

# 3LP: A linear model of locomotion including falling, swing and torso dynamics

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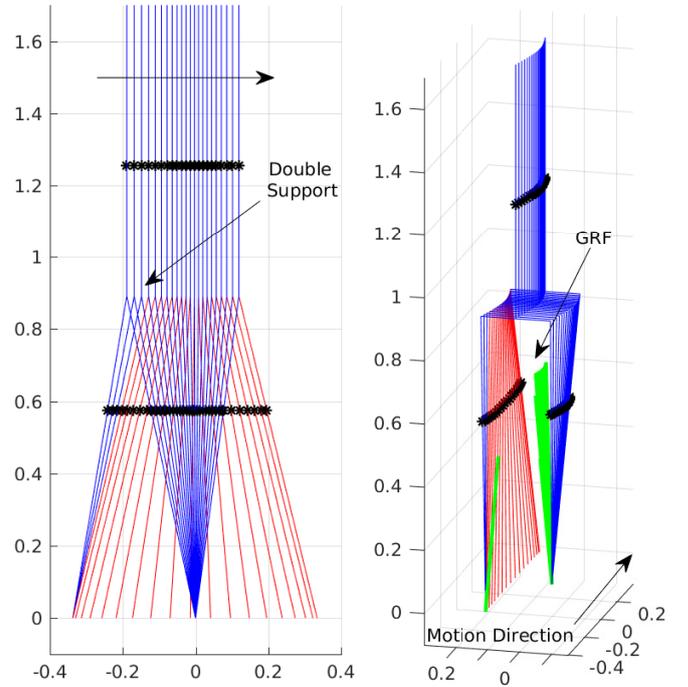
## 1 Summary

In this article, we present a new linear model (3LP) for bipedal locomotion which can describe swing and torso dynamics as well as falling. Compared to traditional inverted-pendulum based models, 3LP produces more human-like center of mass trajectory and swing motion which is missing in point-mass models. Different periodic gaits can be found with 3LP and modulated online without any numerical integration. Using linearity properties, we derive closed-form solutions and a transition matrix which describe state evolution over footsteps. This matrix as well as linear inequalities on inputs and states can be used in online model predictive controllers (MPC) to guaranty feasibility, robustness and optimality of the motion. Besides, 3LP can also be a powerful tool in bio-mechanics as it provides human-like ground reaction and torque profiles.

## 2 Introduction

Locomotion of humanoid robots is a challenging control task due to many complexities in both hardware and software. There are many control approaches proposed in literature for different robot hardware. The category of model based methods provide a solid framework for stability analysis which makes them popular among other methods. Humanoids are however non-linear, high dimensional, floating base and hybrid systems in locomotion. Therefore, setting up control problems that support all aspects of optimality, robustness and versatility seem to be challenging. On the other hand, the interface with the real robot regarding state estimation and precise actuation remains a robot-specific complex task that highly influences the control formulation. In this regard, hierarchical controllers can be proposed to handle the task in separate levels: long-term planning, short-term stabilization, tracking motions and actuator-level control (Faraji et al., 2014; Feng et al., 2013; Dai et al., 2014). Using enough abstraction, each level can update its output as fast as possible to let the robot react to the perturbations. However, regarding available computers and methodologies, it remains impossible to use full models for online planning.

In bipedal locomotion of human-like hardware with similar geometries and degrees of freedom, it is conventional to use Inverted-Pendulum (IP) based models as they mimic human dynamics quite closely. Such abstract models are used in upper levels to plan and stabilize the motion. The linearized



**Figure 1:** The 3D demonstration of a walking gait with contact forces shown in green. The swing leg is shown in red while double support is shown with both legs in blue.

version (LIP) is more popular as it provides closed form solutions, suitable for fast future prediction (Faraji et al., 2014; Herdt et al., 2010). This version can describe double support phase for smooth transition of the force and for avoiding impacts, unlike nonlinear versions. It is also possible to use ankle torques for better stabilization, although it might require time-trajectory optimization which can still be handled online (Feng et al., 2013).

Following our previous work on using hierarchical controllers with linear inverted pendulum (Faraji et al., 2014) and later on using inverse dynamics on the real robot for various balancing and tracking tasks (Faraji et al., 2015), we noticed that it is very important to keep the matching between different levels in the hierarchy. In other words, the abstract planned motion in upper levels can be realized on the real robot only if the full robot has enough control authority, whereas many details about the full model are yet missing in abstract level.

In both original and linearized versions of inverted pendulums, swing and torso dynamics are missing with point-mass assumption. However, the hip torques required for keeping the torso upright and pumping energy into the swing leg play an important role in the center of mass motion. In this article, we introduce a new model that describes these effects in 3D space, while keeping all properties of the LIP as well. This model is still linear, providing closed-form solutions for fast future prediction. Thanks to swing dynamics, it can also produce natural periodic gaits without imposing footsteps. In the following, first we explain the model in details, then mention properties of the out-coming gaits and finally we discuss advantages of this model over inverted pendulums in hierarchical controllers.

### 3 Methods

The proposed model is composed of three linear pendulums, representing the torso, swing and stance legs. Each pendulum has a mass that moves in a constant-height plane. The two legs are separated with a pelvis of certain width, similar to human. Hip and ankle torques are expressed in Cartesian space which keep formulations still linear. To keep calculation of analytical solutions easy, we consider only constant and time-increasing components for control input torques, i.e. swing hip and stance ankle torques. These linear profiles (with respect to time) can describe major shapes observed in human torque profiles (Liu et al., 2008).

The proposed model is also able to model double support phase, when the weight is transferred linearly (with respect to time) to the new stance leg, approximating trapezoidal ground reaction forces of human (Liu et al., 2008). Transition to double support happens in 3LP when the velocity of the swing foot becomes zero at the end of the phase. Note that we assume virtual prismatic actuators that extend the legs to keep pelvis, leg masses and foot tips all move in constant-height planes. With such assumptions, we can solve the model analytically (by symbolic math solvers) and obtain transfer matrices that simply describe state evolution in time. Then, by considering symmetries in locomotion, periodic gaits can be simply obtained by looking at the eigenvectors of a certain matrix which is a function of single and double support durations.

### 4 Results

For any specific choice of timing, there exist infinite gaits of different actuation patterns. Figure.1 demonstrates one of many possible gaits in 3D space. 3LP successfully describes sagittal and lateral motions in a linear fashion. The choice of timings and actuations are related to other criteria like optimality and hardware limitations which are not investigated in this work. However, 3LP can potentially provide many types of gaits like pseudo-passive walking with no hip and ankle actuation. Lateral foot clearance and natural swing dynamics are other important properties of this model. By choosing human-like geometry, mass distribution and gait timing, we

also show that ground reaction profiles and hip/ankle torques are very similar to human as well.

### 5 Discussion

3LP is a model that can significantly improve the matching between abstract and full-model levels in a control hierarchy. Thanks to linear properties, 3LP can simply replace LIP for future planning in online MPC control. Regarding the real hardware, 3LP can encode precise information about actuation limitations which can be used as inequality constraints in MPC optimization. The resulting CoM and swing trajectories are coming from natural dynamics and no trajectories are imposed like in LIP model. This increases the matching and thus, potentially improves human-likeness and walking speed. 3LP is also a very powerful tool in bio-mechanics as it provides human-like dynamic profiles. The geometry however remains yet different, considering knee, heel and toe motions as well as CoM excursion. In future works, we like to investigate different control options for 3LP and analyze motion energetics to discover more aspects of considering swing and torso dynamics. (Khatib et al., 2007)

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