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International Interest Rates and Housing Markets

by

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Abstract

Current account deficits and housing prices showed a strong positive correlation throughout the mid-90s to 2007. This paper studies the effect of a decrease in the international interest rate and in the downpayment requirement to buy a house during that period on the joint behavior of the current account and housing prices. To this end, I build a small open economy model with life-cycle heterogeneous agents and two goods: tradable (non-housing) and non-tradable (housing). I calibrate the model to replicate selected aggregate statistics of the U.S. economy and compute the transition after the decrease in the interest rate and in the downpayment. The model is able to match some relevant facts: the boom and the bust (after 2007) in the housing market, where the bust, as the data show, occurs without a reversal in the interest rate; the increase in the homeownership rate; the simultaneous boom - and bust - in non-housing consumption; and the coexistence of borrowing from abroad with a current account deficit throughout the transition.

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1 Introduction

"In my view... it is impossible to understand this crisis without a reference to the global imbalances in trade and capital flows that began in the latter half of the 1990s."
Ben S. Bernanke (2009)

Current account deficits and housing prices were positively correlated between the mid-90s and 2007.[1] This period was characterized by: first, the magnitude of housing market booms compared with previous experiences, as shown by André (2010); and second, the existence of *global imbalances*,[2] a particular event of this period of time.

Before the Great Recession of 2007-08, housing market booms had been characterized by unprecedented increases in residential investment (Figure 1), housing prices (Figure 2), and homeownership rates (Figure 3). Afterwards, a reversal in all these variables characterized the housing bust period. In addition, a decrease in current account deficits took place (Figure 4) together with the bust (after 2007) in housing markets.

Simultaneously, long-term interest rates have been decreasing over time (Figure 5). Caballero et al. (2008) or Mendoza et al. (2009) study, under different hypotheses, the coexistence of low interest rates and *global imbalances* in the world economy. They address how a huge amount of savings coming from different regions emerged in the international capital markets during the 90's depressing international interest rates.[3] As Caballero et al. (2008) claim, the long-run real interest rate has been steadily declining over the last decade despite the efforts from central banks to raise interest rates - the so called "Greenspan's Conundrum".

In this paper, I develop a theory of housing boom and bust with the following ingredients: small open economy, to analyze shocks to the international interest rate; life-cycle heterogeneous agents model with housing tenure decisions, in order to account for the extensive margin

[1]See Punzi (2007), Aizenman and Jinjarak (2009), André (2010), Gete (2010), or Ferrero (2011) for this evidence in OECD, non-OECD, or emerging economies.

[2]Global imbalances are large and persistent current account deficits run by some countries (e.g. the U.S., Spain) and, simultaneously, current account surpluses in other countries (e.g. Japan, Germany, emerging Asia, some oil exporting countries). See Obstfeld and Rogoff (2010).

[3]As the "saving glut" hypothesis (Bernanke (2005)) suggests.

increase in housing demand (homeownership rate); and residential land, modeled following Davis and Heathcote (2005), and needed to produce new houses. As Davis and Heathcote (2007) show for the U.S. between 1970 and 2003, land governs housing price dynamics.

Two channels fuel a housing boom in this environment: cheap credit and financial innovation. Both of them are taken as given in the model. The first channel translates into a decreasing international interest rate. The second channel is institutional. Strong financial innovation (the development of housing equity withdrawal, subprime loans, the development of securitization), liberalization in mortgage markets, and government support to increase the homeownership rate, resulted in a decrease in the downpayment requirement to buy a house in the U.S.[4] Table 1 shows this evidence for the loan-to-value (LTV) ratio in the U.S. for different years. These data refer to LTV ratios for first time home-buyers, namely, the marginal buyers most affected by borrowing constraints.

A housing boom in this model, after a decrease in the interest rate and in the downpayment requirement to buy a house, implies an increase in housing demand - at both margins -, together with an increase in real house prices and labor used in the construction sector. The decrease in the downpayment requirement to buy a house makes it possible for some renters to become homeowners. The economy borrows from abroad because borrowing is cheaper (due to the decrease in the interest rate) and because more expensive houses are used as collateral against international borrowing. This increase in borrowing from abroad produces a current account deficit. The economy also moves labor from the tradable (non-housing) to the non-tradable (housing) sector, and runs a trade deficit to fulfill the demand for non-housing consumption. As time goes by, and households start to reach their desired stock of housing, the demand for new houses cools down, putting less pressure on new residential land and decreasing housing prices and labor in construction: a bust in the housing market ensues. Non-housing consumption decreases for two reasons: the economy must repay its debt; and the decrease in housing prices makes homeowners poorer than before. This is consistent with a boom in non-housing consumption and its bust. Moreover, the bust in the economy happens without a reversal in low interest rates. Thus, the model also provides some insights for the bust period based on fundamentals.

The modeling strategy for the boom period is in line with Garriga et al. (2012) and Kiyotaki et al. (2011). These papers also find large effects on housing prices of a decrease in the international interest rate in small open economy models calibrated to the U.S. economy. Garriga et al. (2012) develop a representative agent model in which relaxed financial conditions

[4]See for example Chambers et al. (2009), Doms and Krainer (2007), or Green and Wachter (2005).

and a decline in the world interest rate lead housing prices dynamics. Kiyotaki et al. (2011) focus their analysis on the welfare effects of changes in the international interest rate across net buyers and net sellers of houses. Kiyotaki et al. (2011) use an OLG structure but they depart from a law of motion for land as in Davis and Heathcote (2005), which allows my model to qualitatively replicate the behavior of the U.S. economy during the bust period, so that my model is consistent with the development of housing market and the behavior of the current account. As Kiyotaki et al. (2011) show in an OLG framework, a relaxation of the downpayment requirement plays almost no role in house price dynamics. Justiniano et al. (2013) document how relaxed borrowing standards would not be consistent with the spike in the ratio of mortgages to the value of real estate during the housing bust as the data show. In this respect, an OLG structure would be consistent with this dimension of the U.S. data. This result is in line with alternative calibrations and is also consistent with the results presented in the literature by Iacoviello and Neri (2010), who find negligible effects on house price dynamics of a shock to the LTV ratio.

The effects identified in the literature are also qualitatively similar to those in Justiniano et al. (2014a), where the focus is on the interaction between borrowing and lending constraints. The model is built on Iacoviello (2005) with different levels of impatience between borrowers and savers/lenders. In this environment, savers face a lending constraint and the results, qualitatively consistent with different dimensions of the U.S. data, come from a relaxation of this lending constraint (credit supply shock) and not from a relaxation of the borrowing constraint.

Adam et al. (2011), Ferrero (2011), and Justiniano et al. (2014b) also focus their analysis on the correlation between current account deficits and house price dynamics. Adam et al. (2011) depart from the assumption of rational expectations in an asset pricing model with learning. The sharp increase in house prices is due to the presence of agents with subjective beliefs about future prices. Both, Ferrero (2011) and Justiniano et al. (2014b), are built on Iacoviello (2005). Ferrero (2011) frames his analysis within a two-country model with one representative international saver lending to one representative borrower in the U.S.; while Justiniano et al. (2014b) evaluate the interaction of the saving glut and the banking glut on the U.S. economy.

Finally, also connecting current account deficits to the housing market boom before the financial crisis, Favilukis et al. (2013) find no effect of purchases of government debt by foreigners on housing prices when laying down easier financial conditions in a general equilibrium framework with idiosyncratic shocks and incomplete markets. They find that the reduction in the risk-free rate prompted by the flow of international capital into the U.S. economy is compensated by the increase in the housing risk premia because of the reallocation in the domestic agents' portfolio prompted by the foreign purchases of safe assets.

The rest of the paper is organized as follows. The model economy is presented in Section 2. In Section 3, the benchmark model is calibrated to the U.S. economy. Simulations are performed and discussed in Section 4, and a brief summary concludes the paper in Section 5.

2 The Model Economy

I investigate a life-cycle small open economy model populated by heterogeneous agents with three sectors of production. The model strategy follows Gervais (2002), Díaz and Luengo-Prado (2008), and Díaz and Luengo-Prado (2010), who study different issues of the demand for housing in life-cycle closed economies. The main differences stem from the focus of this paper on the ability of an open economy to run a trade deficit; in the feature that housing consumption is considered as a non-tradable good and non-housing consumption as a tradable one; and in the existence of three sectors of production: consumption/tradable sector, residential structures sector, and housing/non-tradable sector. The housing sector supplies new non-tradable houses in the economy by combining residential structures and land where land is a fixed factor of production and is subject to a law of motion as in Davis and Heathcote (2005).

2.1 Households

2.1.1 Preferences

The economy is populated by overlapping generations of individuals with finite lives and age $j \in \{1, ..., J\}$. The utility function in period t of a newborn individual is a CES utility function over consumption of housing services (x_t^j) and non-housing consumption goods (c_t^j):

$$\sum_{j=1}^{J} \beta^{j-1} u(c_{t+j-1}^j, x_{t+j-1}^j) = \sum_{j=1}^{J} \beta^{j-1} \frac{\left(((1-\theta)(c_{t+j-1}^j)^{\frac{\varepsilon-1}{\varepsilon}} + \theta(x_{t+j-1}^j)^{\frac{\varepsilon-1}{\varepsilon}})^{\frac{\varepsilon}{\varepsilon-1}}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}}$$

where $x_t^j = f_t^j + h_t^j$ is housing services, f_t^j being services coming from renting a house, and h_t^j being housing capital (services coming from owning). Renting and owning are perfect substitutes. I assume that one unit of housing capital generates one unit of services, $x_t^j = h_t^j$. c_t^j is non-housing consumption.

2.1.2 Housing Capital and Housing Services

An individual must fulfill in advance at least a minimum downpayment requirement to buy (and thus own) a house. This downpayment requirement is given by a fraction γ_t of the value of the house. The remaining cost can be financed by borrowing against the house, with $(1 - \gamma_t)$ giving the maximum loan-to-value ratio. Housing capital is subject to some degree of indivisibility. This is modeled by assuming a minimum size of housing investment, $\underline{\mathrm{h}}$.

An individual can rent housing services as an alternative to becoming a home-owner. Renting housing services has two advantages over owning: first, it allows individuals to consume housing services and thus avoid the downpayment requirement; and second, rented houses are not subject to the same indivisibility as owner-occupied housing.

The price of one unit of housing services in terms of consumption goods is denoted by $r_t^f p_t^h$, where p_t^h is the price of a house in terms of consumption and r_t^f represents the fraction of that price that an individual has to pay for renting.

Housing capital depreciates at rate δ_h. But rented houses depreciate at a rate δ_f, where $\delta_f > \delta_h$. This difference in depreciation costs is a result of a moral hazard problem in rental markets since renters decide the intensity at which they make use of the house they rent. This moral hazard problem affects the market rate for rental services and this additional cost involved is paid by the renters as a premium.

2.1.3 Income Dynamics

The life of each individual consists of a working period and a retirement period. An exogenously given mandatory retirement age, denoted by j^*, determines the age at which households retire. Individuals are endowed with one unit of working time in each period of their working lives which they supply inelastically. An age-j individual's unit of working time is transformed into z^j efficiency units of labor, and this value depends on age but is constant over time. Each unit of effective labor is paid the wage rate ω_t. This specification allows for individuals to differ both across and within generations. At any point in time, the average productivity level is fixed at unity. The measure of the entire population is also normalized at unity.

During the retirement period ($j > j^*$) households receive a retirement pension, b^j, until the end of their lives. The retirement pensions are paid through the social security system and collected from income taxes on labor.

Another source of income comes through individuals' asset holdings. Individuals accumulate wealth because of life-cycle reasons and to collect the downpayment required to buy a house. Individuals choose between three assets to accumulate wealth: housing, business capital, and deposits at financial institutions.

