Noise and Dynamics in Diffusive Conductors

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***Abstract*—In this article we present two experiments carried on diffusive metallic wires and aiming at understanding better current fluctuations and the informations we can extract from them. The first experiment studies the non-gaussiannity of those fluctuations at low frequency. The second one studies the dynamic response of current fluctuations to an ac exitation. From the frequency dependence of this response function we obtain a direct determination of the inelastic relaxation times.**

Diffusive contacts have attracted a lot of interest over the years [1]–[8]. This is probably due to their simplicity making them good candidates for reliable theoritical predictions of their transport properties. In addition, the ease of tuning the interactions at play by simply changing the length of the sample makes the diffusive wire a perfect platform for experimentalists. It allows the study of electron transport in presence of interactions or the test of new experimental probes in different regimes. In this paper we will demonstrate with two experiments the versatility of such a system. In a first one we will be interested in a short sample for which, at low temperature, interactions are negligible and the transport is elastic and diffusive. It has been predicted that current fluctuations in this elastic regime are non-gaussian and exhibit a third moment $\langle \delta I^3 \rangle$. We will present one of the rare experiments performed on high order moment of current fluctuations. In a second experiment we will demonstrate a new experimental technique allowing a direct determination of relaxation times of electrons. We will distinguish two regimes. The diffusive regime for short wires where relaxation is dominated by the diffusion time τ_D along the sample. And a macroscopic regime at higher temperature and longer wire length, where relaxation is described by electron-phonon interactions allowing the determination of the electron-phonon interaction time τ_{e-ph}.

I. Non-gaussian current fluctuations

At low frequency, the variance of current fluctuations in a diffusive wire has been calculated using many techniques [1]–[3], all providing the same answer for the spectral density of current fluctuations S_{I^2} measured at temperature T with a voltage bias V:

$$S_{I^2} = \frac{2}{3}\frac{2k_BT}{R} + \frac{1}{3}\frac{eV}{R}\coth\frac{eV}{2k_BT}, \tag{1}$$

where R is the sample resistance, e the electron charge and k_B the Boltzmann constant. This result indicates the existence of shot noise with a Fano factor $F_2 = e^{-1}dS_{I^2}/dI = 1/3$ at large bias $V \gg k_BT/e$, which has been confirmed experimentally [8]. The reduction of the Fano factor as compared to that of a tunnel junction, $F_2 = 1$, is interpreted in the quantum theory as stemming from the existence of well transmitting channels and in the semi-classical theory from the existence of a position-dependent distribution function. The third moment of current fluctuations has also been calculated by several theories [3]–[7] which at low frequency all yield to the same spectral density S_{I^3} given by:

$$S_{I^3} = \frac{1}{15}e^2I + \frac{12}{5}k_BT\frac{dS_{I^2}}{dV}. \tag{2}$$

This result differs from that of a tunnel junction $S_{I^3} = e^2I$ on two main factors: first it depends on temperature; second it has a much lower Fano factor at high voltage, $F_3 = e^{-2}dS_{I^3}/dI = 1/15$ instead of $F_3 = 1$ for the tunnel junction. Eq. (2) corresponds to a measurement performed with a noiseless voltage bias and an ammeter, i.e. an apparatus with an input impedance much lower than that of the sample. This situation can be achieved with a high impedance sample, but a typical metallic wire has a low impedance and one has to consider the effects of both the finite impedance of the environment, here a resistance R_A (the input impedance of the amplifier), and the current noise experienced by the sample, here generated by the amplifier used to detect current fluctuations and described by a noise spectral density S_A. Those environmental effects are very subtle on the third moment of voltage fluctuations. They have been thoroughly studied both theoretically [9], [10] and experimentally [11], [12] and obey:

$$S_{V^3} = -R_D^3S_{I^3} + 3R_D^4(S_A + S_{I^2})\frac{dS_{I^2}}{dV}. \tag{3}$$

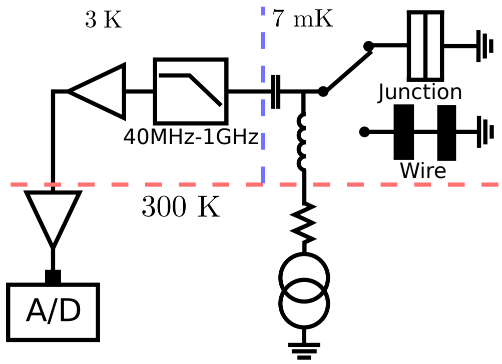

Fig. 1. Schematics of the experimental setup. A/D represents a 14 bits, 400 MSample/s digitizer.

