Review of mode-conversion calculations in toroidal plasmas

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Review of Mode-Conversion Calculations in Toroidal Plasmas†

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ABSTRACT. Recent developments in the gyrokinetic modeling of drift-, Alfvén and Bernstein waves allow for the computation of mode-conversion in two dimensions. Theoretical predictions of global Alfvén eigenmode dampings, for example, are dominated by such processes and agree within 30% with the measurements from the Joint European Torus. Low frequency tokamak heating schemes involve a multitude of Alfvén or ion-hybrid resonances where mode-conversion is possible; the power absorption can however not correctly be determined from the resonance absorption of global fluid wavefields, so that a toroidal description of the conversion is required also for the modeling of the ion-cyclotron heating.
1 INTRODUCTION

The collective interaction of charged particles in a plasma gives rise to a variety of waves which propagate with different velocities depending on the electromagnetic and inertial forces which come into play. In the radio-frequency range for example, plasma compressions across the electric and magnetic fields generate a fast magnetosonic wave which propagates almost independently of the slower Alfvén wave induced by the tension of the mass-loaded magnetic field lines.

Special locations where the phase velocity of two waves match, however, allow for an energy transfer from one wave branch to another in a process called mode-conversion. Simple one-dimensional (1D) slab configurations where this is possible have been studied analytically first using resonance absorption [1, 2] and asymptotic matching [3, 4] with results that were, to a large extent, later confirmed using 1D global gyrokinetic wave codes [5, 6, 7, 8, 9]. Encouraged by the apparent good agreement with resonance absorption, fluid models have been proposed to calculate the power deposition and continuum damping of toroidal wavefields [10, 11, 12]; although it became clear that is not always possible with resonance absorption to determine which species absorbs the power [13], it is not before comparisons became available with 2D global gyrokinetic codes that it was found that resonance absorption is very misleading for weakly damped global wavefields most common in tokamaks [14]. Mode-conversion has therefore to be modeled in toroidal geometry and it has now been verified that it is then also in quantitative agreement with the experiments [15, 16, 17].

To solve the underlying 4th (or higher) order partial differential equation and calculate self-consistently the locations of the conversion regions in toroidal plasmas, the global electromagnetic wave problem has to be solved in two dimensions (2D). Apart from a rather limited number of situations where the structure of the solution can be guessed and solved analytically [18, 20], the calculations have in general to be carried out with sophisticated numerical codes, running today on the largest super-computers.

After a short description in section 2 of the tools that are currently available, the physics of mode-conversion is reviewed qualitatively in increasing order of frequency, starting with drift-kinetic Alfvén Eigenmodes (DKAE) in sect. 3, which, in the 50 kHz range, play a dominant role for the fast particle confinement in the DIII-D tokamak. Mode-conversion calculations have been most thoroughly tested with the comparisons of the Alfvén Eigenmode (AE) damping measured around 200 kHz in the Joint European Torus (JET); they explain in sect. 4 why $\alpha-$particle driven instabilities have only been observed in the Tokamak Fusion Test Reactor (TFTR) and not in JET.

Heating schemes have been tested using Alfvén waves launched around 3 MHz; the calculations reviewed in sect. 5 shed a new light on the measurements from the TCA tokamak and show that it could be difficult to heat plasmas with Alfvén waves to the temperatures required for a fusion reactor. Sect. 6 deals with the ion-cyclotron frequency range (ICRF) in a tokamak around 30 MHz where the 2D character of the mode-conversion appears explicitly. The highest frequency mode-conversion scenario examined in sect. 7 discusses the conversion to ion-Bernstein waves at the second cyclotron harmonic and argues how this might explain the ion-cyclotron emission (ICE) observed in the presence of fast particles.

Most of the results presented in this paper have been obtained using the PENN code [22], which remains the only electromagnetic gyrokinetic toroidal model which has been validated and applied both for stability and heating calculations in tokamaks, with frequencies ranging from drift- up to the second ion-cyclotron harmonic $2\Omega$. 


2 MODELS

Despite early successes in modeling mode-conversion analytically in a 1D slab [3, 23], very few attempts have been made in the two dimensions (2D) in fact required by the tokamak geometry. Part of the reason is that the local dispersion relations valid in the eikonal approximation neglect the complicated interference resulting when multiple reflections are superposed into a single global wavefield. Except for a few peculiar cases [18, 19, 20], the integro-/ differential equations modeling global wave problems cannot be solved using advanced mathematical tools such as asymptotic matching or simplectic analysis. The toroidal mode-conversion problem has therefore to be solved numerically to determine even qualitatively what is the conversion mechanism and where it takes place.

