Quantum 1/f Noise --a Decoherence Phenomenon

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***Abstract*—Our quantum theory of fundamental 1/f noise anticipated since 1975, and used, the theory of decoherence, which was developed two decades later. The quantum theory of the conventional and coherent quantum 1/f effects started precisely from the mixture of states selected by the interaction with the environment, with random phases, states later called "einselected states." The two quantum 1/f effects are thus both "infrared divergence" and "decoherence" phenomena that include the interaction with the rest of the world. This misunderstanding, initiated by van Kampen and two others, delayed progress by 40 years.**

Keywords—1/f noise, decoherence, quantum 1/f effects, quantum 1/f noise, einselection, infrared divergence, phase noise

I. Introduction

The Quantum Theory of 1/f Noise is a new aspect of quantum mechanics, introduced [1] 1975 as an infrared divergence phenomenon in both of its forms. It is also a decoherence phenomenon in both, the conventional and coherent Quantum 1/f Effects (Q1/fE). Indeed, in the 1980's, the Quantum Theory of 1/f Noise was misunderstood by some for many reasons, but *finally for only one reason:* In general, the bremsstrahlung energy loss components emerging from any scattering, or from other processes, are known not to interfere with the main wave function. They are orthogonal, with phases differing by 90 degrees in a certain representation. Therefore, some said that my theory, based on such interference, is not valid. But the experiment always verified our Quantum 1/f (Q1/f) formulas in all domains, universally.

Their mistake was *ignoring the universal phenomenon of decoherence*. Indeed, soon after the superposition state with 90 degrees phase differences is created, decoherence scrambles the phases, randomizing them, as we always assumed in our theory of the conventional and coherent quantum 1/f effects, which we introduced. In general, in times of the order of 10^{-8} s the superposition of states interacts with the environment and is replaced by a mixture of "einselected" states [2]-[4]. The latter are states that are much more stable in the interaction with the environment. This is also why the Schrödinger cat is never found half dead and half alive, and why we don't have good quantum computers!

This may have been understood now by the scientific community, particularly after a 2013 paper [5], and there are no other objections left to our theory and to its simple, practical, universal engineering formulas. These are applicable to all high-tech applications, for materials, devices and systems, existing in any domain in time and space, including also sensors or high stability resonators, oscillators and clocks. It's as basic as time and space, as it also happens in the macroscopic world with gravitons as infraquanta, instead of photons. It shapes all existence in time and space it is *the universal way of existence itself.* It is caused, in final analysis, *by the interaction of our system with the rest of the world.* Alternatively, as we show below, the same result can be obtained in the Quantum Information Theory (QIT) approach, by considering this environment under our observation threshold, i.e., by simply ignoring it and possible negative entropy states left in it. The latter not only fail to provide us with the expected choice in a potential measurement, but have instead a pre-selected outcome.

II. Decoherence

As pointed out by us [6] since 1982, the phases of the bremsstrahlung energy loss components of the outgoing wave function in a scattering process are randomized, as implied by the density matrix formalism used in that paper in anticipation of later results of the theory of Decoherence that was developed later, in the eighties.

Decoherence is the process of continuous, measurement-like, interaction of the quantum system with its environment. It quickly replaces exquisite, often complex (coherent) superpositions of states resulting from interaction processes like scattering, with an (incoherent) mixture of simpler states, that are much more stable in interaction with the environment, and have thus a more objective existence, because one can find them in repeated measurements over times much larger than the decoherence time. These states are called the "einselected pointer states" [2]-[4], [7], and the process in which the environment selects them as the relatively stable descendants of the initial coherent state is known as "einselection." For scattering of an electron or other charged particle on a center of force or arbitrary potential, this mixture of einselected outgoing Heisenberg states is

$$\varphi(\mathbf{x}) = (C/x)e^{iKx}\,[1 + \Sigma_{\mathbf{k}l} b(\mathbf{k},l)a^{+}_{\mathbf{k},l}e^{-iqx}]. \quad (1)$$

Here C is an amplitude factor, K the electron wave vector magnitude, m its mass, x the distance from the scattering center, $b(\mathbf{k},l)=|b(\mathbf{k},l)|\exp(\gamma_{\mathbf{k}l})$ the bremsstrahlung amplitude for photons of wave vector **k** and polarization l, with a random phase $\gamma_{\mathbf{k}l}$, while $a^{+}_{\mathbf{k},l}$ is the corresponding photon creation operator, allowing the photon state to be created from the vacuum if Eq. (1) is inserted into Eq. (9) below. The momentum magnitude loss $q\hbar = mck/K \equiv 2\pi mf/K$ is necessary for energy conservation in the Bremsstrahlung process with hf being the bremsstrahlung energy loss. This yields the quantum 1/f noise as known [1], [8]-[11].

