Development and Benchmarking of the Global Gyrokinetic Code GENE

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Introduction

- The gyrokinetic code GENE [1], has been extended from its original flux-tube version to a global geometry.
- Includes radial variation of temperature and density profiles, as well as of magnetic geometry.
- Non-periodic boundary conditions allow for profile relaxation.
- Heat sources & sinks enable quasi-stationary microturbulence simulations.
- Interface with the MHD equilibrium code CHEASE [2,3].
- Various benchmarks, including comparisons with other global codes are presented.

Global GENE Model

- Field aligned coordinate system $\vec{X} = (x$: radial, y: binormal, z: parallel$)$ $\Longrightarrow$ $\vec{B}_0 = \mathcal{C}(x)\,\vec{\nabla}x \times \vec{\nabla}y$.
- Gyrokinetic equation with radial (x) variations of equilibrium quantities.
- Particle distribution function $f_j(\vec{X}, v_\parallel, \mu) = f_{0j} + f_{1j}$, with f_{0j} a local Maxwellian.
- Gyrokinetic equation is solved for the perturbed distribution function f_{1j}.
- Perturbed electrostatic and vector potentials $(\Phi_1, A_{1\parallel})$ are self-consistently computed through the quasineutrality (Q.N.) equation and parallel component of Ampère's law.
- Gyrokinetic ordering $|k_\parallel| \ll |k_\perp| \Longrightarrow$ Neglect $\partial/\partial z$ compared to $\partial/\partial x$ and $\partial/\partial y$.

The Gyrokinetic Equation

$$-\partial_t g_{1j} = \frac{1}{\mathcal{C}}\frac{B_0}{B^\star_{0\parallel}}\left[\frac{1}{L_{nj}} + \left(\frac{m_j v_\parallel^2}{2T_{0j}} + \frac{\mu B_0}{T_{0j}} - \frac{3}{2}\right)\frac{1}{L_{Tj}}\right] f_{0j}\,\partial_y\bar{\chi}_1 + \frac{1}{\mathcal{C}}\frac{B_0}{B^\star_{0\parallel}}\left(\partial_x\bar{\chi}_1\,\Gamma_{y,j} - \partial_y\bar{\chi}_1\,\Gamma_{x,j}\right)$$
$$+ \frac{B_0}{B^\star_{0\parallel}}\frac{\mu B_0 + m_j v_\parallel^2}{m_j\Omega_j}\left(\mathcal{K}_x\Gamma_{x,j} + \mathcal{K}_y\Gamma_{y,j}\right) - \frac{1}{\mathcal{C}}\frac{B_0}{B^\star_{0\parallel}}\frac{\mu_0 v_\parallel^2}{\Omega_j B_0}\frac{p_0}{L_p}\Gamma_{y,j} + \frac{\mathcal{C}v_\parallel}{B_0 J}\Gamma_{z,j} - \frac{\mathcal{C}\mu}{m_j B_0 J}\partial_z B_0\,\partial_{v_\parallel} f_{1j}\,,$$

- where $g_{1j} = f_{1j} + q_j v_\parallel \bar{A}_{1\parallel} f_{0j}/T_{0j}$, $\bar{\chi}_1 = \bar{\Phi}_1 - v_\parallel \bar{A}_{1\parallel}$, $\Gamma_{\alpha,j} = \partial_\alpha f_{1j} + q_j\partial_\alpha\bar{\Phi}_1 f_{0j}/T_{0j}$ for $\alpha = (x, y, z)$.
- The overbar denotes gyroaveraged quantities.
- Background density, temperature and pressure profiles: $n_{0j}(x)$, $T_{0j}(x)$, $p_0(x)$. Corresponding inverse logarithmic gradients: $L_A(x) = -(d\ln A/dx)^{-1}$ for $A = [n_j, T_j, p]$.
- $\mathcal{K}_x(x,z)$ and $\mathcal{K}_y(x,z)$ are related to curvature and gradients of $\vec{B}_0$. $J(x,z) = [(\vec{\nabla}x \times \vec{\nabla}y)\cdot\vec{\nabla}z]^{-1}$ is the Jacobian.
- $\Omega_j(x,z) = q_j B_0/m_j$, and $B^\star_{0\parallel}(x,z,v_\parallel) = B_0 + (m_j/q_j)v_\parallel(\vec{\nabla}\times\vec{b}_0)\cdot\vec{b}_0$, with $\vec{b}_0 = \vec{B}_0/B_0$.

