A kTC Noise Analysis in a Passive Switched-Capacitor CMS Circuit for CIS

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Abstract—This work presents a noise analysis of a passive switched-capacitor correlated multiple sampling (CMS) circuit based on a kTC noise charge method, developed for periodic switched capacitor (SC) circuits. Two case studies are presented, before a generalized formula for the total output noise voltage is given. Calculated values from the derived formula are compared with noise simulations performed with Spectre® circuit simulator.

Index Terms—Noise analysis, Switched Capacitor circuits, Correlated multiple sampling, CMOS image sensors

I. INTRODUCTION

The CMS is a technique widely used in low noise CMOS image sensors (CISs) and it is used to reduce the thermal, the flicker and the RTS noise [1]. This operation embeds an average of the input signal samples into the correlated double sampling (CDS). The averaging operation reduces the contribution of the thermal noise originated from previous stages in the readout chain [2]. In addition to this effect, the CMS introduces a zero in the transfer function, which decreases the contribution of the flicker noise generated by the in-pixel amplifier [3]. The latter is the dominant noise source in modern ultra low noise CISs for low light applications [2]. Different implementations of the CMS are reported in literature and both analog and digital techniques are exploited. A typical analog implementation of the CMS takes advantage of an integrator element to cumulate the sum of \( M \) consecutive samples of the input signal [1]. However, this solution may suffer from a dynamic range (DR) reduction due to this accumulation process. The digital implementations of the CMS [4, 5] require multiple analog-to-digital conversions, performed by a fast and power-hungry analog-to digital converter (ADC). A SC CMS is based instead on the charge-domain, as shown in [6]. This circuit uses a minimum number of switches and capacitors to average the reset and signal samples, without the need of an additional circuitry and without an impact on the DR.

SC circuits are used for a variety of different applications, among which are signal processing and filter design, where they replace components like large resistors and inductors. The various nonidealties of these components are avoided, like the mismatch, the area and the non-linearity within a range of frequencies. The SC circuits can be implemented either with an active or with a passive implementation. Active SC circuits include an operational amplifier, which can introduce issues with stability and sensitivity, while passive SC circuits require only capacitors, switches and digital control circuits. The main limitations of the passive solution are the analog switches nonidealties, the charge injection, the mismatch and the kTC noise. The latter is the charge noise variance on a capacitor due to thermal noise, where \( k \) is the Boltzmann constant, \( T \) the temperature expressed in Kelvin and \( C \) the value of the capacitor. The theoretical estimation of this SC noise can be arduous, since these circuits are a linear time-variant (LTV) system and the noise transfer functions for each noise source have to be recalculated during each phase [7]. This results in a difficult derivation for many SC circuits. The aim of this work is to provide a complete noise derivation of the total output noise of a SC CMS circuit. This derivation makes use of a charge-based method to define all the involved noise terms. The derivation is first proposed for a CDS circuit and then for a CMS of order four. The obtained analytical formulas and the conclusions drawn by this analysis are verified with noise simulations performed with Spectre® circuit simulator.

This paper is organized as follows: in Section II, the operation principle of the SC CMS circuit is briefly described. In Section III, the noise analysis is presented with a detailed step-by-step derivation of the total output noise. The enlargement of the proposed analysis for a CMS of generic order \( M \) is shown in the same Section. In Section IV, the noise simulation results are shown. Finally, the conclusions of this work are drawn in Section V.

II. PASSIVE SWITCHED-CAPACITOR CMS CIRCUIT

The schematic of the SC CMS and the timing diagram of the control signals (from \( S_1 \) to \( S_{10} \)) applied to the switches are shown in Figs. 1a and 1b. In Fig. 1a, the CMS schematic is divided into two red blocks, which are named the averager and the subtractor. The averager is where the input voltage level, \( V_{in} \), is sampled and the averages are progressively built. The subtractor is the part where the difference between the two average is implemented. The implementation of the subtractor requires only two source follower (SF) buffers, four switches and a bootstrap capacitor, \( C_6 \). The averager takes advantage of the charge sharing principle between equal capacitors (from \( C_1 \) to \( C_5 \)). If the initial voltages of two equal capacitors are respectively \( V_1 \) and \( V_2 \), when connected together the final voltage across them will be equal to \( (V_1 + V_2)/2 \). This is due to the charge sharing between the two capacitors. If \( M \) consecutive samples with a sampling period, \( T_{CMS} \), are stored on \( M \) different capacitors, connecting them all together at the instant \( M \cdot T_{CMS} \), results in averaging the \( M \) samples.
Initially, the switches iterated between capacitors $C_1$ and $C_2$. The average voltage is stored in $T$ opened a sample, $V_1$. Then, $S_1$ is also opened after a time $T_{CMS}$ and the next sample, $V_2$, is stored in $C_1$. $S_3$ is pulsed while $S_1$ is opened and $S_2$ is closed, and the average voltage is stored in $C_4$. The exact same operations are iterated between capacitors $C_1$ and $C_2$, and then the average between fours samples is stored on $C_4$ by closing $S_3$ and $S_5$. Other two iterations on $C_3$ and on $C_2$ allow to store the average of eight samples on $C_4$. The latency introduced by the CMS is minimized if the sampling and the settling time between two consecutive samples are optimized. In a CIS, the additional area of this circuit is not significant, since the control signals for the switches are common to all the implemented readout columns and the logic circuit can be unique. To evaluate the impact of the CMS on the temporal readout noise, the squared absolute value of the CMS transfer function, $|H_{CMS}(f)|^2$, has to be evaluated. The latter is plotted for different values of $M$ in Fig. 2. The area delimited by $|H_{CMS}(f)|^2$ reduces with the increased value of $M$, hence the result of the integration with the constant power spectral density for the thermal noise is inversely proportional to $M$. At low frequencies, the CMS introduces a zero in the transfer function. However, the maximum of $|H_{CMS}(f)|^2$ moves to lower frequencies by increasing $M$. Hence, the $1/f$ noise is also expected to be reduced by increasing $M$ and a plateau for the residual flicker noise can be predicted [2].