All this was found decades after the discovery of the quantum 1/f noise in the Fall of 1974 [1], [8]-[11], anticipated the decoherence theory by decades, introducing for the first time the einselected states ad hoc, with different name, through physical insight, as the true mixture of states expected to emerge from the scattering of a charged particle like an electron, with interaction with the electromagnetic field, including thus the presence of bremsstrahlung and virtual photons of arbitrarily low frequencies. This author gave these anticipated de facto einselected states, the name of "stochastic Schrödinger field," introducing random phases of the bremsstrahlung energy loss components resulting in practice from the outgoing state in any scattering experiment involving an electron or any other charged particle in interaction with the environment.

Aldert van der Ziel had a deep abiding respect for the fundamental 1/f noise. He had struggled his whole life to understand the physical nature and origin of this universal phenomenon. As soon as he realized the conceptual simplicity and universality of the quantum 1/f (Q1/f) derivation, as a property of the new notions of physical cross sections and process rates, he started a good cooperation with this author and with a dozen of graduate students, to verify the Q1/f formulas in all situations in which 1/f fluctuations are observed. In his final invited review paper [12], Aldert wrote: *Our project cannot check the validity or invalidity of Handel's derivation of his predictions for* $\boldsymbol{a_H}$*. This is the domain of the theoreticians. They have every right to criticize the derivation and replace it by a better one. In the latter case, they should see to it that their prediction for* $\boldsymbol{a_H}$ *agrees with Handel's prediction for* $\boldsymbol{a_H}$ *when the latter has been verified experimentally. Up to now this has not been done by them. It is difficult for some scientists to understand how a theory that is in their opinion incorrect can give correct predictions. It must be emphasized that only experiment can decide whether a conclusion is correct or incorrect. In our situation experiments decided that the predictions were right, and I see no way to avoid this conclusion.* We now understand the wisdom here better than ever ....

At the end of the Abstract (No. 145 in Appendix 3) of the paper presented by C.M. Van Vliet at the Xth (ICNF) Int. Conf. on Noise in Physical Systems in Budapest, Aug. 21-25, 1989, C.M. Van Vliet wrote: *"In retrospect we believe that the 'non-conventional' approach in Handel's papers, based on a 'stochastic Schrödinger field' is hereby mainly vindicated; in fact Handel has added a novel aspect of quantum mechanics, which stands out by its simplicity, as compared to the very lengthy perturbation expansions used in the diagrammatic as well as the present quantum field theoretical approaches."* This was the first time that the concept of a "new aspect of quantum mechanics" was introduced. On the solid basis of the theory of decoherence, we see today how its later developments, already included in the conventional and coherent Q1/fE, were also understood and anticipated by Van Vliet and Van der Ziel.

III. Quantum Information Theory Approach

Let's simplify our world and assume only one electromagnetic mode of the universe would be present, with frequency ω and wave vector k. Consider the field mode in its ground state and a pair of 2 incoming identical charged particles with the same well-defined wave vector are being scattered by some potential. Both the initial (incoming) and the final (scattered) state represent a pure state. The initial state is $|++-\rangle_0$, where we reserved the first two arguments of the ket for the two electrons and the last (-) for the field. Due to the interaction with the field, the final state is

$$|f\rangle = (|++-\rangle + \gamma|+-+\rangle/\sqrt{2} + \gamma|-++\rangle/\sqrt{2})/(\sqrt{1+\gamma^2}), \quad (2)$$

where γ is the emission amplitude of a bremsstrahlung photon, i.e., for the excitation of the field oscillator from its ground state (-) to its first excited state (+). The first two arguments label the state of the charged particles, indicating the presence of an energy loss with (-) and the persistence in the same energy state with (+). The third argument of the kets always labels the field oscillator, as mentioned.

The corresponding density operator of the pure state obtained is $\rho = |f\rangle\langle f|$. Its quantum von Neumann entropy $S/k = \sigma = -\mathrm{Tr}(\rho \log \rho)$ is zero.