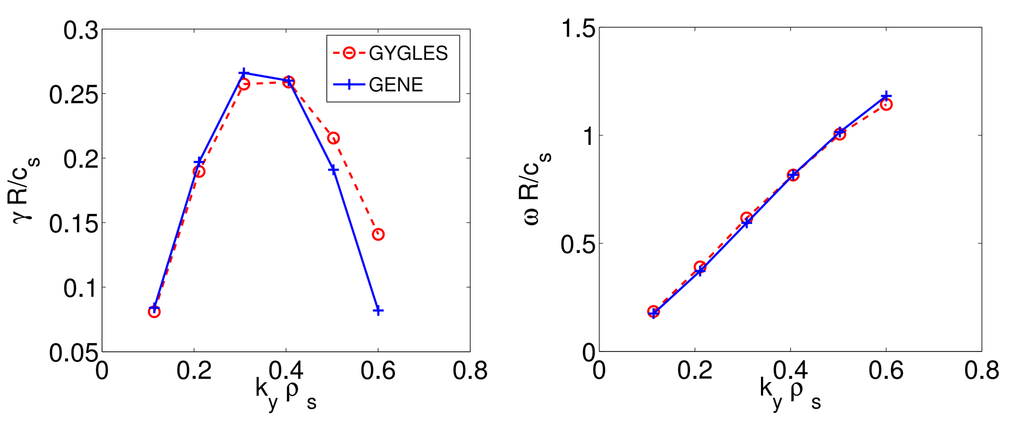
Benchmarking and Code Comparisons

Codes Used for Comparisons

- Comparison with linear and non-linear global PIC codes GYGLES [4] and ORB5 [5] based on δf scheme.
- Analytic, "ad-hoc" equilibrium with circular concentric magnetic surfaces is considered here.
- Global GENE :
  - Solving in direct space except y-direction for which Fourier representation is used.
  - Derivatives in real space computed with finite differences.
  - Dirichlet radial boundary conditions.
  - Direct space anti-aliasing scheme in radial direction.
  - Direct space integral gyroaveraging operator in radial direction.

Linear ITG Spectra for CYCLONE Base Case [6] with Adiabatic Electrons

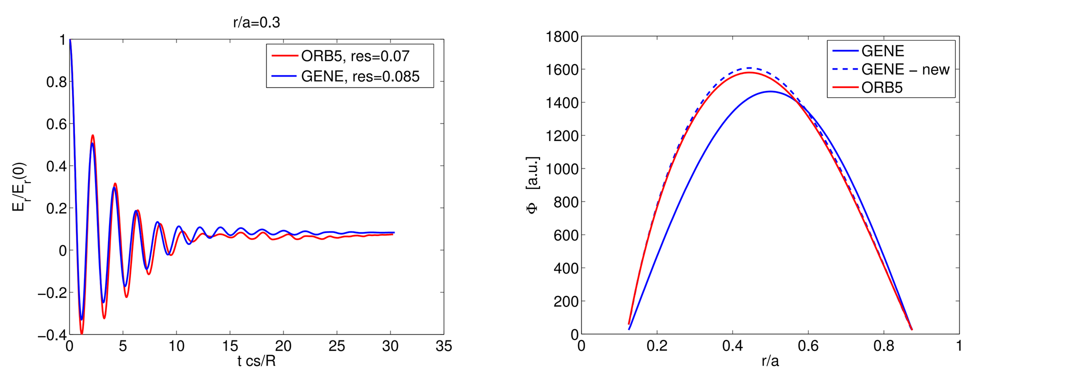
CYCLONE parameters with adiabatic electrons : $a/R = 0.36$, $\rho^\star = \rho_s/a = 1/180$, $q = 0.85 + 2.4(x/a)^2$, $T_i/T_e = 1$, peaked T and n profiles with $R/L_{Ti}(x_0) = 6.96$, $R/L_n(x_0) = 2.2$, and $x_0 = 0.5a$.



- Good agreement on growth rates and real frequencies.
- Remaining discrepancies at high k_y can be assigned to differences in the field solvers (2nd order expansion in $k_\perp\rho_s$ in GYGLES, all orders kept in GENE).

Rosenbluth-Hinton Test

Parameters : $a/R = 0.1$, $\rho^\star = 1/180$, $q = 1 + 0.75(x/a)^2$, $T_i/T_e = 1$, $R/L_T = R/L_n = 0$, $f_1(t=0) = cos(\pi x/lx)$. Adiabatic electrons.