### III. Noise Analysis in SC CMS Circuit

To evaluate analytically the total $kT/C$ output noise of the proposed SC CMS, the noise analysis follows the following steps: first, the noise calculation method based on the charge-domain is explained; then, this method is applied on the basic example of the CDS circuit and to the SC CMS averager of order four; the noise analysis is then extended to a generic order $M$ of the SC CMS. Finally, to derive the formula of the total output noise variance, the contribution of the subtractor is also added.

#### A. Noise Calculation Method

The charge-domain noise calculation method was first introduced in [8], where it was applied to a passive SC low pass filter and to a $N$-path filter. In periodic SC networks, each capacitor can be either connected to a voltage source or to another capacitor, through a switch. When connected to a voltage source, the switch resistance, $R_{sw}$, is responsible for the thermal noise injected in the capacitor, from which it is originated the well-known noise voltage variance, $kT/C$. When connected to another capacitor, a phenomena of charge sharing between the two components takes place. The thermal noise is an uncorrelated type of noise, hence each generated noise term is independent from the others. Each noise contribution can be considered separately, from the generation during the voltage sampling to the propagation, where all operations are ideal since no extra noise is added. In this method, all the different phases in a SC circuit are first determined. Then, all the noise terms are distinguished and propagated in time through all the phases. Finally, all the contributions are summed to obtain the final output noise variance.

![Fig. 2. The term $|H_{CMS}(f)|^2$ as a function of the normalized frequency $fT_{CMS}$, for four different values of the CMS order.](image-url)
B. Correlated Double Sampling

The schematic of the SC CDS is shown in Fig. 3. In a CDS operation, the difference of two input voltage samples is implemented [9]. In this circuit, it is possible to define three phases: \( \phi_1 \), where the first sample is stored on \( C_1 \), \( \phi_2 \), to store the second sample on \( C_2 \), and \( \phi_3 \), to implement the difference between the two samples. The noise charge variance on \( C_1 \) at the end of \( \phi_1 \), \( Q_{\phi_1}^2 \), is equal to \( kTC_1 \). When expressed in terms of voltage noise variance, \( V_{\phi_1}^2 \), it is equal to \( kTC_1 \). This noise value will not be influenced by \( \phi_2 \), while in \( \phi_3 \) the charge sharing between \( C_1 \) and \( C_2 \) will lead to

\[
V_{\phi_1}^2 = \frac{Q_{\phi_1}^2}{(C_1 + C_2)^2} = \frac{kTC_1}{(C_1 + C_2)^2}. \tag{1}
\]

Similarly, the voltage noise variance due to the noise contribution generated during \( \phi_2 \) and shared during \( \phi_3 \), \( V_{\phi_2}^2 \), can be written by inverting \( C_1 \) with \( C_2 \) in \( V_{\phi_1}^2 \). Finally, the last noise contribution is generated during \( \phi_3 \) and can be expressed as

\[
V_{\phi_3}^2 = \frac{kT}{C_1 + C_2}. \tag{2}
\]

All the computed terms can be summed to give the total output noise voltage variance, \( V_{n, out}^2 \), given by

\[
V_{n, out}^2 = \frac{kTC_1}{(C_1 + C_2)^2} + \frac{kTC_2}{(C_1 + C_2)^2} + \frac{kT}{C_1 + C_2}. \tag{3}
\]

If we consider the typical case where \( C_1 \) and \( C_2 \) are both equal to \( C \), the total voltage variance results into \( kTC \).

C. Correlated Multiple Sampling

The schematic of the SC averager circuit for a CMS order equal to four implemented with a minimum number of capacitors is shown in Fig. 4a. The minimum number of phases for sampling and averaging is equal to six (\( \phi_1 \) to \( \phi_6 \)) and they are described in Fig. 4b. The noise generated in \( \phi_1 \) is equal to \( kTC_3 \) and the charge sharing between \( C_1 \) and \( C_3 \) will determine a voltage variance equal to \( kTC_3/(C_1 + C_3)^2 \). By multiplying the previous expression with \( C_3^2 \), it is possible to define a charge noise variance. This noise term will then face a second operation of averaging, which will lead to the following expression

\[
Q_{\phi_1}^2 = \frac{kTC_3^2}{(C_1 + C_3)^2} C_3^2. \tag{4}
\]

Fig. 3. SC CDS circuit implementation.