A. Paradoxical Entropy Increase in 1/f noise

Ignoring the field oscillator, i.e., taking the trace of over the field oscillator label, we obtain a classically correlated system, a mixture of 2 pure states, described by the density operator

$$[|++\rangle\langle++| + \gamma^2(|+-\rangle + |-+\rangle)(\langle+-| + \langle-+|)/2]/(1+\gamma^2). \quad (3)$$

The second term gives the Q1/fE at f=ω/2π, as we show below. The corresponding entropy is

$$\log(1+\gamma^2) - (\gamma^2\log\gamma^2)/(1+\gamma^2) > 0 \text{ for } 0\le\gamma^2\le 1. \quad (4)$$

The entropy of the system thus appears to have increased although according to quantum mechanics it can not increase. Indeed, according to quantum mechanics, time evolution of a state occurs through a unitary transformation. The latter, however, is known to leave $\sigma = -\mathrm{Tr}(\rho \log \rho)$ invariant.

B. Quantum Information Theory

QIT [29] solves this paradox. When two systems A and B with quantum von Neumann entropy $\sigma(A) = -\mathrm{Tr}_A(\rho_A\log\rho_A)$ and $\sigma(B) = -\mathrm{Tr}_B(\rho_B\log\rho_B)$ form a composite system AB of entropy $\sigma(AB) = -\mathrm{Tr}_{AB}(\rho_{AB}\log\rho_{AB})$, we can prove that we have to write

$$\sigma(AB) = \sigma(A) + \sigma(B|A) = \sigma(B) + \sigma(A|B), \quad (5)$$

in perfect analogy with classical entropies or information. We have introduced σ(A|B) as the von Neumann entropy of A conditional on B, or the entropy of A when we know B:

$$\sigma(A|B) = -\mathrm{Tr}_{AB}[\rho_{AB}\log\rho_{A|B}]. \quad (6)$$

Here we have introduced the conditional density matrix $\rho_{A|B} = \rho_{AB}(1_A\otimes\rho_B)^{-1}$, the quantum analog similar to the classical conditional probability, where ⊗ is the tensor product in the joint Hilbert space and $\rho_B=\mathrm{Tr}_A(\rho_{AB})$ is the marginal density matrix obtained by taking a partial trace over the variables associated with A.

C. Negative entropy

The conditional entropy is usually negative in quantum entangled states. Finally, we introduce the quantum mutual entropy which represents the shared entropy, corresponding to the mutual information between A and B:

$$\sigma(A{:}B) = -\mathrm{Tr}[\rho_{AB}\log\rho_{A:B}] \equiv \sigma(A) + \sigma(B) - \sigma(AB), \quad (7)$$

where

$$\rho_{A:B} = \rho_{AB}(\rho_A\otimes\rho_B)^{-1}. \quad (8)$$

Taking all logarithms in the base of 2, the entropies will be expressed in bites. Considering our paradoxical case discussed above for simplicity with $\gamma^2=1$, we obtain the following entropy diagram (Fig. 1) of our quantum mechanically entangled triplet of three systems comprising the charged particles A and B, as well as the field oscillator C.

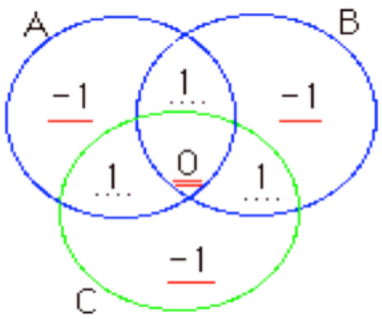

Fig. 1: Entropy diagram of the quantum mechanically entangled triplet comprising the charged particles A and B, as well as the field oscillator C.

Here the quantum conditional entropies are underlined for emphasis, and the mutual quantum entropy has double underlining. Dotted underlining marks the mutual entropy of two of the subsystems, which is, however, not the mutual entropy of the whole system. We see that A, B and C have each 1 bit of entropy, as we expect, since each can be in a + or - state. In addition, $\sigma_{ABC} = 0$ as expected for a pure state. The negative conditional entropies indicate that this state is a pure quantum state which can not be obtained in classical physics.

If we ignore C by tracing over it, we obtain (Fig. 2) the classically correlated system AB with its positive entropy of 1 *and* the negative quantum entropy state of the ignored field oscillator, conditional on AB:

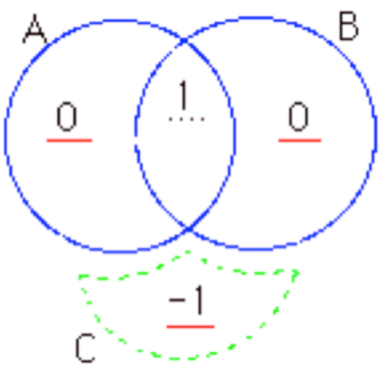

Fig. 2: By tracing over C, we obtain the classically correlated system AB with its positive entropy of 1 and the negative quantum entropy state of the ignored field oscillator, conditional on AB.