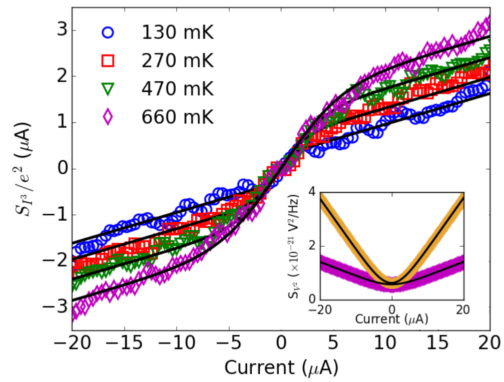

Fig. 2. Intrinsic third moment of current fluctuations S_{I^3} vs. I for the wire at different temperatures. Symbols are experimental data, lines are the theoretical expectations of Eq. (2). Inset: S_{I^2} for the tunnel junction (orange) and the wire (purple) $T \sim 640$ mK.

where $R_D = RR_A/(R + R_A)$ with R being the sample resistance. As a consequence, a reliable way to characterize all the environmental terms is required to extract the intrinsic third moment of current fluctuations S_{I^3}.

Experimental setup. The sample is a 1 μm long, 10 nm wide, 165 nm thick Aluminum (Al) wire of resistance $R_w = 30.5\ \Omega$. Its contacts, also made of Al, are much larger (400 μm×400 μm) and thicker (200 nm) to make sure they behave as good electron reservoirs [13]. An Al tunnel junction of resistance $R_j = 34\ \Omega$ is used as a reference to calibrate the setup. Both samples have been made by e-beam lithography and the metal has been deposited by double angle evaporation [14]. The experimental setup is presented in Fig. 1. The samples are placed on the 7 mK stage of a dilution refrigerator. They are kept in their normal, non superconducting state with the help of a strong Neodymium permanent magnet. The two samples are connected to a cryogenic microwave switch which allows us to measure either of them without changing anything in the detection circuit. They are dc current biased through the dc port of a bias-tee and ac coupled to a cryogenic microwave amplifier in the range 40 MHz-1 GHz. The use of a cryogenic amplifier both optimizes the signal to noise ratio and minimizes the noise experienced by the sample which leads to environmental contributions. The signal is further amplified at room temperature in order to achieve a level high enough for digitization. Non-linearities in the detection are very detrimental since they lead to strong artifacts. Despite the use of ultra-linear amplifiers, non-linearities still give rise to a contribution which is an even function of I in the sample. We simply remove this by considering $[S_{V^3}(I) - S_{V^3}(-I)]/2$. After amplification the signal is digitized by a 14 bit, 400 MS/s digitizer with a 1 GHz analog bandwidth. We measure real-time histograms of the signal from which moments are computed.

Results: elastic transport. In the inset of Fig. 2 we present the measurement of S_{V^2} for the tunnel junction (orange symbols) and the wire (purple). From the high current slope of S_{V^2} vs. I for the tunnel junction we find the gain of the setup. Then, we deduce the Fano factor of the wire $F_2 = 0.35 \pm 0.02$, in good agreement with the theoretical value of $\frac{1}{3}$ in Eq. (1). This ensures that electron transport is elastic in the sample, in agreement with other measurements of similar wires [8], [15]. From S_{V^2} we also deduce the electron temperature for the wire and for the tunnel junction, as well as the noise temperature of the amplifier $T_a \simeq 7.5$ K. Values of the temperature indicated in the various figures correspond to electronic temperatures deduced from the measurements of S_{V^2}.
We then extracted S_{V^3} vs. I for the tunnel junction and the wire at temperatures ranging from 130 K to 660 K. Following the procedure of [16], we use the measurements performed at all temperatures on the tunnel junction to extract the parameters that characterize the environment, i.e. the amplifier impedance $R_A = 44.8\ \Omega$ and the effective environmental noise temperature $T_0^* = 0.54$ K. From the knowledge of the environmental parameters we can extract the intrinsic third moment of current fluctuations in the wire using Eq. (3). The corresponding results are plotted in Fig. 2. The theoretical predictions of Eq. (2) are plotted as solid lines with no fitting parameters. A clear agreement between experiment and theory is achieved at all temperatures for the current range explored. At higher temperature electron-electron interaction start to be important. We also performed this experiment at higher current where interactions have to be taken into account modifying the statistic of electron transport [17].