In the simplest approximation, mode-conversion can be modeled with two lukewarm fluids [24, 25, 26], but the global solutions obtained so far in this manner remain at least inconclusive for the tokamak. Energetic particle modes could in fact also be seen as a special type of mode-conversion, but are not discussed in the present paper [21].

An appealing approach intended for the ion-cyclotron range of frequencies (ICRF) has been derived early on within a Hamiltonian formalism and implemented in the AL-CYON code [27, 28]. Using a linear finite elements (FEM) discretization radially and a Fourier expansion poloidally, a wave equation is solved with two potentials allowing mode-conversion to take place. An artificial viscosity subsequently absorbs the power instead of what should physically be resonant (Landau, cyclotron) wave-particle interactions; in view of the necessity of modeling the mode-conversion layer properly [14], it is however not quite clear if, and under which circumstances, the power deposition calculated in this manner is physically meaningful.

More advanced models for the ICRF have been proposed in [29, 30], with however only the latter implemented so far in the TORIC code. Mode-conversion and the subsequent damping of the slow wave is taken into account using a second order finite Larmor radius (FLR) expansion in the small parameter $k_\perp \rho$, where $k_\perp$ is the characteristic wave-vector across the magnetic field and $\rho = \nu_{th}/\Omega$ is the Larmor radius. The global wavefield is discretized with cubic FEM radially and a Fourier expansion poloidally, which allows for a convenient polynomial representation of the operator $k_\parallel = -i \nabla_\parallel = R^{-1} (n + m/q)$ in terms of the toroidal $n$ and poloidal $m$ mode numbers, $R$ being the major radius and $q$ the safety factor. For a Maxwellian distribution of the bulk species, this formally guarantees a correct evaluation of the Landau damping which is proportional to the plasma dispersion function $Z(\omega/k_\parallel \nu_{th})$; the numerical convergence, however, becomes excessively difficult to achieve in the ICRF, where poloidally localized wavefield structures result in very broad Fourier spectra. To limit the range of poloidal harmonics required and make the calculation practical, an artificial viscosity is added to the dielectric response, which in effect limits the values of $k_\parallel$ and alters the power fraction converted if the slow wave is absorbed close to the conversion layers.

A different approach based entirely in configuration space has been implemented in the PENN code: starting from a second order FLR expansion in $k_\perp \rho$ and keeping the inhomogeneity-induced drifts to first order in $k_\perp L_\perp$, a dielectric tensor has been derived in Ref.[31], assuming a functional dependence of $k_\parallel (\rho, \theta)$ to calculate the resonant wave-particle interactions directly in configuration space. Using a cubic FEM discretization both for the radial and poloidal directions, it is then possible to model also strongly localized wavefields, using an iterative relaxation procedure to approximate $k_\parallel$ everywhere in the plasma [32]. This approach is justified when the wavelength remains sufficiently small to define $\nabla_\parallel$ from one of the wavefield polarizations in an eikonal sense and works relatively well for Alfvén waves and the ICRF where localized slow wave eigenmodes can be resolved numerically without introducing an artificial viscous damping. The iteration
however breaks down with global drift waves such as the ion-temperature gradient (ITG) modes, the existence of which depends critically on a precise representation of $k_{||} \approx 0$ which controls the delicate balance between the bulk species drive and damping.

Of all the toroidal mode-conversion models, PENN is the only one capable so far of dealing with stability and heating problems, with frequencies ranging from drift- up to the second cyclotron harmonic $2\Omega$. Apart from the obvious advantage of developing only one code (which in toroidal geometry are rather heavy), this unified approach allows also for important cross-validations, using for example the detailed comparisons with experiments in the Alfvén frequency range to gain some confidence in the predictions made in the ICRF where direct measurements are difficult.

After this short summary of the models currently available in 2D, the next section begins with the qualitative description of the mode-conversion mechanisms encountered so far in the tokamak, starting with the lowest frequencies around 50 kHz where the global magneto-hydrodynamic (MHD) wavefield couples with an electromagnetic drift wave.
3 MHD AND DRIFT WAVEFIELD

To test if the 3.5 MeV $\alpha -$ particles produced by the fusion reactions can resonantly interact with global electromagnetic wavefields and drive Alfvén eigenmodes (AE) unstable in a reactor, experiments have been carried out in the DIII-D tokamak with a reduced magnetic field bringing the Alfvén velocity $v_A = B/\sqrt{4\pi \sum n_i m_i}$ down to the velocity of the beam deuterons injected with 75 keV into the plasma [33].