D. Solution of the Paradox

In conclusion, the negative conditional entropy of the ignored field oscillator compensates the positive entropy of the system of two particles. Unitarity and entropy conservation are both satisfied. This is the solution of the paradox. It explains why our earlier explanation of the Q1/fE in terms of a two-particle wave function was correct, in spite of the apparent lack of unitarity.

In the rest of the present paper we apply the scheme developed here to the real world with all field oscillators present in $|++-\rangle_o$ and with N incident particles in $|++-\rangle_o$. Note that the 1/f spectrum arises from the dependence of the actual bremsstrahlung coefficient γ on f, and from the summation over all field oscillators, as we see below.

IV. Simplified Derivation of the Conventional Quantum 1/f Effect

We start with the expression of the Heisenberg representation state |f> of N identical bosons of mass M emerging at an angle θ from some scattering process with undetermined bremsstrahlung energy losses reflected in their one-particle waves

$$\varphi_i(\xi_i)|f\rangle=(N!)^{-1/2}\Pi_i\int d^3\xi_i\varphi_i(\xi_i)\psi^+(\xi_i)|0\rangle =\Pi_i\int d^3\xi_i\varphi_i(\xi_i)|f^o\rangle, \qquad (9)$$

where $\psi^+(\xi_i)$ is the field operator creating a boson with position vector ξ_i and |0> is the vacuum state, while $|f^o\rangle$ is the vacuum field state with N bosons of position vectors ξ_i with i = 1...N. All products and sums in this paper run from 1 to N, unless otherwise stated.

To calculate the particle density autocorrelation function in the outgoing scattered wave, we need the expectation value of the operator

$$O(\mathbf{x}_1,\mathbf{x}_2) = \psi^+(\mathbf{x}_1)\psi^+(\mathbf{x}_2)\psi(\mathbf{x}_2)\psi(\mathbf{x}_1), \qquad (10)$$

known as the operator of the pair correlation. Using the commutation properties of the boson field operators, we first calculate the matrix element

$$N!\langle S^o|O|f^o\rangle= \Sigma'_{\mu\nu}\Sigma'_{mn} \delta(\boldsymbol{\eta}_\nu-\mathbf{x}_1)\delta(\boldsymbol{\eta}_\mu-\mathbf{x}_2)\delta(\xi_n-\mathbf{x}_1)\delta(\xi_m-\mathbf{x}_2)\Sigma_{(i,j)}\Pi'_{ij}\delta(\boldsymbol{\eta}_j-\xi_i). \qquad (11)$$

Here the prime excludes μ=ν and m=n in the summations and excludes i=m, i=n, j=μ and j=ν in the product. The summation $\Sigma_{(i,j)}$ runs over all permutations of the remaining N-2 values of i and j. On this basis we now calculate the complete matrix element

$$\langle f|O|f\rangle = [1/N(N-1)]\, \Sigma'_{\mu\nu}\Sigma'_{mn}\int d^3\eta_\mu\int d^3\eta_\nu\int d^3\xi_m\int d^3\xi_n\, \varphi_\mu^*(\boldsymbol{\eta}_\mu)\varphi_\nu^*(\boldsymbol{\eta}_\nu)\varphi_m(\xi_m)\varphi_n(\xi_n)\delta(\boldsymbol{\eta}_\nu-\mathbf{x}_1)\delta(\boldsymbol{\eta}_\mu-\mathbf{x}_2)\delta(\xi_n-\mathbf{x}_1)\delta(\xi_m-\mathbf{x}_2) \qquad (12)$$

$$= [1/N(N-1)]\, \Sigma'_{\mu\nu}\Sigma'_{mn}\, \varphi_\mu^*(\mathbf{x}_2)\varphi_\nu^*(\mathbf{x}_1)\varphi_m(\mathbf{x}_1)\varphi_n(\mathbf{x}_2)$$

The einselected one-particle states are spherical waves emerging from the scattering center located at x=0:

$$\varphi(\mathbf{x}) = (C/x)e^{iKx}\, [1 + \Sigma_{\mathbf{k}l}b(\mathbf{k},l)e^{-iqx}\, a^+_{\mathbf{k},l}]. \qquad (13)$$

