Numerical simulation of viscoplastic avalanches
A general overview

M. Rentschler, S. Wiederseiner, Ch. Aney
martin.rentshler@epfl.ch

École polytechnique fédérale de Lausanne

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Variation in the front position with time for $\theta = 12^\circ$. Experiments done with Carbopol at various concentrations. Dashed curves: theoretical prediction given by a zero-order non-linear convection equation (modeling the behavior of an avalanching mass of Herschel-Bulkley fluid).
Problem statement

Variation in the front position with time for $\theta = 12^\circ$. Experiments done with Carbopol at various concentrations. Dashed curves: theoretical prediction given by a zero-order non-linear convection equation (modeling the behavior of an avalanching mass of Herschel-Bulkley fluid).

**Inertia driven part**

**Gravity driven part**
Problem statement

**Figure:** Experiment: Dambreak of viscoplastic material on an inclined plane (more details in the talk of Steve Cochard)
Governing equations

Incompressibility

\[ \nabla \cdot \mathbf{u} = 0 \quad (1) \]

Conservation of Momentum

\[ \frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} - \frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \tau \quad (2) \]

Newton’s law

\[ \tau = \mu \left( \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right)_{\dot{\gamma}} \quad (3) \]

with viscosity \( \mu(\mathbf{r}, \dot{\gamma}) \), density \( \rho(\mathbf{r}) \), velocity \( \mathbf{u}(\mathbf{r}) \) and pressure \( p \).
Rheological model

Herschel-Bulkley model

\[
\mu(\vec{r}, \dot{\gamma}) = \frac{\tau_0}{|\dot{\gamma}|} \left(1 + \left(\frac{|\dot{\gamma}|}{\dot{\gamma}_C}\right)^n\right) \quad \forall \vec{r} \in \Omega_{\text{liquid}} \subset \mathbb{R}^3
\] (4)
The Solver

- Chorin’s Projection scheme
  BICG to solve the pressure equation
- Semi-implicit,
  the parabolic (stress) term is treated implicitly
- Two phases: Gas./Liquid
- Level-Set and Volume of Fluid to represent the free-surface
- Parallel by domain decomposition (mpi)
- Various rheologies:
  Bingham, Herschel-Bulkley, Coulomb ...
Comparison in 2D

**Figure:** Comparison of the Numerical Result with S. Cochards experiments. Quasi 2D in a channel.
Comparison in 3D

Figure: Comparison of the Numerical Result with S. Cochards experiments. 3 dimensional flow.
Pseudo-plug region

Figure: Permanent uniform flow of a HB-fluid down an inclined channel. \( \dot{\gamma}_C \) defines the critical viscosity to identify pseudo plug regions.
Pseudo-plug regions

Figure: *Pseudo yield surface*: The blue transparent surface represents the free surface, the red the yield surface.\(^2\)

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\(^1\)Inclination 12°, Concentration 0.25%  
\(^2\)Inclination 12°, Concentration 0.25%
Collaps of a sandpile

Figure: Collaps of a column of sand with different layers of sand to visualize the deformation.
Comparison to experimental results

Figure: Comparison of the free-surface with laboratory experiments\(^3\).