From a household's perspective, deposits at financial institutions and business capital are equivalent and a zero-profit condition guarantees that the rates of return on these assets are equalized. As a result, the sum of deposits at financial institutions and business capital constitute a single financial asset, denoted a_t^j.

Homeowners also receive rents from the proportion of total land they have and is needed for new residential development. This proportion of land is exogenously given in each period from the point of view of homeowners. The amount of land they have is a proportion of their housing stock. For a more detailed explanation of this point see Appendix A.

2.2 Financial Institutions

Financial institutions receive individuals' deposits and make use of it for three activities: to finance loans issued to homeowners, purchase residential capital, and borrow/lend in international capital markets. Financial institutions use the same linear technology as homeowners to produce housing services, which they rent out to individuals who do not own a house. Financial institutions borrow from/lend to abroad through the possibility of accessing international capital markets given a fixed interest rate, r_t^*. They have to satisfy the demand for credit from individuals at this interest rate.

Financial institutions, just as homeowners, receive rents from the amount of land that they have available for residential investment.. This amount is proportional to the stock of housing they buy for renting. See Appendix A for a more detailed explanation of this point.

2.3 Technology

2.3.1 Residential Structures and Consumption Sector

Output by residential structures and the consumption sector is produced using a Cobb-Douglas production technology in each sector:

$$f^i(K_t^i, N_t^i) = A_i (K_t^i)^{\alpha_i} (N_t^i)^{1-\alpha_i}$$

where $i \in \{c, s\}$ refers to a specific sector, and c refers to consumption and s to the residential structures sector. A_i is a technology parameter, K_t^i is the total amount of business capital used in each sector and N_t^i represents the share of the working population employed in each sector. The capital factor shares, α_i, are different for each sector with the residential structures sector being more labor intensive, $\alpha_s < \alpha_c$. In each period the stock of business capital depreciates at a rate δ_k. The price of residential structures in terms of the consumption good, which is normalized at unity, is equal to p_t^s.

I assume perfect mobility of factors between sectors such that wages (ω_t) and the price for business capital (r_t^k) are equal in both sectors.

2.3.2 Housing Sector

I am following Davis and Heathcote (2005) in modeling houses. I assume that a constant acreage of new land suitable for residential investment is sold by homeowners and financial institutions to the firms producing houses. In order to produce new houses residential investment must be combined with land. Homeowners and financial institutions own an amount of the acreage of land proportional to the housing stock they own. In each period one new acreage unit of land appears and is sold to firms. This acreage is normalized to one.

Real estate developers combine new residential structures with newly-availabe land to produce new houses according to a Cobb-Douglas technology:

$$f^h(X_t^s, L_t) = (X_t^s)^{1-\phi}(L_t)^{\phi}$$

where X_t^s is the total amount of residential structures used in the production of new houses and L_t represents the amount of land employed. The share of land in the production of new houses is denoted by ϕ. The price of a new house and the price of land, both in terms of consumption good, are represented by p_t^h and p_t^l, respectively.

2.4 Government Expenditure

τ_y is a proportional tax rate on labor income and the return on financial assets. In each period the entire proceeds from taxation on financial assets is used to finance government expenditures. The entire proceeds from taxation on labor income is given back to individuals as pensions when they are retired. The government thus maintains a balanced budget every period.

2.5 The Household's Decision Problem

Households decide on the amount of consumption (c_t^j), housing services (x_t^j), the housing capital stock for the next period (h_{t+1}^{j+1}) and the amount of financial assets (a_{t+1}^{j+1}) by solving this problem:

$$v_t^j\left(a_t, h_t; i\right) = \max_{\left\{c_t^j, x_t^j, h_{t+1}^{j+1}, a_{t+1}^{j+1}\right\}} \left\{u(c_t, x_t) + \beta v_{t+1}^{j+1}\left(a_{t+1}, h_{t+1}; i\right)\right\} \tag{1}$$

$$s.t. \quad c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h h_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + (1+(1-\tau_y)r_t^a)a_t + (1-\delta_h)p_t^h h_t + p_t^l l(h_t, h_{t+1}) \tag{2}$$

$$a_{t+1} \geq -(1-\gamma_t)p_t^h h_{t+1} \tag{3}$$

$$h_t \geq \underline{\mathrm{h}} \ otherwise \ h_t = 0 \tag{4}$$

$$x_t^j = f_t^j + h_t^j \tag{5}$$

Equation (2) is the budget constraint. The term $p_t^l l(h_t, h_{t+1})$ in the budget constraint represents that households own land in this economy. Households have an amount of land proportional to the amount of their housing capital.[5] Equation (3) is the borrowing constraint. Equation (4) is a constraint for the minimum house size available in the housing market. Equation (5) is the value of housing services.

So, in this environment owning is preferred to renting because of three reasons: first, there is a preferential tax treatment for saving in housing rather than in financial assets; second, owning a house allows households to borrow using their housing stock as collateral; and third, the depreciation rate of a rented house is higher than for a house owned, as it is explained in Section 2.1.2..

2.6 The Decision Problem of Financial Institutions

Financial intermediaries issue loans in each period and buy residential capital using the proceeds from deposits they accept and by accessing international capital markets. They have access to the international capital markets through a bond at an international interest rate. They receive payments from rental accommodations, from selling land to the housing sector, and receive the interest on loans issued. They also pay interests on deposits and international bonds. The problem of a new financial institution in period t is as follows:

[5] An explanation of how households hold the proportion of land relative to their homes is offered in Appendix A.

$$\Psi\left(F_t,B_t,A_t,K_t\right) = \max_{\{F_{t+1},B_{t+1},A_{t+1},K_{t+1}\}} \Big\{ r_t^f p_t^h F_t + p_t^l l(F_t) + X_t^A +$$
$$+ r_t^k K_t - X_t^K - p_t^h X_t^f - X_t^B + \frac{1}{1+r_t}\Psi\left(F_{t+1},B_{t+1},A_{t+1},K_{t+1}\right)\}$$

$$s.t.\ \ X_t^K + p_t^h X_t^f + X_t^B \leq r_t^f p_t^h F_t + p_t^l l(F_t) + \ X_t^A + r_t^k K_t \tag{6}$$

$$X_t^A = A_{t+1} - (1+r_t^a)A_t$$

$$X_t^B = B_{t+1} - (1+r_t^*)B_t$$

$$X_t^f = F_{t+1} - (1-\delta_f)F_t$$

$$X_t^K = K_{t+1} - (1-\delta_k)K_t$$

where F is the amount of houses rented to households, A is the deposits made by households, B is the international borrowing/lending and K is the business capital rented to firms.

From this problem I get the dynamics for the rental price:

$$r_t^f p_t^h = (1-\phi)[(1+r_t)p_{t-1}^h - (1-\delta_f)p_t^h]$$

and the zero-profit condition: $r_t = r_t^* = r_t^a = r_t^k$.

2.7 Competitive Equilibrium

I am interested in the transition of this model since I want to replicate a trade deficit together with borrowing from abroad as it was the case for the U.S. during the period under study. To replicate both facts at the same time, it is necessary to evaluate the transition since in a steady state, and without any kind of exogenous growth, a country will be paying debt interest through exports with a small enough international interest rate. For a definition of the steady-state competitive equilibrium see Appendix B.

Denote $q = \{a_t, h_t, i\}$, q $\in$ Q.

Definition *A* ***competitive equilibrium*** *for a given government policy,* τ_y*, downpayment requirement,* γ_t*, and an age-dependent measure of agents type,* $\lambda_j(q)$*, is a collection* *of relative prices* $\left\{p_t^h, p_t^s, p_t^l, r_t^f, r_t, w_t\right\}$, *a collection of functions for the household problem* $\left\{v_t^j(q), c_t^j(q), f_t^j(q), h_t^j(q), a_t^j(q)\right\}$, *a value function for financial institutions* $\Psi\left(F_t, B_t, A_t, K_t\right)$, *and aggregate quantities for the whole economy* $\left\{Y_t^c, Y_t^h, Y_t^s, X_t^s, L_t, K_t^c, K_t^s, N_t^c, N_t^s, F_t, B_t, A_t\right\}$ *such that:*

1. *Inputs are priced competitively every period.*

2. *Given* τ_y, γ_t *and prices, the functions* $\left\{v_t^j(q), c_t^j(q), f_t^j(q), h_t^j(q), a_t^j(q)\right\}$ *solve the dynamic program in the household problem.*

3. *Given prices and the function* $\Psi\left(F_t, B_t, A_t, K_t\right)$, $\{F_{t+1}, B_{t+1}, A_{t+1}, K_{t+1}\}$, *solves the financial institutions' problem.*

4. *Individual and aggregate decisions are consistent:* $C_t = \sum_{j=1}^J \int_Q c_t^j d\lambda_j(q)$, $H_t = \sum_{j=1}^J \int_Q h_t^j d\lambda_j(q)$, $F_t = \sum_{j=1}^J \int_Q f_t^j d\lambda_j(q)$, $A_t = \sum_{j=1}^J \int_Q a_t^j d\lambda_j(q)$.

5. *The government maintains a balanced budget every period:*

$$G_t + b_t = \sum_{j=1}^{J} \int_Q \left[\tau_y w z_t^j + \tau_y r a_t^j\right] d\lambda_j(q)$$

   where $b_t = \sum_{j=1}^J \int_Q b_t^j d\lambda_j(q) = \tau_y w_t \overline{N}_t$.

6. *Labor market clears every period:* $N_t^c + N_t^s = \overline{N}_t$.

7. *Capital market clears every period:* $K_t^c + K_t^s = K_t$.

8. *Land market clears every period:* $L_t = \overline{L}_t$.

9. *Residential structures market clears every period:* $X_t^s = Y_t^s$.

10. *Housing market clears every period:*

$$Y_t^h = X_t^h + X_t^f$$

   where $X_t^h = H_{t+1} - (1-\delta_h)H_t$.

11. *Trade balance is determined every period:*

$$TB_t = Y_t^c - C_t - X_t^k - G_t$$

   where $X_t^k = K_{t+1} - (1-\delta_k)K_t$.

12. Net foreign asset position is determined in every period:

$$B_{t+1} = TB_t + (1 + r_t^*)B_t$$

2.7.1 Characterization

The price for structures becomes:

$$p_t^s = \frac{A_c}{A_s}\frac{\alpha_c^{\alpha_c}(1-\alpha_c)^{1-\alpha_c}}{\alpha_s^{\alpha_s}(1-\alpha_s)^{1-\alpha_s}}\left(\frac{w_t}{r_t+\delta_k}\right)^{\alpha_c-\alpha_s}$$

From this equation, after a shock to the international interest rate, the effect on the price of structures depends on the difference between the capital shares of the consumption sector and the residential structures sector $(\alpha_c - \alpha_s)$. As it will be shown in the calibration section, it is true for the U.S. economy that the residential structures sector is more labor intensive than the consumption sector. This means that after an exogenous decrease in the international interest rate, wages will increase, since labor becomes relatively scarce, and as a consequence of the bigger capital share in the consumption sector, this will imply an increase in the price of residential structures.

Housing prices become:

$$p_{ht} = \frac{p_{st}^{1-\phi} p_{lt}^{\phi}}{(1-\phi)^{1-\phi}\,\phi^{\phi}}$$

From this equation for housing prices it can be inferred that after a positive shock (interest rate and downpayment requirement) to the demand for housing, an increase in the demand for land will occur and, given the fixed supply of land in the economy each period, the price of land will increase. Both the increase in the price of structures and the increase in the price for land will increase housing prices in this model.