Here C is an amplitude factor, K the boson wave vector magnitude, $b(\mathbf{k},l)=|b(\mathbf{k},l)|\exp(\gamma_{kl})$ the bremsstrahlung amplitude for photons of wave vector **k** and polarization l, with a random phase γ_{kl}., while $a^+_{\mathbf{k},l}$ is the corresponding photon creation operator, allowing the photon state to be created from the vacuum if Eq. (13) is inserted into Eq. (9). The momentum magnitude loss q=Mck/hK≡Mf/hK is necessary for energy conservation in the Bremsstrahlung process. Substituting Eq, (13) into Eq. (12), we obtain

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)\Sigma_{\mathbf{k},l}|b(\mathbf{k},l)|^2[1+\cos q(x_1-x_2)]\}, \qquad (14)$$

where we neglected a small term of higher order in b(**k**,l). To perform the angular part of the summation in Eq. (14), we calculate the current expectation value of the state in Eq. (13), and compare it to the well known cross section without and with bremsstrahlung

$$\mathbf{j} = (h\mathbf{K}/Mx^2)[1 + \Sigma_{kl}|b(k,l)|^2] = \mathbf{j}_o[1 + \int \alpha A df/f], \quad (15)$$

where the quantum fluctuations have disappeared, where $\alpha=e^2/hc$ is the fine structure constant, $\alpha A=(2\alpha/3\pi)(\Delta\mathbf{v}/c)^2$ is the fractional bremsstrahlung rate coefficient, also known in QED as the infrared exponent, and the 1/f dependence of the bremsstrahlung part displays the well-known infrared catastrophe, i.e., the emission of a logarithmically divergent number of photons in the low frequency limit. Here $\Delta\mathbf{v}$ is the velocity change $h(\mathbf{K}-\mathbf{K}_o)/M$ of the scattered boson, and $f=ck/2\pi$ the photon frequency. Eq. (14) gives

$$\langle f|O|f\rangle = |C/x|^4\{N(N-1) + 2(N-1)(\int \alpha A df/f)[1+\cos q(x_1-x_2)]\}, \quad (16)$$

which is the pair correlation function, or density autocorrelation function along the scattered beam with df/f=dq/q. The spatial distribution fluctuations along the scattered beam will also be observed as fluctuations in time at the detector, at any frequency f. According to the Wiener-Khintchine theorem, we obtain the spectral density of fractional scattered particle density ρ, (or current j, or cross section σ) fluctuations in frequency f or wave number q by dividing the coefficient of the cosine by the constant term N(N-1):

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/fN \quad [\text{or } =2\alpha A/f(N-1) \text{ for fermions}], \quad (17)$$

where N is the number of particles or current carriers used to define the current j whose fluctuations we are studying. Quantum 1/f noise is thus a fundamental 1/N effect.

The exact value of the exponent of f in Eq. (17) can be determined by including the contributions from all real and virtual multiphoton processes of any order, and turns out to be αA-1, rather than -1, which is important only philosophically, since αA«1. The spectral integral is thus convergent.

For fermions we repeat the calculation replacing in the derivation of Eq. (11) the commutators of field operators by anticommutators, which finally yields in the same way

$$\rho^{-2}S_\rho(f) = j^{-2}S_j(f) = \sigma^{-2}S_\sigma(f) = 2\alpha A/f(N-1), \quad (18)$$

which causes no difficulties, since N≥2 for particle correlations to be defined, and which is practically the same as Eq. (17), since usually N»1. Eqs. (17) and (18) suggest a new notion of physical cross sections and process rates which contain 1/f noise, and express a fundamental law of physics, important in most high-tech applications [11] - [21].

V. Discussion

We introduce the degree D of quantum 1/f decoherence and will focus on the determination of the decoherence time scales. We determine D from comparison of the calculated and measured 1/f noise, or phase noise levels close to carrier. These D values can be determined in a wider class of materials, devices and systems, at various temperatures, in order to use the D values for a new, independent method of investigating special materials, devices and systems, e.g., materials with lower dimensionality, for their transport properties and reliability, including the study of low-temperature behavior of D. A focus of future research is the application of these results to the improve quantum computers, by opening a new way of studying the decoherence..

In materials, devices and systems, D is usually close to 1, or 100% for QED quantum 1/f noise. However, decoherence happens only to about 10% in beta radioactive decay [14], i.e., D=0.1 in this case. It is also practically important to find how partial decoherence can reduce the observed Q1/f noise in devices.

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