PEAK PROBABILITY FUNCTIONS FOR RANDOM DYNAMIC RESPONSE OF COMPOSITE PLATE WITH INITIAL GEOMETRIC IMPERFECTION RESTING ON ELASTIC FOUNDATIONS

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1. INTRODUCTION

For high performance military aircrafts and future high-speed civil transport planes, certain structural skin components are subjected to very large acoustic loads under in thermal environments [1]. Some structural components are excited by the intense acoustic and thermal-mechanical loads (~1650 °C temperature and ~180 dB Overall Sound Pressure Level, OASPL [2]). In order to protect substructure of hypersonic flight vehicle, a sandwich structure with two thin stiff and strong ceramic facesheets separated and a relatively thick, lightweight, and compliant aerogel thermal insulator has been developed as a thermal protection system (TPS). The facesheets and the relatively thick aerogel thermal insulator are bonded tightly. The current investigation focuses on the random dynamic response of the top facesheet for the TPS sandwich structure since the ceramic-based composite plate is structurally weak with low fatigue resistance and can fail easily. Thus the contribution of the relatively thick and compliant core and the bottom facesheet are treated as an equivalent elastic foundation, on which the top facesheet is resting on. Very few analyses and results have been reported regarding characterizations of strain process for thin plates subject to a combination of thermal and acoustic loadings. Peak probability function of strain process is related to acoustic fatigue life prediction [3-5]. Therefore it is of practical interest to understand in-plane strain process and peak probability function of thin top facesheet in the TPS sandwich structure under thermal and acoustic loadings. The influence of the thermal sealing material around the top facesheet in the thermal environments has been considered as the in-plane boundary constraints with the equivalent boundary compliance. Transverse initial geometric imperfection w* in a stress-free state has been considered in the model to represent the imperfection due to plate manufacture [6].

2. THEORETICAL FORMULATION

Plate Model

The length, width and the thickness of the top facesheet plate are a, b and h, and the schematic model is shown in Fig. 1. A Cartesian coordinate Oxyz is located in the middle surface of the rectangular plate with its origin at the left corner. Assume that (u, v, w) represent the displacements of an arbitrary point in the x, y and z directions, respectively. In-plane boundary constraint condition is specified only on the top facesheet of the TPS sandwich structure due to thermal sealing materials around it in thermal environments. The load-displacement relationship of the foundation is assumed to be

\[ F = K_w w - K_s v^2 w \]

where \( F \) is the force per unit area, \( K_w \) is the Winkler foundation stiffness and \( K_s \) is the shearing layer stiffness, and \( \nabla^2 \) is the Laplace operator in x and y.

![Fig. 1: The model configuration. The thin top facesheet of the sandwich TPS has been modeled as a thin plate resting on a two-parameter elastic foundation.](image)

Modal Equations by Galerkin Procedure

The deflection function \( w(x,y,t) \) is assumed to be:

\[ w(x,y,t) = \sum_{i,j} q_{ij}(t) \sin(\pi x/a)\sin(\pi y/b) \]

(1)

where \( x/a \) and \( y/b \) and \( i,j = 1,2,3... \) are numbers of half waves in x and y directions, respectively. \( q_{ij}(t) \) is the modal amplitude of the plate mode. For sake of simplicity, only sine type global type of imperfection is studied in the work. Thus,

\[ w^*(x,y) = \eta \sin(\pi x/a)\sin(\pi y/b) \]

(2)

The midplane nonuniform temperature field in the present work is assumed as a simplified illustration:

\[ \bar{T}(x,y) = T_0 + \theta_0 \delta \sin^2(\pi x/a)\sin^2(\pi y/b) \]

(3)
Substituting Airy stress function $F(x, y, t)$ and the transverse deflection function into the governing equation, and a set of coupled nonlinear ordinary differential equations (ODEs) is developed utilizing Galerkin procedure under the classical theory of thin plates and the von Kármán type kinematic relation, and the reduced-order modal equation takes the form as following:

$$\ddot{q} + C\dot{q} + (K + G + H+\Theta)q + f(q) = p$$  \hspace{1cm} (4)

Some noteworthy statements can be given from the explicit expression of the matrix term in the modal equation. For instance, $K$ is the diagonal linear structural stiffness matrix. The diagonal linear stiffness matrix $H$ is induced by the uniform temperature rising $t_0$. The symmetric and non-diagonal linear stiffness matrix $G$ is introduced by the non-uniform temperature rising term $\eta\delta$. It is worth noting that the compliances $S_x$ and $S_y$ are only included in the linear stiffness matrices $H$ and $G$. The symmetric and non-diagonal linear stiffness matrix $\Theta$ is induced by the geometric imperfection, and the terms are related to the imperfection $\eta$ size linearly. The column vector $f(q_j)$ consists of the coupled nonlinear quadratic and cubic modal amplitude terms, which can be written as:

$$f(q_j) = \sum_{i,j,k,l,m,n,r,s} R_{ijklmns} q_i q_j q_k q_l q_m q_n q_r q_s + \sum_{i,j,k,l,m,n,r,s} S_{ijklmns} q_i q_j q_k q_l q_m q_n q_r q_s$$  \hspace{1cm} (5)

$R_{ijklmns}$ is the coefficient of the nonlinear quadratic modal amplitude term induced by the initial geometric imperfection, and it is linearly related to the geometric imperfection size $\eta$. $S_{ijklmns}$ is the coefficient of the nonlinear cubic modal amplitude term, which is induced by the coupling between in-plane stretching and transverse deflection. All nonlinear modal amplitude terms are independent with the equivalent elastic foundation stiffness and thermal effects.