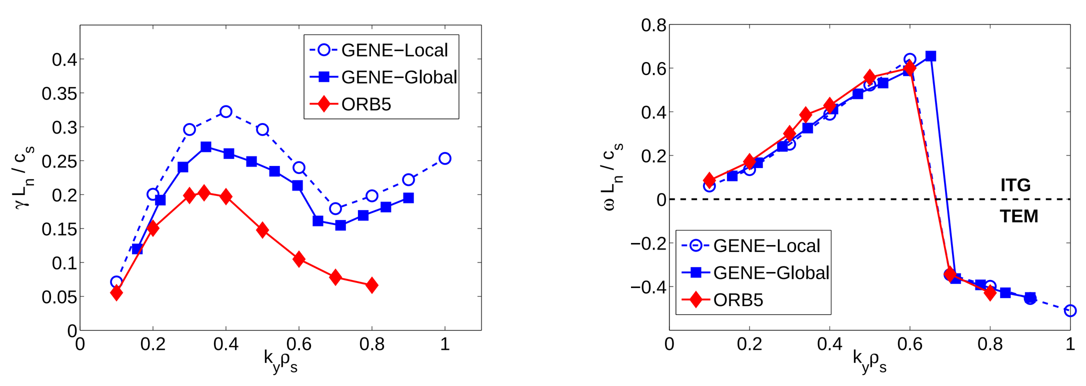


Evolution of the flux-surface-averaged, radial electric field. Initial electrostatic potential, solution to the Q.N. equation.

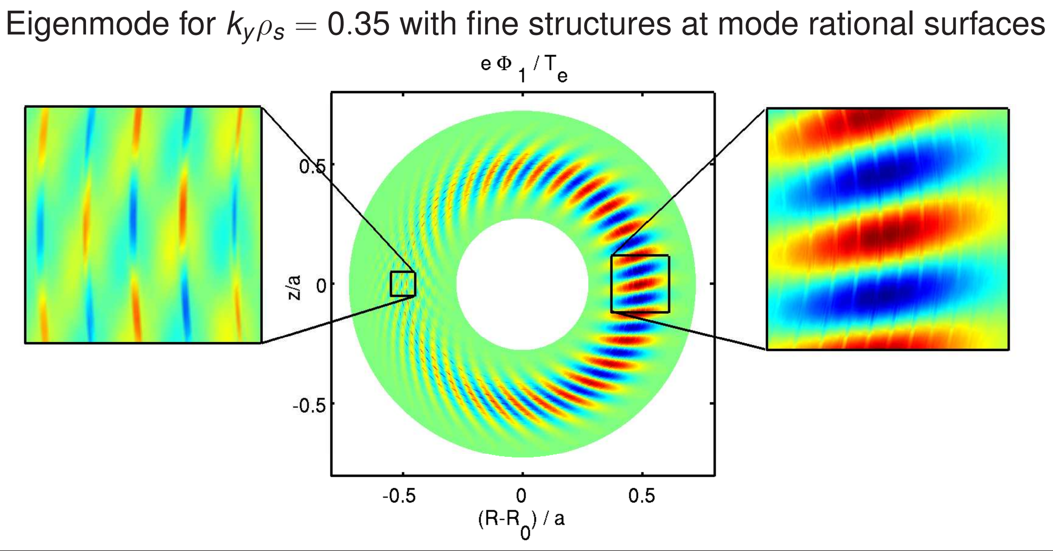
- Good agreement obtained for GAM frequency and damping rate, as well as for residual.
- Remaining discrepancies related to $\rho^\star$ approximations in GENE, in particular in the gyroaveraging appearing in Q.N. equation.
- After correcting these $\rho^\star$ approximations on gyroaveraging:
  - Very good agreement is reached on the Q.N. solution.
  - However, zonal modes become unstable! (under investigation).
- Current simulation results are thus still obtained using the uncorrected gyroaveraging operator.

Linear ITG-TEM Spectra for CYCLONE Base Case with Kinetic Electrons

- CYCLONE parameters with kinetic electrons ($m_i/m_e = 400$).