3 Calibration

My calibration strategy closely follows Gervais (2002) and Díaz and Luengo-Prado (2008). The benchmark model is calibrated to the U.S. economy as a closed economy in the initial steady state. After the shocks I allow the economy to have access to international borrowing and lending and to have a trade balance different from zero. Thus, the U.S. economy becomes a small open economy during the transition. I present the calibration of my benchmark economy in the following order: demographics and labor income distribution, technology, and preferences and market arrangements.

I need to perform the calibration of parameters associated with preferences $(\beta, \theta, \sigma, \varepsilon)$, the income tax rate (τ_y), the downpayment fraction (γ_t) in the initial steady state, and parameters associated with technology $(A_c, A_c, \alpha_c, \alpha_s, \phi)$ as well as depreciation rates $(\delta_h, \delta_f, \delta_k)$. The distribution of productivity levels within and across generations also needs to be specified. The U.S. economy in 1994 is the target for the calibration of the initial steady state.

3.1 Demographics and Labor Income Distribution

A model period is taken to correspond to one full year. Individuals are assumed to live for $J = 60$ model periods. One can think of members of a new generation as being born at real-life age 24 (model period one) and having an average life expectancy of 83 years (60 model periods). The retirement age is set at age 63 (model period 40).

The distribution of productivity levels (z^j) governs labor income. I calibrate this process using the CPS survey for 1994. More precisely, I calculate the mean labor income for each quintile in the data at each age and assign this value directly to the five individuals making up the population in the model. The normalized labor income profile for each individual, each representing a quintile, is shown in Figure 6.

The retirement age is obtained using the same data. The median labor income for the entire population becomes zero at age 63 (model age 40). When individuals become retired they start to draw their pensions collected from taxes on income over their working life.

3.2 Technology

I need to construct measures of output by the consumption sector and by the residential sector, capital, the stock of houses and their investment counterparts $(Y^c, p^h Y^h, K, p^h(H + F), X^k, p^h(X^h + X^f))$. I use data from the National Income and Product Accounts (henceforth NIPA) and the Fixed Assets Tables (henceforth FAT), both from the Bureau of Economic Analysis. Capital is defined as the sum of non-residential private fixed assets plus the stock of inventories plus consumer durables. Investment in capital, X^k, is defined accordingly.[6] $p^h(H + F)$ is private residential stock in the data and $p^h(X^h + X^f)$ is private residential investment. Output in the consumption sector is defined as labor income plus income from non-residential capital, $Y^c = F(K^c, N^c) = wN^c + rK^c = C + I^k + G$, output in the residential sector is labor income plus income from non-residential capital plus land income $Y^h = F(X^s, L^c) =$

[6] I include net exports in my measure of capital investment since the benchmark economy is a closed economy.

$wN^s + rK^s + p^l L = I^h + I^f$, and total output is defined as the sum of output from each final sector, $Y = Y^c + p^h Y^h$, or measured GDP minus imputed housing services.[7] I do not make any imputation to output for government owned capital since my focus is only on privately held wealth.

The business capital share for the residential structures sector is $\alpha_s = 0.132$, and I take this value from Davis and Heathcote (2005). I also take from Davis and Heathcote (2005) the land share in new housing, $\phi = 0.106$. Calculations in Davis and Heathcote (2005) are in the same context as the model presented here.

I proceed as Cooley and Prescott (1995) and calculate the implied share of capital in output in the consumption sector, which is $\alpha_c = 0.26$. The capital-output ratio (K/Y) is 1.66 and the housing-output ratio $((p^h(H+F))/Y)$ is 1.07.[8] I set the depreciation rate of capital, δ_k, so that it matches the investment to capital ratio in NIPA, $\delta_k = 0.12$. The implied steady state interest rate is 3.4 percent, in line with similar calibrations presented by, for example, Díaz and Luengo-Prado (2008).

The value of the implied capital share in the consumption sector may seem low, but it is not very different from typical values in the literature when given as a function of GDP instead of output. GDP is output plus the imputed value of housing services: $GDP = Y + (r + \delta_h)p^h H + r^f p^h F$. The capital-GDP ratio (K/GDP) is 1.53, the housing-GDP ratio $(Kp^h(H+F))/GDP$ is 0.98, and the aggregate ratio $(K + p^h(H+F))/GDP$ is 2.51. The resulting share of capital income to GDP is 31.52 percent, only slightly lower than the value estimated by Prescott (1986).

The technology level for consumption sector (A_c) and residential structures sector (A_s) is such that $A_s < A_c$ as suggested by evidence. I set $A_c = 2$ and $A_s = 0.9$ to replicate the aggregate ratio $(K + p^h(H+F))/GDP = 2.51$ and the housing-output ratio of 1.07.

The minimum house size is such that the homeownership rate in the economy is 64 percent as in the U.S. economy in 1994.[9] With minimum size equal to 1.4775 for owner-occupied houses and given a downpayment fraction of 20 percent, the model replicates the above homeownership rate for the U.S..

I borrow the values for the depreciation rates from Díaz and Luengo-Prado (2010), given that my benchmark economy is a closed economy and those values are consistent with general

[7] C is output in the consumption sector minus the sum of investment in physical capital and government expenditures.

[8] All figures I report are averages in NIPA/FAT for the sample period 1954-1994.

[9] Data from United States Statistical Abstract and Housing Vacancies and Homeownership (CPS/HVS) for 1994.

equilibrium. The values are $\delta_h = 0.0424$, $\delta_f = 0.0483$, and $\delta_k = 0.12$. I also borrow an income tax rate τ_y equal to 0.2 from Díaz and Luengo-Prado (2008).

3.3 Preferences and Market Arrangements

The value of the discount factor (β) is chosen to make the capital-output ratio equal to 1.66. It should be noted that capital refers to the total amount of capital which includes housing and business capital, and that output corresponds to the sum of output goods and the value of housing services. The discount factor which achieves the desired capital-output ratio is 0.9757.

The share of housing services in total expenditures is controlled by θ. I set this parameter in order to replicate the ratio $p^h(H + F)/C$ in the U.S. economy. In the data this ratio is equal to 1.4, and the θ that matches it is 0.0765. With this value for θ I also replicate the ratio $p^h(H + F)/GDP$ in the U.S. of 0.98.

I set the risk aversion parameter in the utility function at $\sigma = 0.5$, since this is the value usually employed in the business cycle literature. This parameter determines the inter-temporal elasticity of substitution. The last parameter referred to preferences is ε. This value determines the intra-temporal elasticity of substitution between consumption goods and housing services. Most of the literature uses a Cobb-Douglas utility function in order to reconcile the differences found in the literature for this parameter above and below one.[10] Recent studies that rely on structural analysis in order to estimate those parameters suggest that housing services and consumption goods are complements. See, for example, Bajari et al. (2010) or Li et al. (2009). So I set this parameter at a value slightly smaller than one and choose a conservative value of $\varepsilon = 0.9$ as in Gete (2010).

I require a minimum downpayment of 20 percent for the initial steady state. Thus individuals can borrow up to 80 percent of the value of the house. While in reality households may be able to acquire houses with lower downpayment, it is also the case that these households face higher marginal borrowing costs (including a higher interest rate and the purchase of mortgage insurance). To keep the model tractable, the downpayment parameter is the same for all consumers and the borrowing rate is not a function of γ_t.

All parameter values for the benchmark calibration are summarized in Table 3.

[10] See Fernandez-Villaverde and Krueger (2011) for a discussion.

4 Results

4.1 The Benchmark Economy

The typical behavior of individuals in the initial steady state is similar to the one in Gervais (2002). I also break it down into three categories of individuals: poor and lower middle class (first and second quintiles), middle-class (third quintile) and upper-middle class (fourth and fifth quintiles). The behavior of poor and lower middle-class individuals is quite simple: they rarely own their home, they set up cooperatives[11] in which a small proportion of households (around 17-20% in the first quintile and 50-55% in the second) own the house in which they live, and always consume a small amount of housing services. Middle-class individuals initially consume small amounts of housing services and consumption goods in order to save enough to eventually become homeowners. When they become homeowners, at age 33, they move into the smallest possible house. At that time, these individuals are constrained both by the downpayment and the minimum house size constraints. During their first few years as homeowners, they use their extra income both to increase consumption and pay off their mortgages. As they get wealthier, they eventually move into bigger houses.

The consumption level of young upper middle-class individuals is also constrained as they accumulate wealth to cover the downpayment on a house. After 2-3 years of accumulating wealth, at age 29-30, they move into the largest house their downpayment allows. Unlike lower middle-class individuals, they are not constrained by the minimum house size: they keep living in the largest house their downpayment allows for 2-3 years. After this constrained period, they increase housing services and consumption.

All individuals accumulate assets during their productive years which they deplete to provide for consumption once retired instead of having a retirement pension. Homeowners thus partially revert to debt financing their house (rather than holding the entire asset as equity) in order to consume goods as well as housing services during the last few periods of their life. Without uncertainty, all individuals die with zero net worth.

4.2 Experiment

I evaluate the behavior of the model after a permanent and unanticipated shock to the international interest rate and to the downpayment requirement to buy a house. I compute the transition of the model and analyze the joint effect of these two shocks happening once and

[11] Detailed explanation in Appendix A.

for all. Since the model is calibrated to the U.S. economy, the size of the shocks is taken from data for this country.

The initial steady state is a closed economy where the trade balance and borrowing from abroad are equal to zero. The final steady state that the economy reaches is then computed, where the country is borrowing from abroad and paying debt interests by exporting goods. At this stage, the economy is holding a sustainable level of debt. Thereafter, I compute the transition between these two points and look at the behavior of different variables. A detailed explanation of the computational procedure can be found in Appendix A.

I lower the shock for the international interest rate from the value obtained in the initial calibration to a new value of 0.6 percentage points smaller. This value is the mean of the annual decreases in the federal founds effective rate of the U.S. between 1990 and 2003.[12,13]

For the downpayment requirement, the shock is the mean of the annual decreases in the U.S. between 1994 and 2003. This will imply a fall from its initial steady state value from 0.2 to 0.18. The magnitude of the change is in line with the evidence presented in the introduction.

Since both shocks happen once and for all, the boom period in the model corresponds to the first period of the transition. Also, given that the size of the shocks comes from the annual average variation, my quantitative targets for this experiment are the annual average changes observed in data over the boom period.

4.2.1 The Joint Effect

In this section I investigate the effect of both shocks taking place at the same time. Figure 7 shows how both shocks occur once and for all.

Both shocks drive up the demand for housing services - at both margins. The interest rate decrease makes, on the one hand, mortgages[14] cheaper, and, on the other hand, makes it more attractive to save in a house than in financial assets. The downpayment shock allows easier access to the housing market for households that would otherwise rent, and makes it possible for previous owners to buy bigger houses.

[12] Kiyotaki et al. (2011) set a shock of 1 percentage point decrease in the international interest rate with no particular focus in any interest rate series for the U.S. Garriga et al. (2012) do their own calculations and choose a path for the world interest rate that would be equivalent to an average annual decrease of 0.3 percentage points from 1990 to 2004. In the case of Ferrero (2011), the decrease found in the data for the long-term real interest rate would be equivalent to 0.8 percentage points between 2000 and 2004.

[13] I choose the federal founds effective rate that being the reference rate.

[14] The model presented can be rewritten to include a simple mortgage into the definition of financial assets. See Gervais (2002).

The first result is plotted in Figure 8 and shows that the model replicates the evolution of housing prices and the current account balance. The model generates a housing prices boom in the period of the shock of 3.4% together with a current account deficit. The mean annual increase between mid-90s and 2007 in the U.S. data was also 3.4%. The model predicts a large current account deficit (around a 10.97% of GDP) compared with the size of the shocks in the period of the boom since there is no adjustment cost and all variables move freely after both shocks. The bust in housing prices after the first period of the transition happens together with a reduction in the current account deficit, but without turning into a surplus.