As the frequency reproduced the value expected for toroidal Alfvén Eigenmodes $\omega_{TAE} \simeq v_A/2qR$, the instabilities have first been interpreted as TAEs. Surprisingly large particle losses of up to 70% of the fast population confined mainly in the plasma core ($s < 0.3$) [34] can however not be explained with TAE perturbations dominant in the external regions of the DIII-D plasma with a normalized minor radius $s \simeq \tau/a > 0.6$.

Figure 1: Unstable drift-kinetic Alfvén eigenmode (DKAE) in the DIII-D tokamak.

Using a gyrokinetic analysis first performed in Ref.[35] with the PENN code, Fig.1 shows that the **external kinetic-Alfvén wavefield** $s > 0.6$ gets coupled with an internal drift wave in the neighborhood of the rational surface $q = 1$ with dominant harmonics $n = -m = 5$ so as to guarantee a small value $k_\parallel \simeq 0$ and a frequency $\omega = 71kHz \simeq 6\omega_{ci}$. In contrast to the Alfvén ITG modes [36], global drift-kinetic Alfvén eigenmodes (DKAE) are driven by the beam deuterons; mode-conversion then transfers the drift wave energy from the plasma core to the global kinetic-Alfvén wavefield which reaches the plasma edge where it can be detected with magnetic probes [37].

Further calculations and detailed comparisons with experiments are required to establish the physics controlling the stability of DKAE and assess if their saturation is always accompanied by large losses as observed in the DIII-D tokamak. With the present knowledge of this instability, it is however advisable to avoid planing advanced scenarios with strongly reversed magnetic shear, since they yield large values of the safety factor and bring the TAE frequency $\omega_{TAE} \propto B/q$ right down to the drift frequency range.
MHD AND KINETIC-ALFVÉN WAVEFIELD

The plasma perturbations for which mode-conversion processes are currently best understood are Alfvén eigenmodes (AE) around 150 kHz, for which quantitative frequency and damping rate predictions [16, 38, 39] have been tested against active measurements from JET [40, 41].

Early models examined the possibility of converting to kinetic Alfvén waves (KAW) in the vicinity of neighboring fluid resonances coupled by the toroidicity, or within a toroidicity gap to approximate the amount of power absorbed by the Landau damping of the KAW coupled to a TAE gap mode. This lead to the prediction of the so-called kinetic TAE modes (KTAE) [18] with frequencies slightly above $\omega_{TAE}$, continuum damping [11, 12] and radiative damping [42]. These are all special features of what we call kinetic Alfvén eigenmodes (KAE). KAEs comprise all the Landau damped/driven eigenmodes of global kinetic Alfvén wavefields [38, 16].

In retrospect, it is interesting (and perhaps symptomatic for thermonuclear fusion research) that none of the conversion mechanisms which could be modeled simply and solved analytically is now thought to be generally dominant in the interpretation of the AE stability in JET.

So what are these dominant mode-conversion mechanisms?

1. Most important for AE reaching out to the plasma edge is the large edge magnetic shear associated with the plasma shaping (X-point), which squeezes the wavefield radially until the spatial scale of the global mode reaches the scale length of the KAW [16]. The mode-conversion and subsequent Landau damping that follow provide a damping rate $|\gamma/\omega| > 0.1$ sufficient to stabilize an $\alpha-$particle pressure gradient drive $\gamma/\omega < 0.005$ typical of a reactor plasma [43].

2. If the shear is weak in the plasma core where the aspect ratio is large, the KAW expands radially until it meets the global AE scale length and converts [16]. Even if the $\alpha-$particle pressure is largest in the core, the gradient is weak, and the mechanism is generally stabilizing with a damping in JET typically around $|\gamma/\omega| \simeq 0.01$ [44].

3. In contrast with fluid models which predict that core-localized TAE wavefields get more peaked when the bulk $\beta$ rises [45], recent calculations from the PENN code show that by matching the global and KAW scales, mode-conversion is enhanced with $\beta$ and results in a KAE wavefield which is finally more global [39]. Under circumstances, this can make $\beta$ a stabilizing factor when the AE wavefield reaches the plasma core or the edge.

4. Unless combined with one of the previous mechanisms, mode-conversion induced by an Alfvén resonance is generally not very efficient, making the now widely used shear-Alfvén gap structures a poor indicator to predict the damping or even guess the existence of AEs. In this context, it is worth repeating that the continuum damping of global wavefields such as TAEs [11, 12] is very misleading as shown theoretically in Ref.[14] and verified experimentally in the comparisons with measurements from JET [16, 15].

