epsilon-1) \left(\Pi_{t+1} \right)^{\epsilon-2} \right] + \phi_3 \left[\theta\delta K \epsilon (1+\phi) (\Pi)^{\epsilon(1+\phi)-1} \right] \\ & +\phi_4 \left[\theta \Delta \epsilon (1+\phi) \Pi^{\epsilon(1+\phi)-1} + (1-\theta) \frac{\epsilon(1+\phi)}{\epsilon-1} w'(\Pi) w(\Pi)^{\frac{\epsilon(1+\phi)}{\epsilon-1}-1} \right] \end{array}$$

Solving the optimal rate of inflation

The previous FOC can be rewritten as

$$\phi_1 F\left[\epsilon(1+\phi)\left(\frac{\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\delta\Pi^{\epsilon(1+\phi)}}-\frac{\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right)+(\epsilon-1)\left(\frac{\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}-\frac{\delta\Pi^{\epsilon-1}}{(1-\theta\delta\Pi^{\epsilon-1})}\right)\right]$$
$$=\phi_4\Delta\epsilon(1+\phi)\left[\Pi^{\epsilon(1+\phi)}-\Pi^{\epsilon-1}\frac{1-\theta\Pi^{\epsilon(1+\phi)}}{1-\theta\Pi^{\epsilon-1}}\right]$$

Now express every variable as a function of Π to later solve Π as a function of μ

$$w(\Pi)=\frac{1-\theta\Pi^{\epsilon-1}}{1-\theta}$$

$$Y^{\frac{\phi+\alpha}{1-\alpha}+\sigma}=\left[\frac{1-\theta\beta\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\beta\delta\Pi^{\epsilon-1}}w(\Pi)^{-\frac{1+\phi\epsilon}{\epsilon-1}}\right]$$

$$F=\frac{Y^{1-\sigma}}{1-\theta\beta\delta\Pi^{\epsilon-1}}$$

$$\Delta=\frac{(1-\theta)w(\Pi)^{\frac{\epsilon(1+\phi)}{\epsilon-1}}}{1-\theta\Pi^{\epsilon(1+\phi)}}$$

$$\phi_4=\frac{1}{\mu}\frac{1-\alpha}{1+\phi}\frac{Y^{\frac{1+\phi}{1-\alpha}}}{1-\beta\theta\Pi^{\epsilon(1+\phi)}}$$

$$\phi_1=\frac{\phi_4\Delta\epsilon(1+\phi)\left[\Pi^{\epsilon(1+\phi)}-\Pi^{\epsilon-1}\frac{1-\theta\Pi^{\epsilon(1+\phi)}}{1-\theta\Pi^{\epsilon-1}}\right]}{F\epsilon(1+\phi)\left[\frac{\delta\Pi^{\epsilon(1+\phi)}}{1-\theta\delta\Pi^{\epsilon(1+\phi)}}-\frac{\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}\right]+F(\epsilon-1)\left[\frac{\Pi^{\epsilon-1}}{1-\theta\Pi^{\epsilon-1}}-\frac{\delta\Pi^{\epsilon-1}}{(1-\theta\delta\Pi^{\epsilon-1})}\right]}$$

Having everything expressed in terms of Π, I can solve Π as a function of μ

$$\left[Y^{-\sigma}-\frac{1}{\mu}\Delta Y^{\frac{\phi+\alpha}{1-\alpha}}\right]=\phi_1\frac{(1-\sigma)Y^{-\sigma}}{1-\theta\delta\Pi^{\epsilon-1}}-\phi_1\frac{\left(\frac{1+\phi}{1-\alpha}\right)Y^{\frac{\phi+\alpha}{1-\alpha}}}{1-\theta\delta\Pi^{\epsilon(1+\phi)}}w(\Pi)^{\frac{1+\phi\epsilon}{\epsilon-1}}$$