In-Plane Strain Response

Total in-plane strain can be evaluated explicitly. As an illustration, the in-plane normal strain along $x$ direction is shown as following:

$$\varepsilon_x = l_0 + \sum_{i,j,k,l,m,n} \tilde{r}_{ijkl} q_i q_j q_k q_l q_m q_n$$  \hspace{1cm} (6)

in which constant, linear and coupled quadratic modal amplitude terms are included. It is interesting to note that the uniform temperature rising $t_0$ can induce a constant in-plane strain due to the in-plane boundary constraints. The linear term is induced by the linear strain distribution due to Kirchhoff hypotheses. It includes two parts, the first part is linearly related to the out-of-plane coordinate $z$, and the second part is linearly related to the initial geometric imperfection size $\eta$. The foundation stiffness and the geometric imperfection effect are not included in the quadratic coefficients explicitly. It is worthy of noting that the thermal effects only contribute to the constant term $l_0$ and are not included in the linear and quadratic coefficients. Displacement and strain process can be given by solving the set of coupled nonlinear ODEs (Eq. 4). It is noteworthy that as the linear term $\tilde{r}_{ijkl} q_i q_j$ dominates strain process, symmetric Gaussian displacement process results in Gaussian strain process. Thus spectral-based approaches such as Bendat’s model and Dirlik narrow band model can be used to predict the sonic fatigue life. On the other hand, if strain process is governed by the quadratic term, Gaussian displacement process leads to non-Gaussian skewed strain process. Therefore the peak probability function of the strain process needs to be estimated for acoustic fatigue life prediction.

3. RESULTS

The number of physical degrees-of-freedom for the structure has been reduced in the coupled modal equations using a series of basis functions to expand the deflection displacement field (Eq. 1). Thus the accuracy and efficiency of the present model depend on the number of series included in the displacement expansion function. Results showed the maximum normalized difference for the displacement is no more than 3% if four symmetric modes are included. In order to demonstrate the capability of the reduced four-mode model to yield accurate dynamic response, Fig. 2 shows the time history of the displacement $w_{max}$ and the associated $\varepsilon_x$ of the postbuckled plate at under the periodic excitation load. It is
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noted that the displacement response as calculated by the four-degree reduced-order model is nearly identical to the solution of full-order FEA model, and only small difference can be noticed for the displacement and strain process.

Fig.4: The displacement $w_{\text{max}}$ and the associated in-plane strain.  

Fig.5: Histogram of in-plane strain process for the top facesheet with initial geometric imperfection and in-plane constraints. (a) 700°C/140dB; (b) 700°C/180dB.

It is apparent that the thermal buckling of the perfect plate occurs when the resultant in-plane compressive force increases with an increasing of thermal effects. Both give rise to thermal expansion and induce a negative thermal stiffness under in-plane boundary constraints of the panel. The perfect plate remains flat as the applied temperature increases until it experiences a critical bifurcation (it buckles) at the critical point. Fig. 3 shows the critical buckling temperature $T_{c}$ variation with the equivalent in-plane boundary stiffness $K_{b}$ with $S_{b} = 1/K_{b}$. It is clear that $T_{c}$ decreases with increasing $K_{b}$ and it converges to the buckling temperature of simply supported plate with four immovable edges. The variation of the maximum deflection with the uniform temperature rising is provided for an imperfect plate in Fig. 4. Due to the effect of the initial geometric imperfection, the plate experiences a gradual deflection with $t_{0}$. Above a certain point, which is another critical state, a secondary stable equilibrium branch appears. The equilibria are unsymmetric and the occurrence of the imperfection actually postpones the onset of the critical state until a higher $t_{0}$. Fig. 5 demonstrates that due to dominant contribution of the quadratic modal amplitude terms in Eq. (5), the skewed non-Gaussian strain process can be observed for the top facesheet of a sandwich TPS structure subjected to a combination of thermal and acoustic loadings. Therefore the peak probability function of the strain process is not Rayleigh distribution and a new peak probability function has to be determined.

4. CONCLUSIONS

This paper investigates the dynamic response of a thin facesheet of a TPS sandwich structure subjected to a combination of thermal and acoustic excitation. A simply-supported thin plate with initial geometric imperfection under in-plane boundary constraints resting on a two-parameter elastic foundation is proposed to characterize the behavior of the facesheet of the sandwich TPS for simplification. A theoretical reduced-order model is developed based on the thin-plate theory and the von Kármán-type relationship. Quadratic terms are included in the coupled modal equations due to initial geometric imperfection (initial curvature). It is clear that the critical buckling temperature decreases with increasing in-plane boundary stiffness. Due to the effect of the initial geometric imperfection, the plate experiences a gradual deflection with $t_{0}$. A secondary stable equilibrium branch appears above a certain point. The equilibria are unsymmetric and the occurrence of the imperfection actually postpones the onset of the critical state until a higher $t_{0}$. A skewed non-Gaussian strain process occurs due to the contribution of the quadratic term in the strain distribution. Thus the peak probability function of the steady-state strain process has to be estimated carefully with the extensive numerical analysis using the reduce-order model.

REFERENCES