5. When large values of the safety factor $q$ and high toroidal mode numbers $n$ yield large poloidal mode numbers $m$ since $k_{||} = R^{-1}(n + m/q) \simeq \omega/v_A$ remains approximately fixed, or when every previous damping mechanism is weak, multiple mode-conversion regions appear and interact to form a strongly kinetic AE [41]. A local radiative damping model is then however not sufficient to properly
account for the resonant interaction even with the bulk species. In addition, it becomes imperative to evaluate also the power transfer between the KAE wavefield and the fast particles, which can provide for either drive or damping [43, 39].

For illustration, Fig.2 shows the wavefield of a stable KAE calculated in the 12MW fusion yield JET discharge 42677, where mode-conversion is induced by the high magnetic shear in the edge region and where the global wavefield develops short scale lengths comparable with the kinetic Alfvén wavelength.

An important aspect of the mode-conversion studies in the 100-300 kHz range is that actual predictions could be tested against the measurements from JET, with an agreement around 5% for the frequency and 30% for the damping [16], and showing that mode-conversion is a dominant damping mechanism with an isotope scaling argument [44].

5 FAST AND KINETIC-ALFVEN WAVE

Frequencies from approximately 1 MHz up to the first cyclotron harmonic around 20 MHz can be used for Alfvén wave heating scenarios. These rely on a fast magneto-sonic wave to carry the electromagnetic power from an external antenna into the plasma, where, after mode-conversion occurs in the neighborhood of fluid resonances, the power is absorbed mainly by electron Landau damping [46].

Extensive studies have been carried out in Ref.[22] using parameters relevant for the TCA tokamak, aiming at the validation of the new 2D toroidal gyrokinetic PENN code with the 1D cylindrical gyrokinetic SEMENE and the 2D toroidal fluid LION code [45]. Perhaps the most important and surprising finding is the strength of the mode-conversion when the plasma is warm enough, irrespective almost of how well the poloidal mode
numbers of the antenna and that of the fluid resonance are matched. As a rule of thumb, the mode-conversion and the power deposition simply occur in the neighborhood of the most external resonance where the plasma is warm enough to propagate a KAW [22], i.e. where $\omega/k_{||}v_{th,e} < 1$ [46]. This suggests that the power depositions calculated for the TCA tokamak are typically much more localized in the edge region than previously obtained with a fluid resonance absorption model [47].

Fig.3 illustrates the toroidal mode-conversion in a heating scenario with fluid resonances located around $s = 0.33, 0.59, 0.79, 0.91$ and labelled $m = -1, 6, 7, 0$ in the cylindrical approximation. Using an antenna with helical currents ($n_a = -2, m_a = -1$), the fluid model predicts 75% power absorption on the the $(n = -2, m = -1)$ resonance surface in the plasma core. The gyrokinetic calculation from the PENN code shows in Ref.[22] that mode-conversion occurs already around $s = 0.79$ where the electron temperature is sufficient to satisfy the condition $\omega/k_{||}v_{th,e} < 0.8$ and most of the power is then deposited externally $s > 0.5$.

Another study documented in Ref.[48] examined the damping rate of the global Alfvén eigenmode (GAE) measured in TCA [49] with $|\gamma/\omega| > 0.015$. This is in good agreement with the toroidal mode-conversion calculations from PENN $|\gamma/\omega| > 0.01$, but an order of magnitude larger than the cylindrical conversion from the ISMENE code, showing clearly the importance of the toroidal effects.

A more striking example of power losses in the edge region has been given in Ref.[14] with a current drive scenario originally proposed for the International Thermonuclear Experimental Reactor (ITER). Although it remains speculative to draw general conclusions from only a couple of studies, it is fair to say that with the present theoretical knowledge, it looks rather hopeless to penetrate and deposit power with Alfvén waves any reasonable distance into a large and hot fusion plasma.
6 ION-ION HYBRID REGIME

Starting from a fluid model’s perspective, the ion-hybrid regime is probably the most obvious to check when looking for a strongly toroidal mode-conversion mechanism: indeed, the magnetic flux-surface alignment of the Alfvén resonances is then broken by the vertical ion-hybrid resonances \[13\] and results in a wavefield which is explicitly singular in two dimensions (2D).

In spite of the recent analytical developments aiming at a complete description of the mode-conversion process in 2D \[20, 50\], all the wavefields which are presently available in tokamak geometry have been obtained numerically with one of the codes described in section 2. The largest efforts have probably been made using the TORIC code \[30\], for a variety of scenarios which have recently been reviewed in Ref.\[51\]. Good agreement is reported with the power deposition measured in the ALCATOR C-MOD tokamak \[52\]. From the wavefields displayed, it is however not possible to say exactly where and how the mode-conversion takes place; since it is not clear either if the solutions can be converged poloidally without adding an ad-hoc viscous damping, it remains unclear if the wavefields and the power deposition calculated in this manner are always physically meaningful.