Figure 9 shows how the model is able to replicate the behavior of the extensive margin increase. The increase in the homeownership rate in the boom period is 0.79%, in line with the average annual increase of the homeownership in the U.S. economy since 1994 (0.5%). It is interesting to note that after the boom in the homeownership rate, there is also a decrease in this variable during some of the bust periods. This happens because there are two effects at work after both shocks: first, houses are valued more highly as collateral; and second, houses are more expensive. Households at the margin decide to invest in housing when prices go up because they can use their houses as collateral. After the boom period, the first effect starts to drive down the homeownership rate (the value of the collateral starts to decrease), but after some periods housing prices start to fall and it is this second effect (houses becoming cheaper as time goes by) that drives homeownership up again. Notice that this happens without a reversal in the shocks, especially without a reversal in the downpayment requirement to buy a house.

The housing stock increases around 1.7% in the period of he boom and it is still increasing, at a smaller rate, after some periods (0.5% in the first year after the boom period). In the U.S. economy the average annual growth rate of net residential stock was 2.6% between 1994 and 2007; in the bust period (2008-2012) this rate fell to 0.18% according to the data.[15]

The increase in non-housing consumption is due to two reasons: the increase in the value of the collateral, and the decrease in the interest rate. As it can be seen, the increase in non-housing consumption is consistent with the average annual increase in personal consumption expenditures for the U.S. economy, and is around a 3%.[16] The overshooting in non-housing consumption is related to the small open economy assumption, but the quantitative response in the boom period of non-housing consumption is also determined by the increase in the value of the collateral. Thus, after the boom in non-housing consumption there is a bust because of two reasons: the debt must be repaid, since it is a small open economy borrowing from

[15] Data from the Fixed Assets Tables, from the Bureau of Economic Analysis.

[16] Data for Personal Consumption Expenditures in NIPA tables.

abroad in the period of the boom; and, because of a decrease in the value of the collateral, since housing prices start to decrease after the boom in the first period.

Figure 10 represents the ability of an open economy model to replicate the pattern observed in the data. An open economy moves resources from the tradable to the non-tradable sector (labor in the construction sector), thus increasing the production of the non-tradable good (housing production). Since there is an increase in the demand for non-housing consumption, the economy has to satisfy this increase in the demand by running a trade deficit. Gete (2010) also points out this result after a positive shock to the demand for housing.

In Figure 11, I show the decomposition of housing prices together with the rent-to-price ratio. As it can be seen, the price of structures increases once and for all. This is due to a converse result of the Stolper-Samuelson theorem. After the decrease in the interest rate, the capital-labor ratio of the economy increases, causing wages to rise. Since structures are more labor intensive compared to the consumption sector, this increase in wages drives up the price of structures. Land prices go up more in the period of the shock, and then start to fall. The pressure on land is grater in the period of the shock and then decreases gradually over time. This effect brings about house price changes that follow the same pattern. The fact that there is a new fixed amount of land available for construction in each period (as in Davis and Heathcote (2007)) makes housing prices demand driven.

The rent-to-price ratio decreases in the period of the boom and then increases following the non-arbitrage condition of the financial institutions. The change in the rent-to-price ratio is in line with what we observe in the data during the boom period. However, during the bust the rent-to-price ratio goes in the opposite direction than borne out in the data. The intuition behind this result arises from the non-arbitrage condition for the rental price of financial institutions. The non-arbitrage condition is a function of the international interest rate and housing prices. After the decrease in the international interest rate the rent-to-price ratio collapses, but afterwards the evolution of housing prices governs the evolution of the rent-to-price ratio.

It is important to note that the bust happens neither with a reversal in the interest rate after the boom period, nor with a reversal in the downpayment requirement. As the data show, interest rates remained low during the bust period, but also the downpayment requirement to buy a house was reversed throughout the bust years (see Favilukis et al. (2013)). A branch of the literature pointed out how the increasing difficulties to borrow by households could have driven a bust in credit that precipitated the Great Recession of 2007-08 (see Huo and Rıos-Rull (2014)), a mechanism of which this model abstracts. The housing bust in this model does take place but is quantitatively at odds with the US data. As for homeownership rate and the price of structures over the bust period the model does not account for the data in the U.S. However, a reversal in the downpayment requirement and a recession of the economy after the boom (with low interest rates maintained) would worsen the bust predictions of the model for housing variables and would predict a decrease in the homeownership rate and the price of structures during the bust in line with the data.

Figure 12 shows what happens to all the housing variables taken together and how the model is able to generate a boom and a bust in the housing market. The housing stock shows the biggest increase in the boom period, thereby increasing the pressure on land and increasing housing prices, and as a result, the economy moves labor to the production of structures in this period. After the boom period, the pressure on land starts to ease off, thus decreasing housing prices and the production of new houses in the economy. It is at this stage that the bust hits the economy and this is the mechanism behind the overshooting in housing prices. The increase in housing production, i.e. investment in housing, puts pressure on land prices replicating the overshooting in this variable. For this reason housing prices drop to a level above the initial steady state. The investment in housing, in a steady state with a lower interest rate and downpayment, is bigger than the investment in housing at the initial steady state. Once all new houses have been produced, the depreciation of a bigger stock of housing must be built in a plot of land of the same size each period.

Finally, Figure 13 shows how, during the bust period, there is a reversal in the trend of the current account balance but a current account deficit still remains. In the period of the shocks the model predicts a big current account deficit, and a big deficit (around 2.7% of GDP) still exists in the first period of the bust and during some periods ahead. The net foreign asset position is deteriorating until the final steady state is reached. Moreover, and consistent with the evidence for this period, the bust happens without a reversal in interest rates.

5 Conclusions

In this paper, I have studied the transition of a life-cycle heterogeneous agents small open economy model in which the economy goes to a new steady state with lower interest rates and a lower downpayment requirement to buy a house.

The model is able to replicate some important facts of the U.S. economy between the mid-90s and 2007 such as the boom in non-housing consumption, the increase in the extensive margin demand for housing in this period, and the current account deficit together with borrowing from abroad during the transition. Two characteristics of the model contribute to successfully replicating those facts: one is that demand drives up housing prices in the economy, and the other is the life-cycle heterogeneous agents structure. Households want to build up their desired stock of housing and this pushes up housing prices. The model is also able to replicate (qualitatively) the bust period after 2007. The bust in housing prices occurs when there is a fall in demand. The degree of heterogeneity in the economy allows some households to access the housing because of the decrease in the downpayment requirement to buy a house. The decreasing interest rates allow households to get cheaper credit for consumption due to the increasing market value of their homes.

The model generates a surge of housing and non-housing consumption at the beginning of the transition. Everybody prefers an expensive house in order to have access to credit. When the pressure of new housing investment on land decreases, the value of houses drops, debt must be repaid, and a bust takes place in the housing market. Therefore, the current account deficit decreases with low interest rates maintained, as it seems to be case for the U.S. after the bust in housing markets. It is true that the downpayment requirement in the U.S. after the bust in housing markets became higher, but given the intuition of the model, this positive shock to the downpayment requirement would imply a sharper bust in the housing variables presented here even with low interest rates.

Table 1: Loan to Value Ratio (LTV).

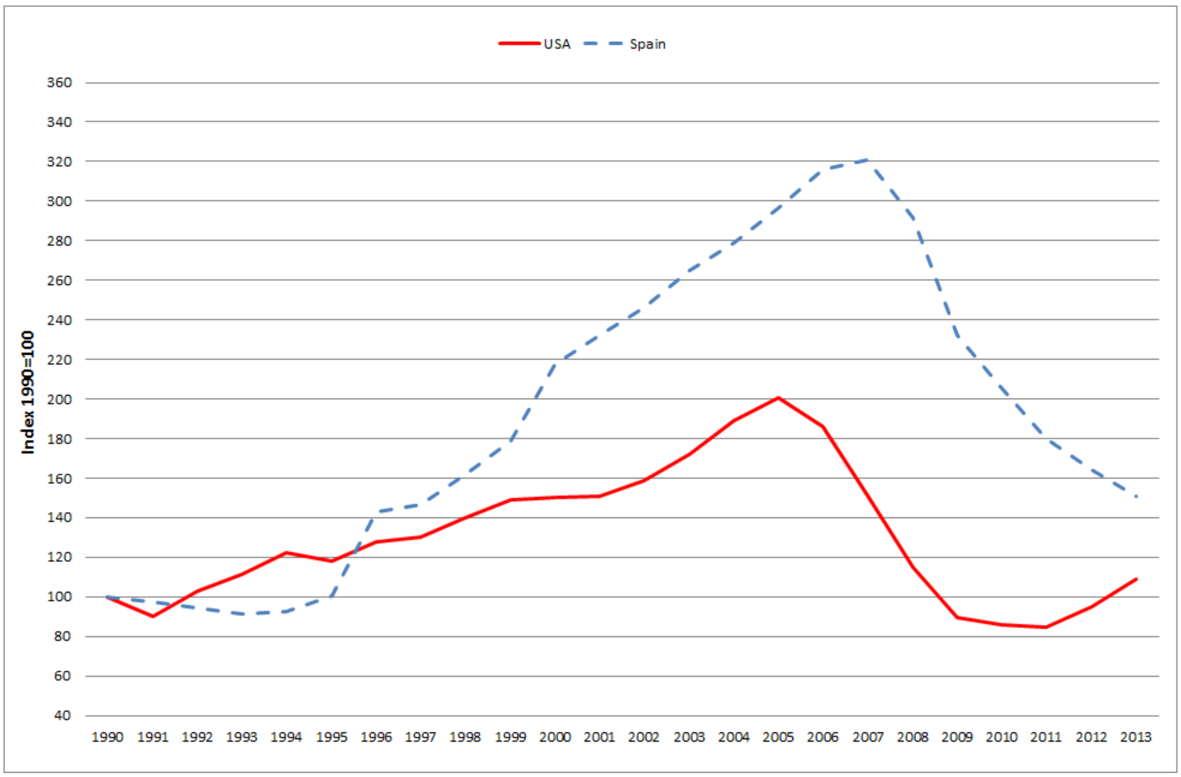
Year	LTV
1995	78.4
1998	86.2
2003	94.4

Source: Duca et al. (2011).

Table 2: Model Parameters.