II. Determination of relaxation times

The environmental terms we have subtracted correspond to a feedback mechanism due to the non-linearity of the second moment of current fluctuation. In fact the intrinsic third cumulant itself is explained by such a feedback in the quasi-classical theory. At a given frequency the effect of the feedback is to modulate the noise, hence one could think of using a controlled excitation power $\delta P(f)$ to effectively reproduce, in a controlled manner, this modulation. This would permit to easily measure a response function $R(f) = \delta S_2/\delta P(f)$. At low frequency $R(0)$ correspond to the noise equivalent to the environmental terms of the third cumulant. At higher

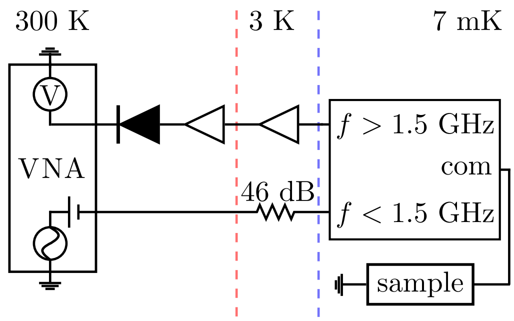

Fig. 3. Experimental setup. Diode symbol represents a power detector. VNA=Vector Network Analyser

frequency $R(f)$ should exhibit a cutoff frequency revealing any relaxation dynamic present in the sample.

The frequency dependence of $R(f)$ has been calculated for a metallic wire in different regimes [18]. For long enough samples the energy relaxation of the electron gas is dominated by electron-phonon interactions. This occurs when $L \gg L_{e-ph}$ where L_{e-ph} is the electron-phonon scattering length given by $L_{e-ph}^2 = D\tau_{e-ph}$ with D the diffusion coefficient. For shorter samples $L \ll L_{e-ph}$ electron-phonon processes are inefficient and the energy relaxation is dominated by diffusion of hot electrons into the contacts. In both regimes the frequency dependence of $|R(f)|^2$ is extremely well approximated by a Lorentzian decay:

$$|R(f)|^2 = \frac{R(0)^2}{1 + (2\pi f/\Gamma(T_e))^2} \tag{4}$$

where Γ the energy relaxation rate depends on the relaxation process. The frequency dependence of $R(f)$ is a direct probe of Γ without any assumption about the specific heat C_e as in previous work [19]. In the presence of several relaxation processes, the fastest relaxation usually dominates. Since τ_{e-ph} is strongly temperature dependent and diverges at low temperature whereas τ_D is temperature independent, the energy relaxation is dominated by electron-phonon coupling at high temperature ($\tau_{e-ph} \ll \tau_D$) and diffusion at low temperature ($\tau_{e-ph} \gg \tau_D$). Our measurement allows continuous monitoring of Γ as a function of temperature.

We have measured $\Gamma(T_e)$ for six samples made of different metals (Al, Ag) and different geometries. The wires have length L ranging from 5 μm to 300 μm and thickness d of 10 nm for the shortest and 20 nm for the others. The width has been adjusted to obtain a resistance of the order of 50 Ω for impedance matching purpose. The experimental setup is presented in Fig. 3. The sample, placed at the 10 mK stage of a dilution refrigerator, is dc and ac biased through the low frequency port of a diplexer by a time dependent voltage $V = V_0 + \delta V \cos(2\pi f t)$ with $\delta V < V_0$. The dc part V_0 is used to control the sample mean electron temperature through a constant Joule heating $P_J = GV_0^2$ and allowed us to work between $\sim$ 50 mK and $\sim$ 2 K. The superimposed ac power at frequency f, $\delta P_J(t) = 2GV_0\delta V \cos(2\pi f t)$ modulates the electron temperature of the sample. To detect

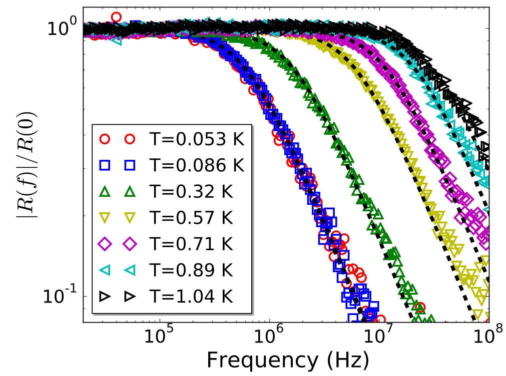

Fig. 4. Amplitude of the normalized thermal impedance as a function of frequency for sample 2. The symbols are the experimental data and the dashed lines are fits according to Eq. (4). The different curves correspond to different electron temperatures from $\simeq$50 mK to $\simeq$1 K.