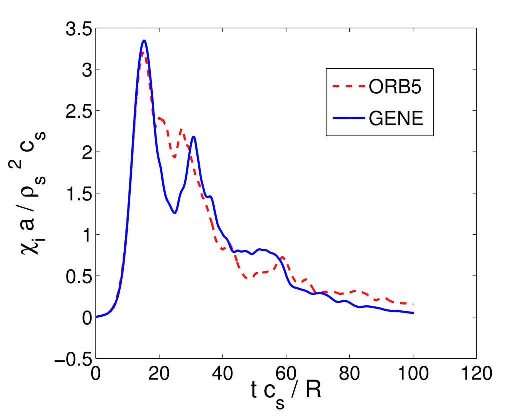


Eigenmode for $k_y\rho_s = 0.35$ with fine structures at mode rational surfaces

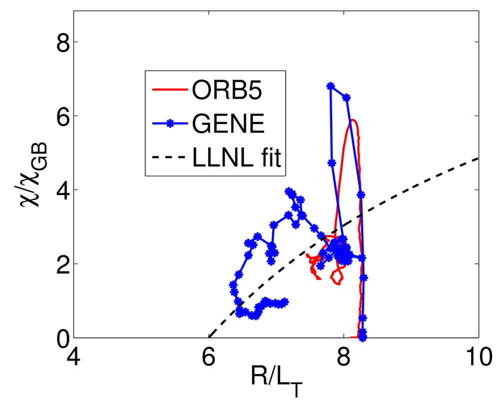


- Transition from ITG to TEM at higher $k_y\rho_i$.
- Differences between global GENE and ORB5 results may be related to ORB5 treating only trapped electrons kinetically (adiabatic response for passing), while GENE treats electrons fully kinetically.
- Resolution for global GENE simulations: $(320 \times 64 \times 64 \times 32)$ in the $(x, z, v_\parallel, \mu)$ directions $\Longrightarrow$ High resolutions in $(x, v_\parallel, \mu)$ required for resolving non-adiabatic response of passing electrons at mode rational surfaces.
- Do the corresponding radial fine structures in the linear eigenmodes survive in the non-linear regime? In particular, do they affect the non-linear fluxes?

Non-Linear ITG Simulations without Sources $\Longrightarrow$ Relaxation



Evolution of ion heat diffusivity χ_i for CYCLONE parameters with peaked gradient profiles.



$(R/L_T, \chi_i)$ trace for CYCLONE parameters with flat gradient profiles [7].

- Same initial conditions $\Longrightarrow$ Remarkable agreement: Time traces of the first burst are essentially identical.
- Global GENE recovers well the non-linear relaxation traces in the $(R/L_T, \chi_i)$ plane published in [7].

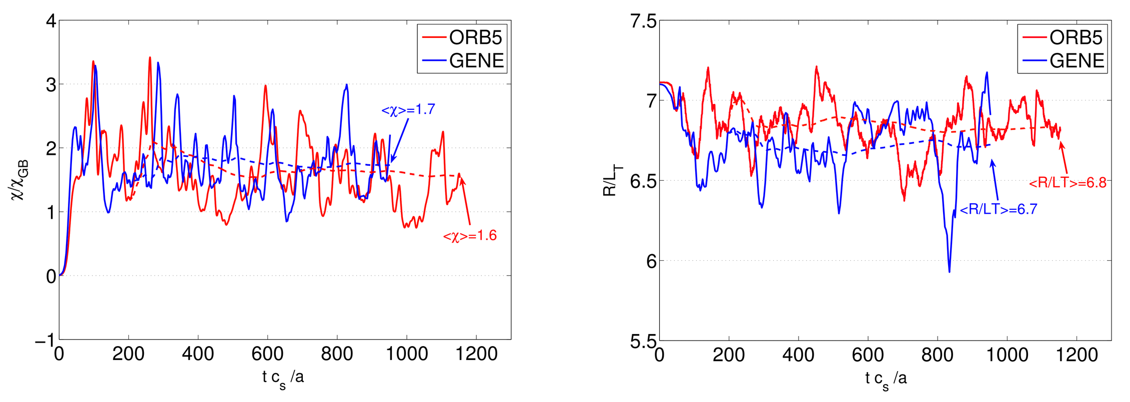
References :
[1] F. Jenko, *et al.*, *Phys. Plasmas* 7, 1904 (2000).
[2] H. Lütjens, *et al.*, Comp. Phys. Comm. 97, 219 (1996).
[3] X. Lapillonne, *et al.*, *Phys. of Plasmas* 16, 032308 (2009).
[4] M. Fivaz, *et al.*, *Comp. Phys. Comm.* 111, 27 (1998).
[5] S. Jolliet, *et al.*, *Comput. Phys. Comm.* 177, 409 (2007).
[6] A. M. Dimits, *et al.*, *Phys. Plasmas* 7, 969, (2000).
[7] G. L. Falchetto, *et al.*, *Plasma Phys. and Control. Fusion* 50, 124015 (2008).