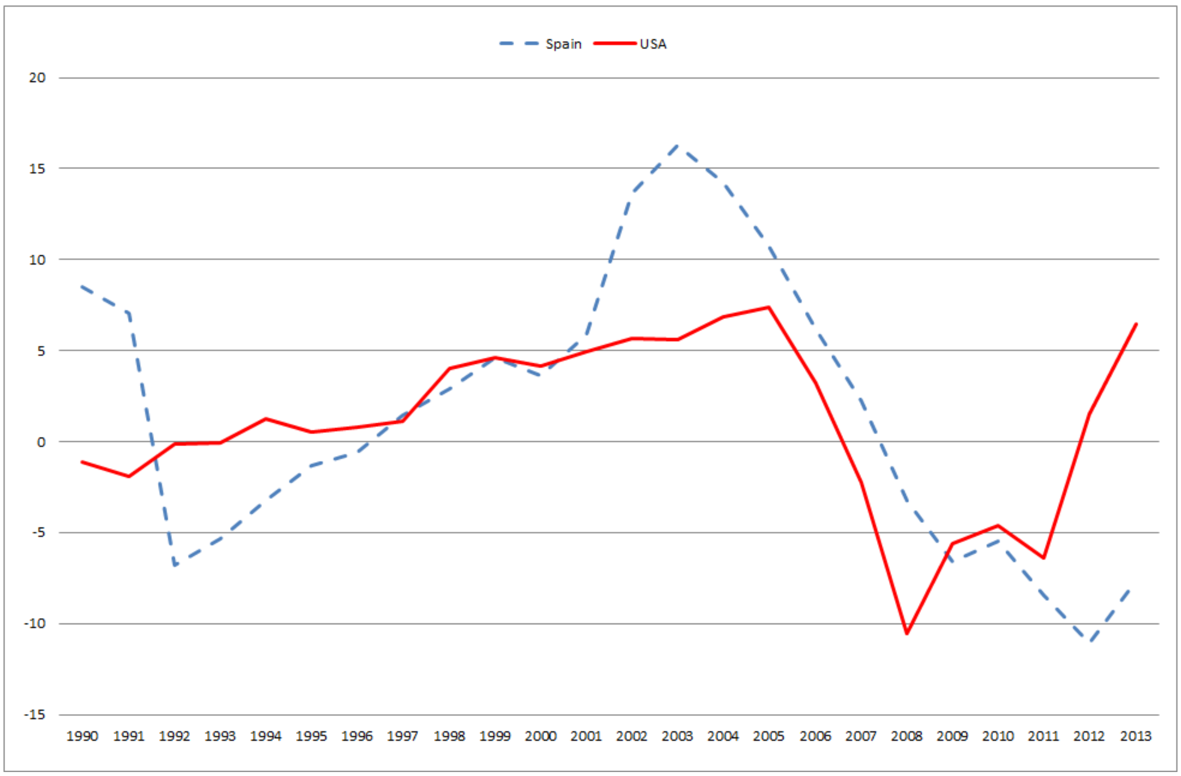
Parameter	Value
β	0.9757
θ	0.0765
σ	0.5
ε	0.9
γ	0.2
$\underline{\mathrm{h}}$	1.4775
A_c	2
A_s	0.9
α_c	0.2616
α_s	0.132
ϕ	0.106
δ_k	0.117
δ_h	0.0424
δ_f	0.0483
τ_y	0.2

Figure 1: Residential Investment.



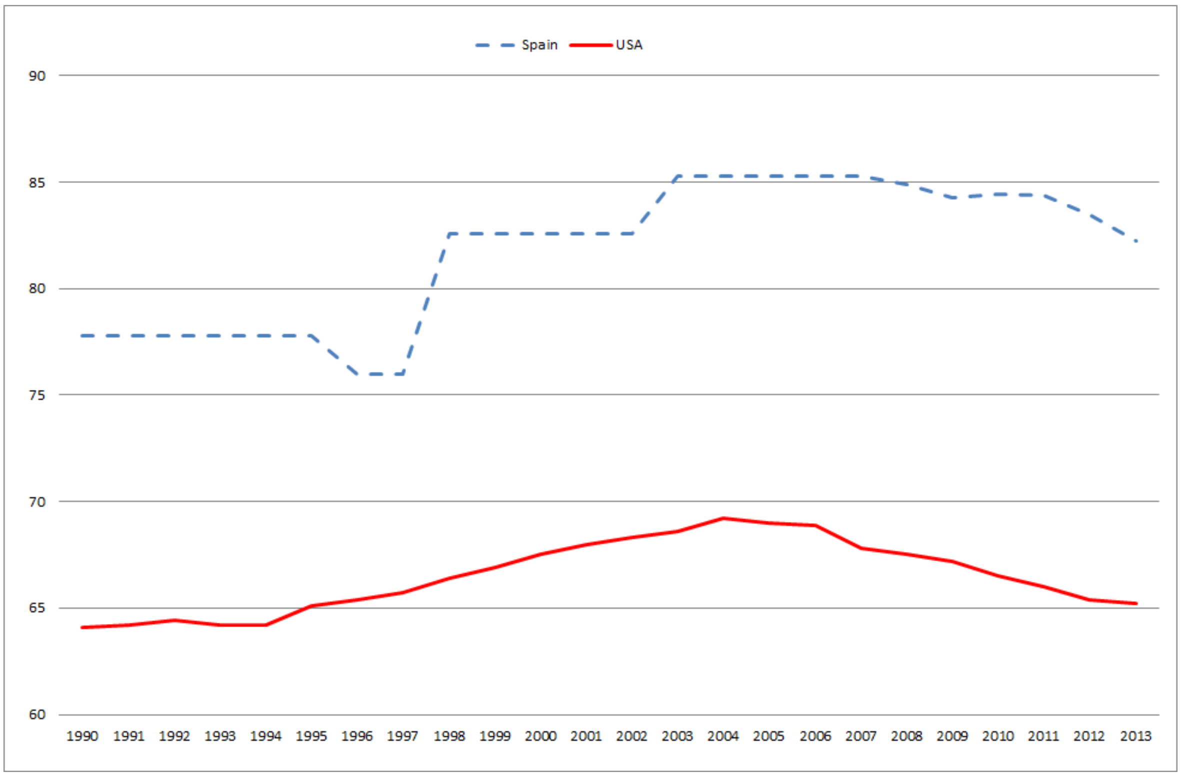
Source: OECD, Eurostat.

Figure 2: Real House Prices (% Change).



Source: OECD.

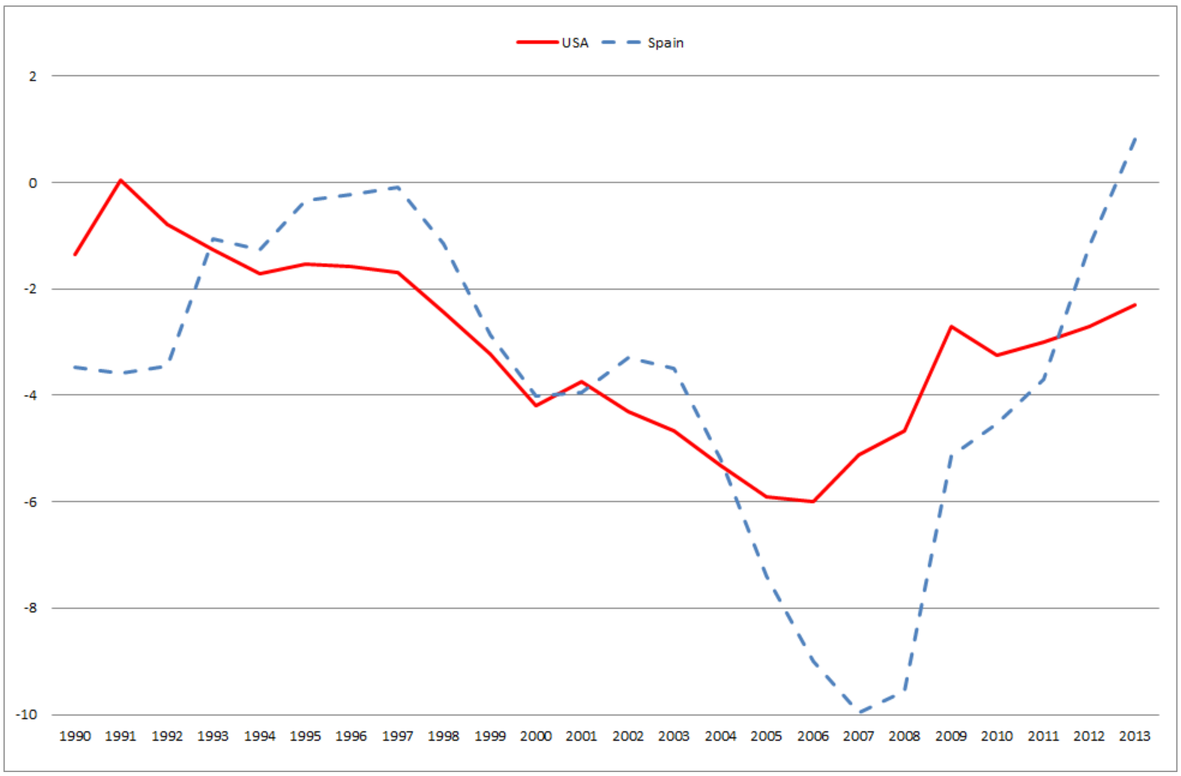
Figure 3: Homeownership Rates.



Source: USA, US Census Bureau. Spain, OECD, Banco de España, and Eurostat.

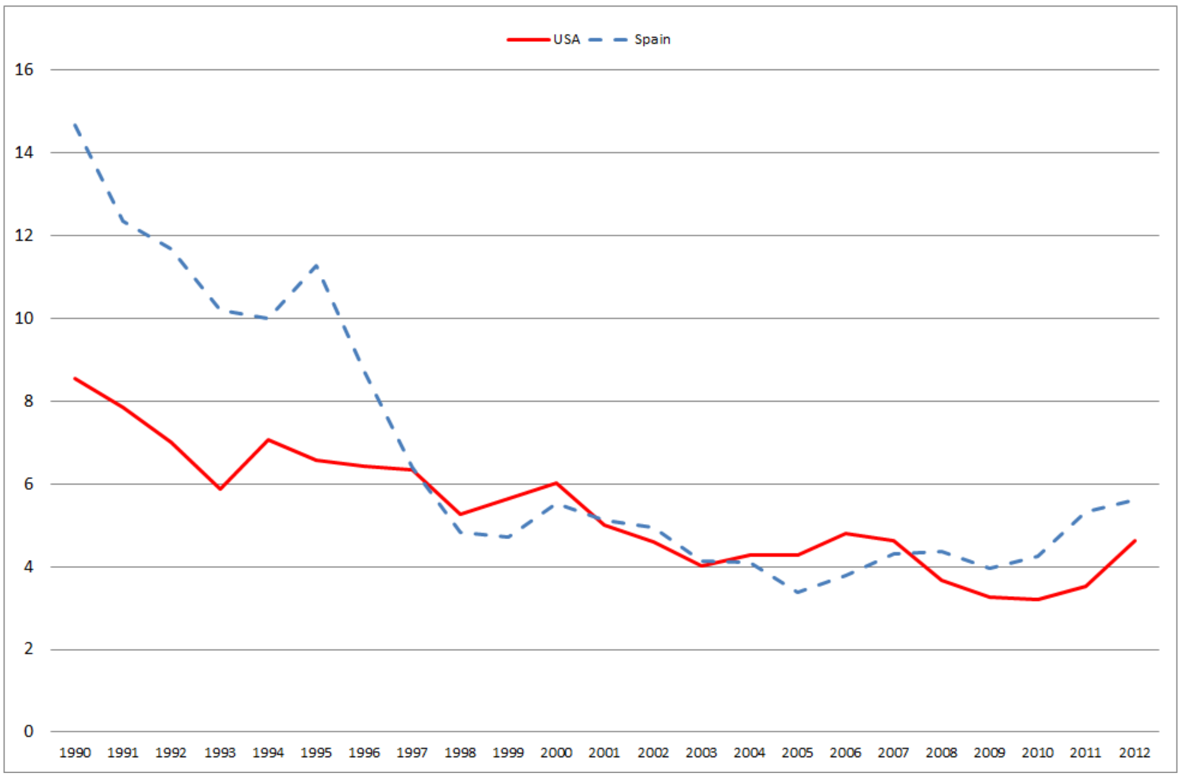
Note: Spanish homeownership rate from data for 1990 (77.8), 1996 (76), 1998 (82.57), 2003 (85.3), and applying Eurostat growth rates from 2007 up ahead.

Figure 4: Current Account (% GDP).