Using the PENN code for a scenario in a hydrogen - deuterium plasma with varying concentrations, Ref.\[32\] shows that mode-conversion occurs

1. **at the ion-hybrid layer**, where nearly vertical slow wavefronts develop and propagate in a manner similar to the solutions calculated in the mid-plane using 1D slab gyrokinetic models \[3, 5\].

2. **in the vicinity of Alfvénic resonant magnetic surfaces**, kinetic-Alfvén waves propagate mainly along the magnetic field and escape radially if an evanescence suddenly prevents them of going any further.

3. In regions of strong toroidal coupling where \(\epsilon_\perp \simeq 0\), **localized slow wavefields** are predicted to form weakly damped eigenmodes which could be driven to large amplitudes \[32\]. Recent measurements of the ion-cyclotron emission (ICE) spectra in JET show that an instability driven by fast particles does indeed exists in the ion-hybrid frequency range and is localized radially around a normalized radius \(r/a \simeq 0.3\) \[53\]; further calculation balancing the fast particle drive with the electron Landau damping are however necessary to assess if such eigenmodes can also be driven unstable and explain the experimental observations.

4. If a second cyclotron harmonic layer \(\omega = 2\Omega\) is present, mode-conversion becomes possible also to the ion-Bernstein wave and will be discussed in more details in the coming section.

To illustrate the second mechanism with an example similar to Fig.4 of Ref.\[32\] and show the gyrokinetic counterpart of a study suggested by Appert et al. in Ref.\[13\], an Alfvén wave mode-conversion scenario in a hydrogen model plasma is here perturbed with a tiny amount (4\%) of tritium. Figure 4 suggests how the slow kinetic Alfvén wave converted at a resonance around \(s = 0.7\) follows first the magnetic field lines counterclockwise in the plasma cross-section until it hits the cyclotron layer of the tritium; reflected poloidally, the slow wave then escapes radially inwards and deposits power by electron Landau damping and cyclotron interactions with the tritium.

Further studies are required to complete the understanding of the mode-conversion in the ion-ion hybrid regime and assess in particular the formation of slow wave eigenmodes.
Comparisons between different models and experimental measurements are clearly desirable and should aim at defining a range of parameters for which theoretical predictions can be trusted.

7 FAST AND ION-BERNSTEIN WAVE

We call ion-Bernstein (IBW) the slow wave which is converted at the second or every higher harmonics of the ion-cyclotron frequency to distinguish it from the kinetic Alfvén wave converted at the ion-hybrid resonance. To our knowledge, only one toroidal study documented in Ref.[54] has been performed so far in this regime, using the PENN code to compute the location, the amplitude and wave-vector of the mode-converted IBW. Because of the second order FLR expansion used in the model, the study was limited to the second cyclotron harmonic, in a case where the expansion parameter remained sufficiently small $k_{\perp} \rho < 0.5$.

Figure 5 illustrates the toroidal conversion mechanism in a JET-like plasma, when the temperature of a deuterium plasma with 4% hydrogen reaches 10 keV without creating a fast-ion tail. Launched from the low-field side of the torus, a low amplitude fast wave (invisible in the plot) propagates into the plasma, sets-up a partially standing wavefield and reaches the second cyclotron harmonic layer of the deuterium where $\omega = 2\Omega_D$. Mode-conversion to IBWs occurs in regions along the second harmonic cyclotron layer where the wavelength of the fast and slow wave match. This happens at different radial locations corresponding to different poloidal mode numbers, for a normalized radius $s \simeq 0.3, 0.6$ in Fig.5. The IBW then propagates radially along the high-field side of the $\omega = 2\Omega_D$ layer, and, due to a relatively long poloidal evanescence length, extends also somewhat towards the low-field side of the torus. Because of the change in polarization, the power is absorbed by electron Landau damping and second harmonic cyclotron harmonic interaction with the majority deuterons instead of what would simply be a minority heating of hydrogen if mode-conversion were absent.
Figure 5: Bernstein wave heating in a hot D(4%H) JET plasma. The dashed line corresponds to the layer where $\omega = 2\Omega_D = \Omega_H$.

8 CONCLUSION

With this short review spanning over more than three orders of magnitudes in frequency, it has hopefully been possible to convince the reader that mode-conversion is a very rich physical phenomenon explaining a variety of observations and measurements currently made in fusion plasmas. The excellent quantitative agreement achieved in particular for the damping of Alfvén eigenmodes and the power deposition in the ICRF show that fundamental features have indeed been captured, justifying some of the predictions now made for a tokamak experimental reactor.

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