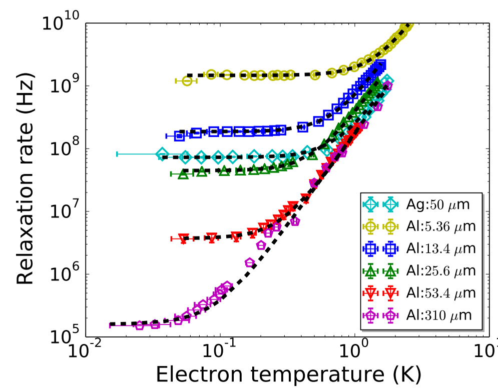

Fig. 5. Energy relaxation rate as a function of electron temperature for all the samples. Dashed lines are fits according to Eq. (5)

this temperature, we measure the rms amplitude of the voltage fluctuations (Johnson noise) generated by the sample. Indeed, the noise spectral density of voltage fluctuations S_V is related to the electron temperature by $S_V = 4k_B T_e/G$. The voltage fluctuations are measured in the frequency band $\Delta F \simeq 1.5-5$ GHz (high frequency port of the diplexer) and amplified by a cryogenic amplifier placed at the 3 K stage of the dilution refrigerator. Their rms amplitude is detected by a power meter (diode symbol in Fig. 3) whose response time $\tau_{det} \sim 1$ ns limits the maximum frequency at which the noise modulation can be detected, $f \lesssim 1$ GHz. Experiments have been performed at a phonon temperature of 10 mK. In Fig. 4, we present the normalized thermal impedance versus frequency for sample 2 for electron temperatures between 53 mK and 1.04 K. The symbols are the experimental data and the black dashed lines the fits according to Eq. (4). The frequency dependence of $|R(f)|^2$ is very well fitted by a Lorentzian, $\Gamma(T_e)$ being the only fitting parameter. We have performed this experiment for different samples and extracted $\Gamma(T_e)$ on 5 orders of magnitude.

We present in Fig. 5 the measured relaxation rates as a function of electron temperature for all the wires. At low temperature we observe a plateau, the relaxation rate does not depend on temperature. In this limit only diffusion cooling occurs, and $\Gamma(T_e) \simeq 10.01/\tau_D$. At high temperature the observed T_e^n dependency is characteristic of an electron-phonon cooling process [20]. In [21] the dynamic has only been calculated in the electron-phonon cooling or diffusion cooling regimes and not during the crossover. We thus assume that the frequency dependence of $|R(f)|^2$ follows a Lorentzian even during the crossover between the two regimes with a relaxation rate given by the sum of the relaxation rates of the two processes:

$$\Gamma(T_e) \simeq \frac{10.01}{\tau_D} + \mathrm{A}T_e^n. \tag{5}$$

Dashed lines in Fig. 5 are fits according to Eq. (5). The plateau observed in $\Gamma(T_e)$ at low temperature, see Fig. 5, provides a direct determination of the diffusion time τ_D as a function of sample length. At high temperature the relaxation is dominated by electron-phonon interaction , the expected value for three dimensional phonon bath in the clean metallic limit [20], [22]. In previous experiments, τ_{e-ph}^{-1} has been reported to behave as T_e^n with n ranging from 2 to 4 depending on the nature of the disorder [19], [23]–[27]. Disordered gold wires have been observed to behave as $T_e^{2.9}$ below 1K [26]. As far as we know, electron-phonon relaxation rates of Al and Ag have not been measured below 1K, a temperature range hardly explored [19], [26], [27]. From this experiment we are also able to extract the specific heat of electrons, for more information see [28].

III. Conclusion

We have measured the third moment of current fluctuations in a wire, thus demonstrating that even the simplest conductor exhibits non-Gaussian noise. Our data at low voltage are in very good quantitative agreement with the theory. In particular we have found a Fano factor $F_3 = 1/15$ characteristic of elastic transport in diffusive conductors. We have also demonstrated a sub-kelvin direct measurement of inelastic times in wires made of simple metals, which provides the determination of the electron-phonon scattering time, the diffusion time and the electron heat capacity of the sample. Our approach is however extremely versatile, and of great interest to study interactions and electron diffusion in modern materials.

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