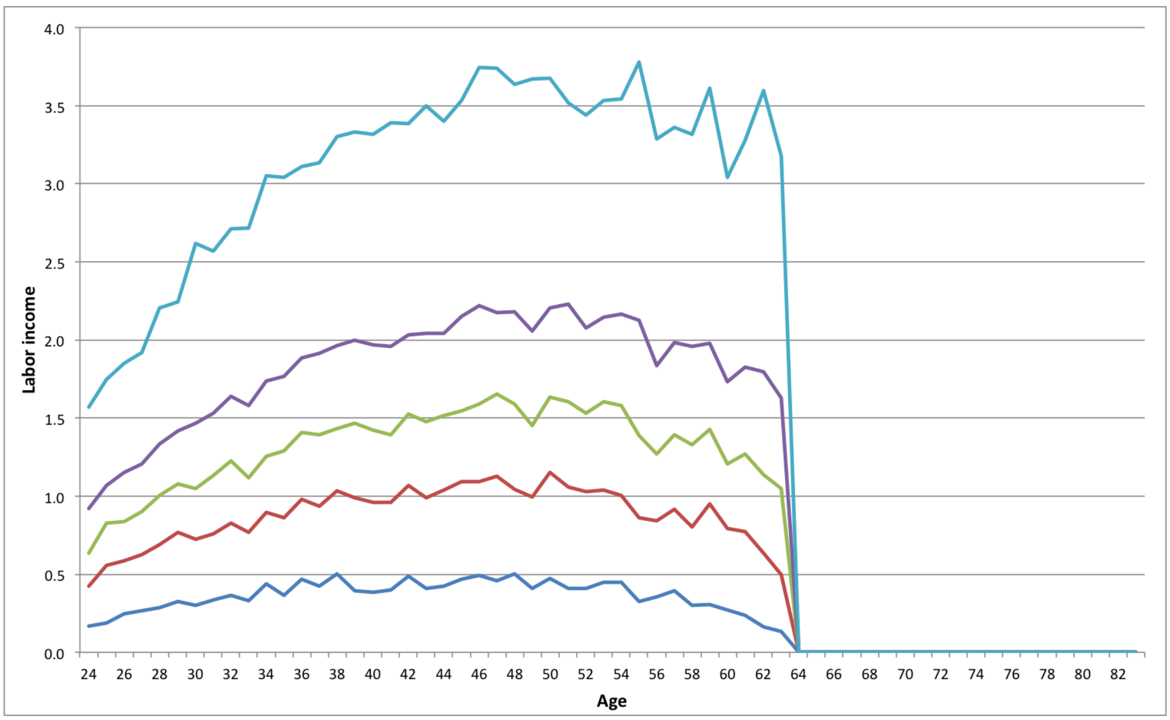
Source: OECD.

Figure 5: Long-term Interest Rate on Government Bonds.



Source: OECD.

Figure 6: Labor Income.



Labor income process for each quintile.

Source: Current Population Survey (CPS).

Figure 7: Interest Rate and Downpayment Shock.

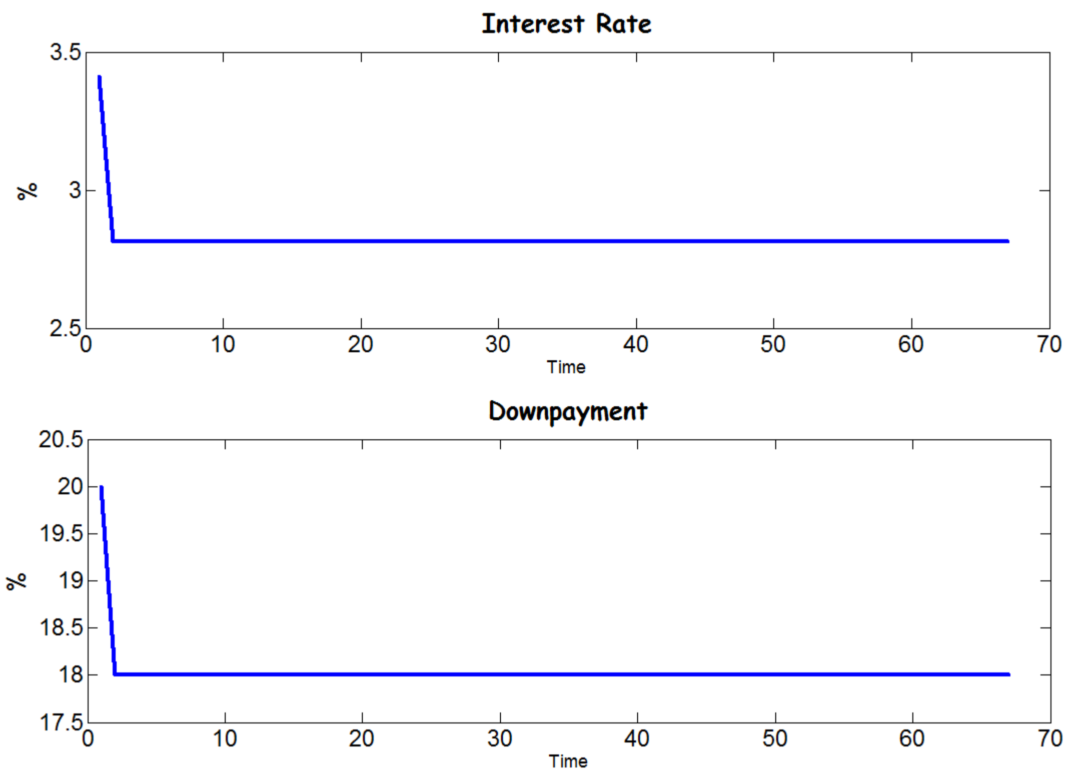


Figure 8: Negative Correlation between House Prices and Current Account Balance.

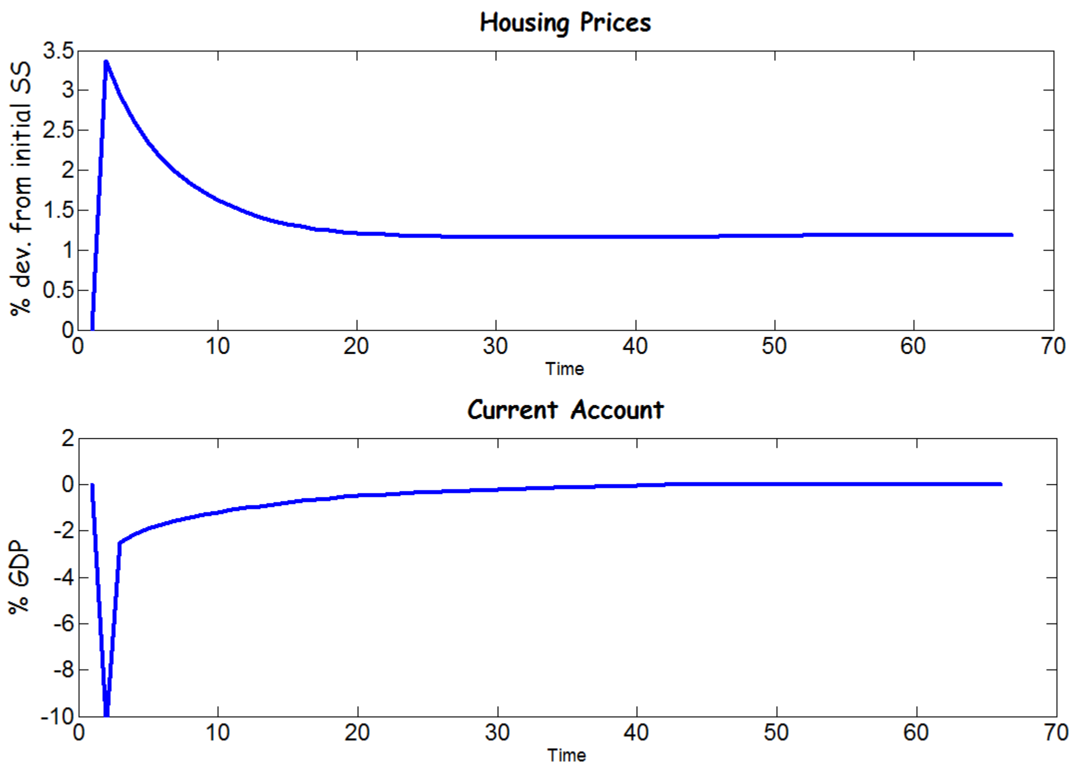


Figure 9: Extensive Margin and Non-Housing Consumption.

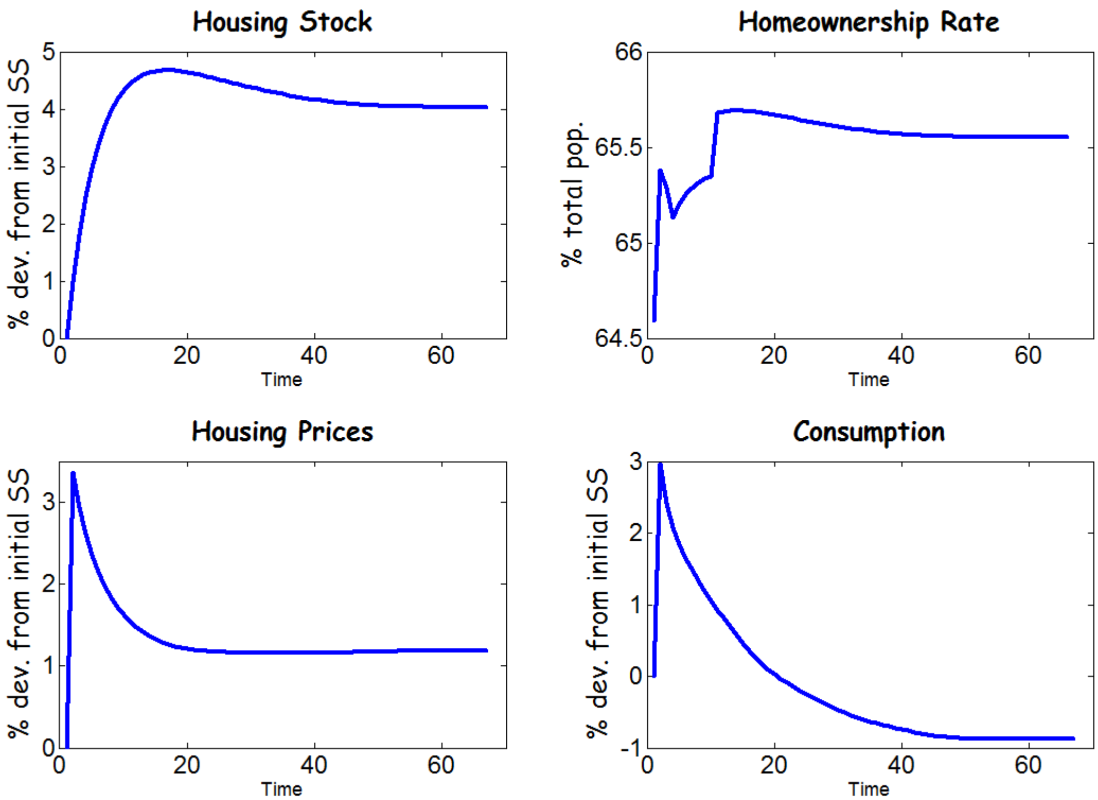


Figure 10: International Interest Rate Shock and the Housing Boom.