Non-Linear ITG Simulations with Sources $\Longrightarrow$ Quasi-Stationary Microturbulence

- Radially dependent heat source/sink over whole system, conserving surface-averaged density and parallel momentum:

$$\frac{df_1}{dt} = -\gamma_h\left[\langle f_1(\vec{X}, |v_\parallel|, \mu)\rangle - \langle f_0(\vec{X}, |v_\parallel|, \mu)\rangle\frac{\langle\int d\vec{v}\,\langle f_1(\vec{X}, |v_\parallel|, \mu)\rangle\rangle}{\langle\int d\vec{v}\,\langle f_0(\vec{X}, |v_\parallel|, \mu)\rangle\rangle}\right].$$

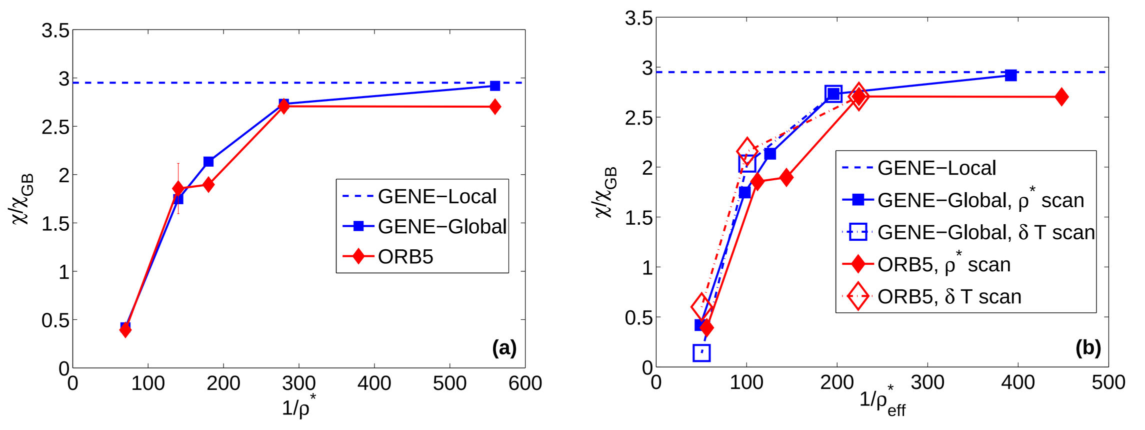
- Relaxation coefficient $\gamma_h \sim 10^{-1}\gamma_{ITG}$ $\Longrightarrow$ Background temperature profile is approximately maintained, while avoiding direct effect on microturbulence.
- CYCLONE parameters with flat gradient profiles.
- Numerical resolution for GENE: $(120 \times 48 \times 16 \times 48 \times 16)$ in the $(x, y, z, v_\parallel, \mu)$ directions.



Time evolution of (a) heat diffusivity χ_i, and (b) temperature gradient R/L_{T_i} for CYCLONE parameters with heat sources/sinks.

Dependance of Ion Heat Diffusivity on System Size and Gradient Profile Width $\Longrightarrow$ Effective $\rho^\star$

- Nonlinear electrostatic simulations of ITG turbulence with heat sources, assuming adiabatic electrons. CYCLONE Base Case equilibrium parameters.
- Study of global effects by carrying out both a scan in $\rho^\star = \rho_s/a$ at fixed relative temperature gradient profile width Δ_T/a, as well as in Δ_T/a at fixed $\rho^\star$.



Heat diffusivity χ_i in Gyro-Bohm units ($\chi_{GB} = \rho_s^2 c_s/a$) as a function of (a) $1/\rho^\star = a/\rho_s$ at fixed Δ_T/a, and (b) as a function of $1/\rho^\star_{eff} = \Delta_T/\rho_s$ varying both $\rho^\star$ at fixed Δ_T/a and Δ_T/a at fixed $\rho^\star$.

- The main variation of χ_i from global effects is caught by its dependence with respect to the effective parameter $\rho^\star_{eff} = \rho_s/\Delta_T = \rho^\star(\Delta_T/a)^{-1}$, which represents the width of the strong gradient region in gyroradius units.
- Global results converge towards local, flux-tube results for $1/\rho^\star_{eff} \to \infty$: Agreement within less than 10% for $1/\rho^\star_{eff} \gtrsim 200$.
- The reduction of the heat diffusivity due to global effects thus does not appear to result from profile shearing but rather from the constriction of non-linear turbulent structures within the unstable gradient region.
- Global effects may not only be important in small machines (i.e. low $1/\rho^\star$) but also in larger machines with short gradient lengths such as found in transport barriers.

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