Fig. 4. (a) Averager circuit for a SC CMS order equal to four in a minimum number of capacitors. (b) Timeline of the CMS \( M = 4 \) with all the phases and the operations for each phase.

If we consider all capacitors to be equal to \( C \), \( Q_{\phi_1}^2 \) is equal to \( kTC_1/16 \). The same noise contributions can be obtained for the other three samples, generated during \( \phi_2 \), \( \phi_3 \) and \( \phi_4 \). In \( \phi_5 \) and \( \phi_6 \), two different averages are generated, which are averaged a second time during \( \phi_6 \). The two additional noise contributions, \( Q_{\phi_5}^2 \) and \( Q_{\phi_6}^2 \), can be written as

\[
Q_{\phi_5/6}^2 = \frac{kTC_1^2}{C_1 + C_3} + \frac{kTC_2^2}{C_2 + C_3} \frac{C_1^2}{(C_1 + C_3)^2}. \tag{5}
\]

Both of these variances are equal to \( kTC \Phi \), in the case of equal capacitors. The last noise contribution to be taken into account is the one generated by the average implemented during \( \phi_6 \), which results in

\[
Q_{\phi_6}^2 = \frac{kTC_1^2}{C_1 + C_3}. \tag{6}
\]

The latter is equal to \( kTC/2 \) when \( C_1 = C_3 \) is assumed. All the noise terms can be summed to obtain the total output charge noise variance, which is equal to \( kTC \). The noise analysis for the SC averager can now be generalized for an order \( M \) of the CMS. As shown in Fig. 5, every averager in a CMS of order \( M \) can be divide into the average of two averagers of order \( M/2 \). Based on the previous examples, if this process is iterated, each averager can be considered to generate a noise voltage variance equal to \( kTC \). Under these assumptions, the total noise output voltage, \( V_{n, avg}^2 \), is given by

\[
V_{n, avg}^2 = \frac{1}{4} \left( \frac{kT}{C} + \frac{kT}{C} + \frac{kT}{2C} \right) = \frac{kT}{C}. \tag{7}
\]

Fig. 5. Generic CMS order \( M \) splitted into two blocks of order \( M/2 \) and a simple averager.
The first noise term in (7) is originated by the noise sharing, while the second is the contribution of the average block. $V_{n,\text{avg}}^2$ is equal to $kT/C$ for each order $M$ of the passive SC CMS circuit. The output noise contribution of the averager is independent from the number of input voltage samples that are processed. To give a formula for the total output voltage noise of the SC CMS circuit, the impact of the subtractor and the noise generated by the subtractor itself have to be added. The operation of subtraction doubles the input noise variance, while each voltage buffer will contribute with a noise variance equal to $\gamma kT/C_6$, where $\gamma$ is the noise excess factor [10]. The latter assumes a low output impedance equal to $1/G_m$ for the voltage buffers. Finally, the contribution from the switching operation is equal to $kT/C_6$, and the final output voltage variance results into

$$V_{n,\text{out}}^2 = \frac{2kT}{C} + \frac{(2\gamma + 1)kT}{C_6}. \quad (8)$$

IV. SIMULATIONS RESULTS

The proposed formula for the output voltage noise of the SC CMS is verified with periodic noise (PNOISE) and transient noise simulations in the Spectre® circuit simulator. These simulations are performed with a time between the samples, $T_{CMS}$, equal to 1μs. Figs. 6a and 6b show the root-mean-square of the CMS output voltage, $V_{n,\text{out}}$. In Fig. 6a, $V_{n,\text{out}}$ is computed for four different values of the averager capacitor, $C$, from 200 fF to 1 pF. The noise contributions from the averager and the subtractor are shown separately and in dotted lines. The averager noise reduction with an increase in $C$ is predicted by the $kT/C$ term in (8). The noise of the subtractor is instead not influenced by the value of $C$. In Fig. 6b, the output voltage noise is evaluated for four different CMS orders. The simulation results confirm that the noise contribution of the SC CMS is independent of $M$, hence it is not influenced by the needed number of samples. The PNOISE and transient noise simulations results show a good matching and validate the presented noise analysis results.

V. CONCLUSION

A method for $kTC$ noise calculation based on the charge-domain noise is used to derive an analytical expression for the output noise of a passive SC CMS circuit. This method is applied to the CDS and the CMS with $M = 4$, before enlarging to the CMS of a generic order $M$. The derived formulas are validated by the good matching between the calculated values and the Spectre® PNOISE and transient noise simulation results. The analytical and simulation results show the noise contribution of this SC CMS circuit is not influenced by the CMS order.

REFERENCES