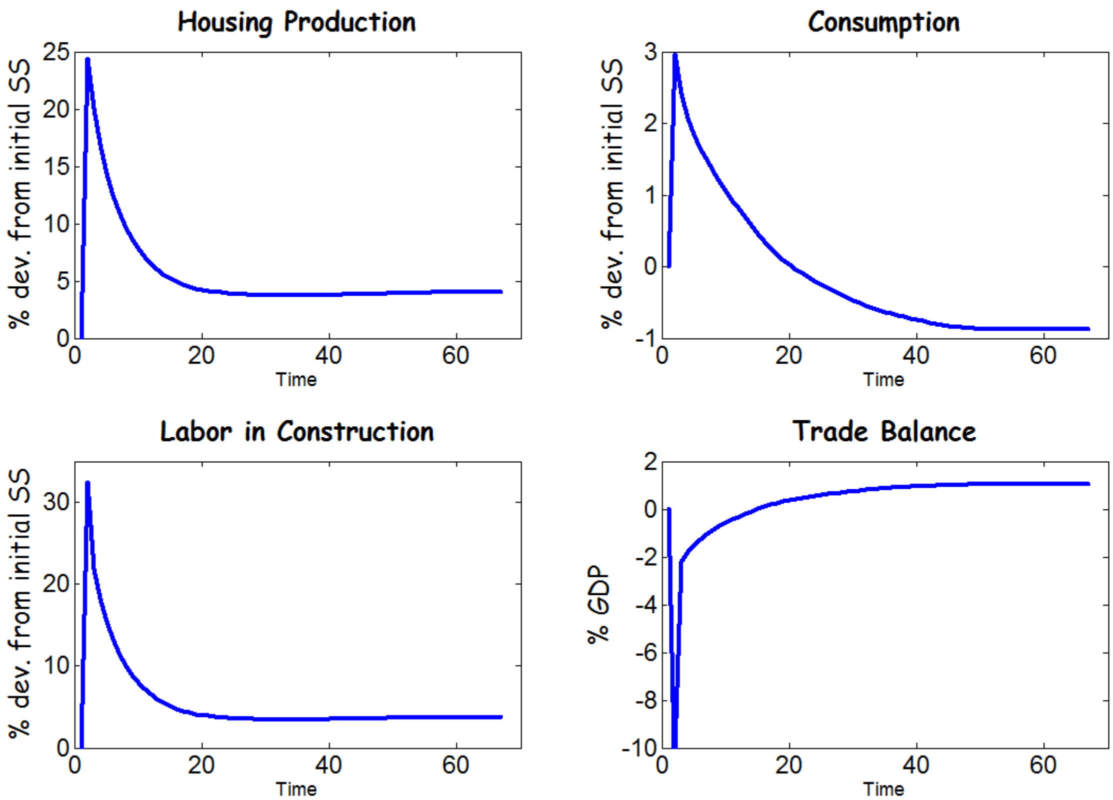


Figure 11: Housing Prices and Land Price.

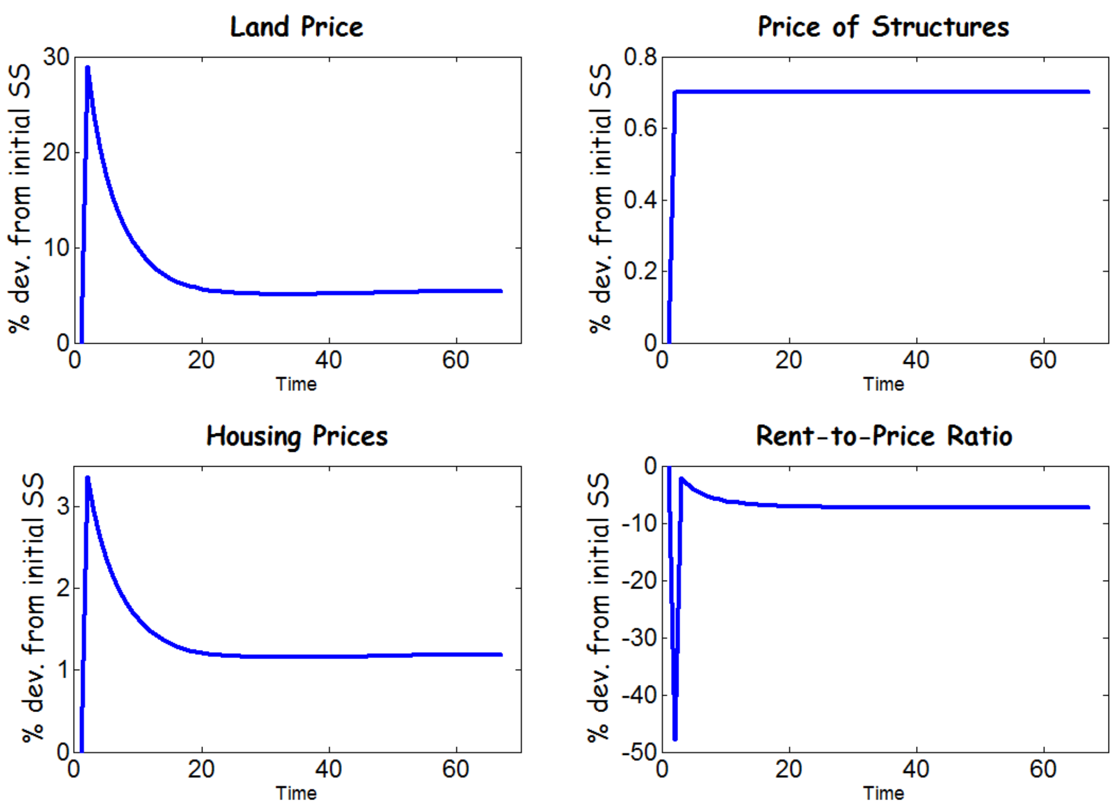


Figure 12: Housing Market.

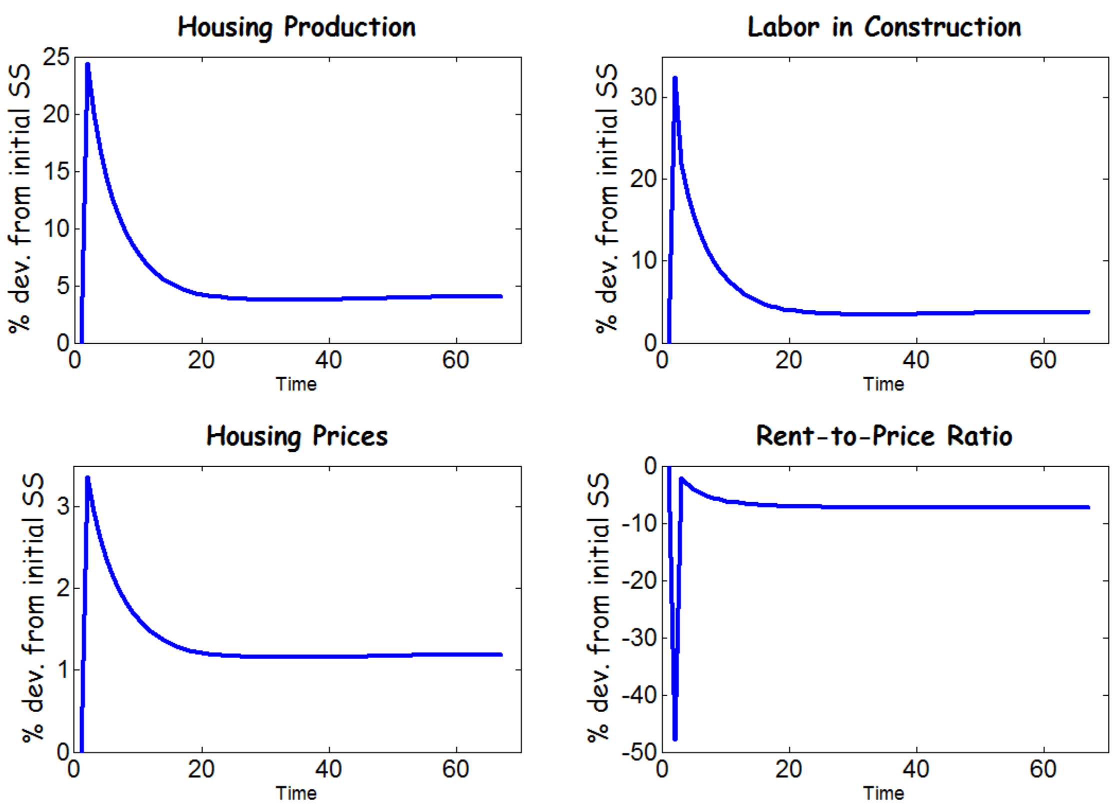
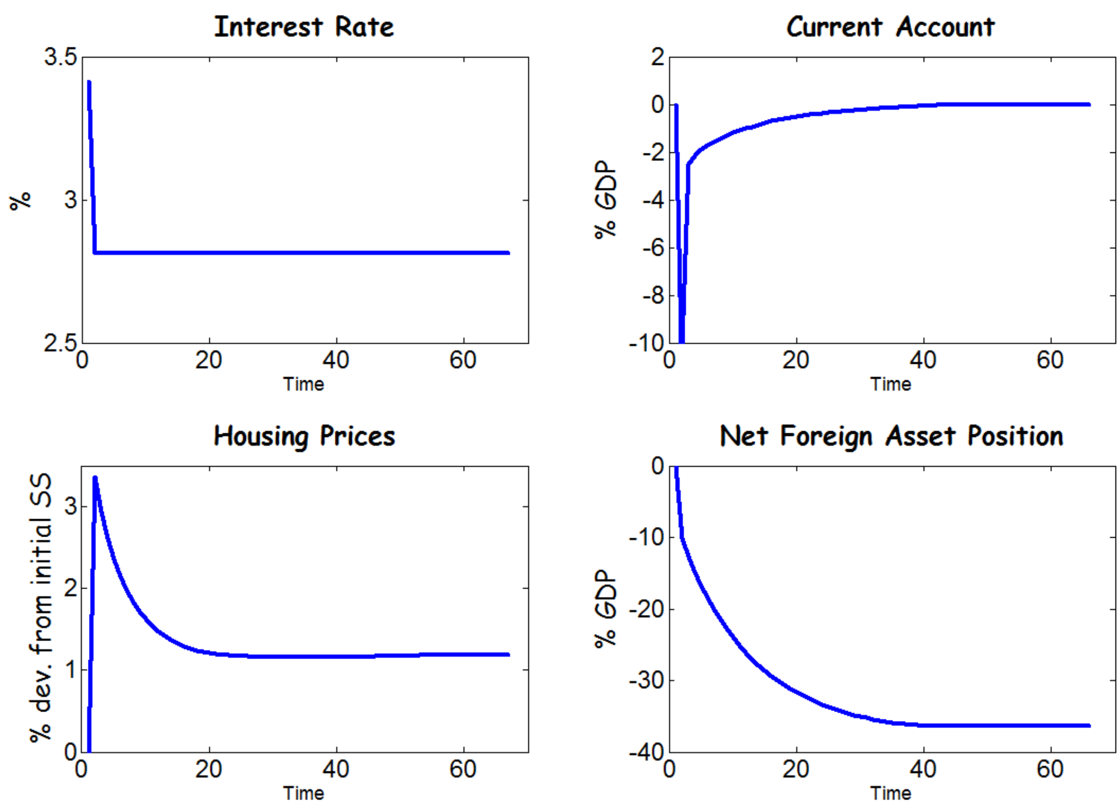


Figure 13: Housing Boom and International Interest Rates.



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Appendix A: Computational Procedures

The first thing I do is to calculate the initial steady state and the final one. The computational procedure for the transition is based on Heer and Maussner (2008).

Households Own Land

The budget constraint is reduced with some assumptions. The first one is relative to the amount of land households hold in the economy. I assume that households have an amount of the total land in the economy proportional to their housing stock, and they receive this amount of land exogenously each period. To have an intuition of this assumption think of the steady state. In each period households have to cover the amount of housing capital depreciated and construct it on the new plot of land that they receive in that period. So, rents from land are received by households in a proportional amount to their housing stock. This assumption makes it possible to have land in the economy owned by individuals.

I make use of the Cobb-Douglas production function to derive the relationship between land and the housing stock. From the marginal product of land I get:

$$p_t^l l(h_t, h_{t+1}) = \phi p_t^h (h_{t+1} - (1 - \delta_h) h_t)$$

Using this expression the budget constraint becomes:

$$c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h (1 - \phi) h_{t+1} \leq$$

$$\leq z^i (1 - \tau_y) w_t + b_t^{j>j^*} + (1 + (1 - \tau_y) r_t^a) a_t + (1 - \delta_h) p_t^h (1 - \phi) h_t$$

Now, I add the term $p_{t-1}^h (1 - \phi) h_t$ to both sides of the budget constraint. Rearranging I get:

$$c_t + r_t^f p_t^h f_t + a_{t+1} + p_t^h (1 - \phi) h_{t+1} \leq$$

$$\leq z^i (1 - \tau_y) w_t + b_t^{j>j^*} + a_t + p_{t-1}^h (1 - \phi) h_t + (1 - \tau_y) r_t^a a_t - \delta_h p_t^h (1 - \phi) h_t + (p_t^h - p_{t-1}^h)(1 - \phi) h_t$$

Voluntary Equity

In order to compute the equilibrium of the model, it is convenient to reformulate the household problem. Define voluntary equity as the wealth held less the proportional amount of land, $y_t^j \equiv a_t^j + p_{t-1}^h(1-\phi)h_t^j$. So:

$$c_t + r_t^f p_t^h f_t + y_{t+1} \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t - \delta_h p_t^h(1-\phi)h_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

Where the term $(p_t^h - p_{t-1}^h)(1-\phi)h_t$ refers to the capital gains made by a household because of a change in housing prices from one period to the next.

The Borrowing Constraint

The borrowing constraint is:

$$a_{t+1} \geq -(1-\gamma_t)p_t^h h_{t+1}$$

which, making use of the definition for voluntary equity, can be written in the following way:

$$y_{t+1} + p_t^h \phi h_{t+1} \geq \gamma_t p_t^h h_{t+1}$$

becoming a constraint on next period's net worth:

$$y_{t+1} \geq (\gamma_t - \phi)p_t^h h_{t+1}$$

Solution Method

I closely follow the solution method in Gervais (2002). The state variables for the household problem are the earnings process, the quintile to which households belong, and voluntary equity, $\{z^j, i, y_t^j\}$. With this reformulation, I deal with one state variable. This greatly simplifies the problem imposed by the endogenous liquidity constraint in the solution of the household problem.

The household problem can be broken down into intra-period and inter-temporal decisions. The inter-temporal decision consists of choosing the amount of savings to carry over to the next period. Once the inter-temporal decision is made, households choose the amount of composite goods and housing services to consume during the current period, as well as the composition of savings carried over from the previous period.

This possibility derives from the fact that only one state variable is needed to describe the situation of an age-j individual with productivity level z_j. This state variable is today's net worth or, alternatively, yesterday's savings. Without uncertainty, the make up of the composition of today's savings is irrelevant since the same composition will be the result whether the decision is made today or tomorrow. Hence, the only information needed as an individual enters a period is the total amount of savings carried over from the previous period, as opposed to its split between financial assets and housing. In other words, today's amount of savings is chosen knowing that its composition will be optimized tomorrow.

These points become obvious in the following recursive formulation of the households' problem. Let $v_t^j\left(y_t;i\right)$ denote the value of behaving optimally from period j until period J for an individual who enters period j with net worth y_t, productivity level z, and belonging to quintile i, in each period of time t. Given a net worth position at age j; a household chooses the next period's net worth to maximize total future discounted utility. The value function of an age-j individual is defined as:

$$v_t^j\left(y_t;i\right) \equiv \max_{\left\{y_{t+1}^{j+1}\in\Gamma\right\}}\left\{G^j(y_t,y_{t+1};i)+\beta v_{t+1}^{j+1}\left(y_{t+1};i\right)\right\}$$

where Γ is the feasible set from which tomorrow's net worth is chosen. The return function of an age-j individual, G^j; is defined as the maximum utility level a household can achieve given today's and tomorrow's level of net worth. In other words, the return function is the one which solves the following intra-temporal problem:

$$G^j(y_t,y_{t+1};i) \equiv \max_{\left\{c_t^j,x_t^j,f_t^j,h_t^j,a_t^j\right\}}\left\{u(c_t,x_t)\right\}$$

$$s.t. \quad c_t + r_t^f p_t^h f_t + y_{t+1} + \delta_h p_t^h(1-\phi)h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

$$y_t = a_t + p_{t-1}^h(1-\phi)h_t$$

$$x_t = f_t + h_t$$

$$y_t \geq (\gamma_{t-1} - \phi) p^h_{t-1} h_t$$

$$h_t \geq \underline{\mathrm{h}} \ otherwise \ h_t = 0$$

For the results presented in this paper, I use 200 grid points for voluntary equity and linear interpolation in order to get more accuracy (the grid points are not equally spaced to maximize efficiency). Households in each quintile are born with zero financial assets ($a^1_t = 0 \ \ \forall i, \forall t$) and zero housing stock ($h^1_t = 0 \ \ \forall i, \forall t$).

Different Households in the Economy

This economy is populated by three types of households.

Renters: Households without enough net worth to buy the minimum house size are forced to rent; other households prefer to save for a longer period of time to attain the desired level of owned housing by renting for some time before buying a house. In this case, they solve the following problem:

$$G(y_t, y_{t+1}; i) = \max_{\{c^j_t, f^j_t, a^j_t\}} \{u(c_t, f_t)\}$$

$$s.t. \quad c_t + r^f_t p^h_t f_t + y_{t+1} \leq$$

$$\leq z^i (1 - \tau_y) w_t + b^{j > j^*}_t + y_t + (1 - \tau_y) r^a_t a_t$$

$$y_t = a_t$$

$$y_t \geq 0$$

Home owners: Households with enough net worth to purchase a house bigger than the minimum house size solve:

$$G^j(y_t, y_{t+1}; i) = \max_{\{c^j_t, h^j_t, a^j_t\}} \{u(c_t, h_t)\}$$

$$s.t. \quad c_t + y_{t+1} + \delta_h p^h_t (1 - \phi) h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi)h_t$$

$$y_t = a_t + p_{t-1}^h(1-\phi)h_t$$

$$y_t \geq (\gamma_{t-1} - \phi)p_{t-1}^h h_t$$

$$h_t > \underline{\mathrm{h}}$$

Households in the margin (Cooperatives): There are some households with enough resources to buy the minimum house size and they would be constrained by this election. Here I make the assumption of allowing these households to make a convex combination between the minimum house size and the amount of housing services they would rent. The problem with the non-convexity of the minimum house size is that, along the transition and for the calibrated model, I always find some individuals switching from owning the minimum house size to renting making it impossible to clear the housing market. This happens for a very small fraction of individuals and could be solved by some others techniques like linear interpolation in ages. The existence of the minimum house size is key in this model since I do not have adjustment costs in housing capital and I want to model the homeownership rate. In my model, without a minimum house size, all individuals would own a small fraction of housing capital in all periods but the first one, in which they have zero assets by assumption. Because of the preferential tax treatment and the access to credit that allows households to get credit, owning a house is always preferred to renting. Both assumptions are in line with the available evidence.

By assuming that households can do a convex combination between renting and owning the minimum house, I can solve the problem of clearing the housing market and have a realistic homeownership rate in the economy.

The problem I solve for these households is as follows:

$$G^j(y_t, y_{t+1}; i) = \max_{\left\{c_t^j, x_t^j, q_t^j, f_t^j, h_t^j, a_t^j\right\}} \{u(c_t, x_t)\}$$

$$s.t. \quad c_t + r_t^f p_t^h q_t f_t + y_{t+1} + \delta_h p_t^h (1-\phi)(1-q_t)h_t \leq$$

$$\leq z^i(1-\tau_y)w_t + b_t^{j>j^*} + y_t + (1-\tau_y)r_t^a a_t + (p_t^h - p_{t-1}^h)(1-\phi)(1-q_t)h_t$$

$$y_t = a_t + p^h_{t-1}(1-\phi)(1-q_t)h_t$$

$$x_t = q_t f_t + (1-q_t)h_t$$

$$y_t \geq (\gamma_{t-1} - \phi)p^h_{t-1}(1-q_t)h_t$$

$$h_t = \underline{\mathrm{h}}$$

No more households in the economy will do a convex combination of this kind if it is not with the minimum house size. The reason is that they would always prefer owning to renting in cases other than the minimum house size.

An intuition for this problem would be to consider it as cooperatives, i.e. that some households, belonging to the same age and quintile, were allowed to establish a cooperative. Then, only some of them would live in the house, while the others would rent. However, they can use this house as a collateral for credit in the capital markets.

An alternative interpretation can be that households deposit their savings in a financial intermediary and that the probability of buying a house depends on the fraction of the downpayment deposited. If a household deposits half of the required downpayment to buy a house, then the household is allowed to buy a house with 50% probability. If the household does not win the lottery, it does not lose its assets. Next period he will make a new deposit and get a new chance to buy a house.

Appendix B: Stationary Competitive Equilibrium

Denote $q = \{a, h, i\}$, q $\in$ Q.

Definition *A* ***stationary competitive equilibrium*** *for a given government policy, τ_y, and downpayment requirement, γ, is a collection of relative prices $\left\{p^h, p^s, p^l, r^f, r, w\right\}$, a collection of functions for the household problem $\{v^j(q), c^j(q), f^j(q), h^j(q), a^j(q)\}$, an age-dependent measure of agents type, $\lambda_j(q)$, a value function for financial institutions $\Psi(F, B, A, K)$, and aggregate quantities for the whole economy $\left\{Y^c, Y^h, Y^s, X^s, L, K^c, K^s, N^c, N^s, F, B, A\right\}$ such that:*

1. *Inputs are priced competitively.*
2. *Given τ_y, γ and prices, the functions $\{v^j(q), c^j(q), f^j(q), h^j(q), a^j(q)\}$ solve the dynamic program from the household problem.*
3. *Given prices and the function $\Psi(F, B, A, K)$, $\{F', B', A', K'\}$, solves the financial institutions' problem.*
4. *Individual and aggregate decisions are consistent: $C = \sum_{j=1}^{J} \int_Q c^j d\lambda_j(q)$, $H = \sum_{j=1}^{J} \int_Q h^j d\lambda_j(q)$, $F = \sum_{j=1}^{J} \int_Q f^j d\lambda_j(q)$, $A = \sum_{j=1}^{J} \int_Q a^j d\lambda_j(q)$.*
5. *The government maintains a balanced budget:*
$$G + b = \sum_{j=1}^{J} \int_Q \left[\tau_y w z^j + \tau_y r a^j\right] d\lambda_j(q)$$
where $b = \sum_{j=1}^{J} \int_Q b^j d\lambda_j(q) = \tau_y w \overline{N}$.
6. *Labor market clears: $N^c + N^s = \overline{N}$.*
7. *Capital market clears: $K^c + K^s = K$.*
8. *Land market clears: $L = \overline{L}$.*
9. *Residential structures market clears: $X^s = Y^s$.*
10. *Housing market clears:*
$$Y^h = X^h + X^f$$
where $X^h = \delta_h H$, and $X^f = \delta_f F$.

11. Trade balance is determined:

$$TB = Y^c - C - X^k - G$$

where $X^k = \delta_k K$.

12. Net foreign asset position is determined:

$$B = -\left(\frac{TB}{r}\right)$$