Pressure drop and convective heat transfer in different SiSiC structures fabricated by indirect additive manufacturing

Ehsan Rezaei
MEMTI-SUPSI
6928 Manno, Switzerland

Institute of Mechanical Engineering, EPFL
1015 Lausanne, Switzerland
e-mail: ehsan.rezaei@epfl.ch

Maurizio Barbato
MEMTI-SUPSI
6928 Manno, Switzerland
e-mail: maurizio.barbato@supsi.ch

Sandro Gianella
EngiCer SA
Viale Pereda 22, Balerna 6828, Switzerland
e-mail: sandro@engicer.com

Alberto Ortona
MEMTI-SUPSI
6928 Manno, Switzerland
e-mail: alberto.ortona@supsi.ch

Sophia Haussener
Institute of Mechanical Engineering, EPFL
1015 Lausanne, Switzerland
e-mail: sophia.haussener@epfl.ch

1 Corresponding author.
The microstructure of porous materials has a significant effect on their transport properties. Engineered cellular ceramics can be designed to exhibit properties at will, thanks to the advances in additive manufacturing. We investigated the heat and mass transport characteristics of SiSiC lattices produced by 3D printing and replication, with three different morphologies: rotated cube, Weaire-Phelan and tetrakaidecahedron lattices, and a commercially available ceramic foam. The pressure gradients were measured experimentally for various velocities. The convective heat transfer coefficients were determined through a steady state experimental technique in combination with numerical analysis. The numerical model was a volume-averaged model based on local thermal non-equilibrium assumption of the two homogeneous phases. The results showed that for tetrakaidecahedron and Weaire-Phelan structures, undesirable manufacturing anomalies (specifically window clogging) led to unexpectedly higher pressure drops across the samples and increased thermal dispersion. Compared to the tetrakaidecahedron and Weaire-Phelan structures, the manufactured rotated cube lattice and the random foam had lower heat transfer rates but also lower pressure drops. These lower values for the rotated cube lattice and foam are also a result of their lower specific surface areas.

1. INTRODUCTION

It is well known that the properties of porous media not only depend on their bulk material properties but also on their microstructure [1]. The traditional production of open-cell porous ceramics results in assemblies of edges (called cell struts) and faces (called cell windows) that form randomly organized cells connected together through open faces [2]. Recently, efforts have been devoted to investigating novel additive manufacturing methodologies to directly or indirectly fabricate ceramic structures [3-5]. As a result, researchers have been working on design and characterization of structures...
with engineered unit-cells, called lattices [6-9]. Compared to traditional random foams, lattices produced by additive manufacturing techniques are reproducible and offer more design variability, which can result in structures with tailored properties such as high stiffness/weight ratio [10], zero thermal expansion coefficient [11], and negative Poisson’s ratio [12]. However, while the multi-physical behavior of the ceramic foams is extensively investigated, there is a lack of studies on the behavior of lattices and other novel structures.

Porous ceramics are often used in high temperature applications given by their high-temperature stability, their inertness to acidic or base environments, their possibility to withstand oxidative environments, and their relatively high mechanical strength and thermal shock resistance [2], [13-15]. Furthermore, they can be manufactured to exhibit high specific surface areas and a percolating fluid phase, making them particularly interesting for applications including heat exchangers, radiant burners, reformers, solar receivers, or heat storage systems [16-19]. The study of the thermal and fluid transport in porous ceramic structures is essential for a subsequent optimization of their application [20, 21].

The heat transfer from the solid phase to a fluid passing through the porous media is often characterized by the volumetric convective heat transfer coefficient. In the literature, two experimental approaches are usually followed for the determination of the volumetric convective heat transfer in porous media. Younis and Viskanta [22] calculated the volumetric convective heat transfer coefficient for different velocities in ceramic foams by passing steam into the cold foam and measuring the transient gas
temperature at the outlet. In a different approach, Calmidi and Mahajan [23] investigated aluminum foams by passing cold air into the foams, which were heated by an electric source. They calculated the average convective heat transfer coefficient at different velocities using the steady state temperature readings of the thermocouples. Fuller et al. [24] illustrated that the transient method is a more accurate technique as it excludes the conduction effects on the samples. However, the results of the two methods will converge for thin samples with high thermal conductivities. Various studies used both methods to determine the heat transfer rate in the ceramic foams, however only few correlations have been proposed. Younis and Viskanta [22] investigated the heat transfer in a set of 10-65 PPI alumina and cordierite foams with porosities ranging between 0.83-0.87. They proposed a correlation based on the foam’s pore diameter and sample length. Kamiuto and Yee [25] proposed a correlation based on a collection of results from 16 foams reported in literature. Dietrich [26] tested 10-45 PPI foams made of alumina, mullite and oxidic-bonded SiC, with porosities ranging between 0.75-0.85, and proposed a Nusselt-Hagen correlation valid for a large range of Reynolds numbers. Xia et al. [27] proposed their own correlation based on a set of metallic and ceramic foams and performed a comparison between different correlations proposed in the literature. They found a large discrepancy between various correlations that were attributed to different conditions at which the studies were carried on.

The pressure drop within a porous medium has been particularly well studied due to the simplicity and low cost of the measurement procedure. It consists of passing fluid into a duct filled with the porous medium and measuring the pressure at the inlet and
outlet of the sample [28-31]. The accuracy of this method is high as long as the sample is thick enough or exhibits low permeability. Several groups studied the pressure drop of ceramic or metal foams [29-35]. Edouard et al. [36] published a summary of the correlations reported in these studies. Dietrich et al. [28] proposed a correlation based on experiments of different ceramic foams and compared their correlation with large number of experimental data from the literature, demonstrating that their correlation could predict most of the literature values within an error range of ±40%.

Advances in computational power have allowed advanced numerical characterization of porous structures utilizing their exact microstructures (obtained, for example, by computed tomography and subsequent digitalization through various segmentation approaches) in pore-level simulation to calculate the transport properties and structural parameters [37], [38]. Transport characteristics of regular lattices (such as tetrakaidecahedron and Weaire-Phelan structures) have also been modeled, claiming that they are the closest representative lattices to the structure of the random foams [39, 40].

In the literature, the experimental studies on transport properties have been mainly performed on random foams. To the author’s knowledge, there is no study in the literature that systematically analyzes pressure drop and heat transfer behavior of ceramic structures with different unit-cells made by the same manufacturing procedure. This is a particularly important task for the case of SiC-based structures as they have been widely used for high-temperature applications involving fluid flow and heat transfer [16, 17, 19, 41]. Here, we aim at analyzing the effect of the porous medium’s
unit-cell shape on the transport behavior. For this reason, lattices with three different microstructures are designed and manufactured. A random foam with similar porosity is also produced for comparison. The specimens were used in a combined experimental-numerical approach to quantify their pressure drop and convective heat transfer behavior. The morphology of the produced specimens was characterized and the advantages and disadvantages of the manufacturing technique are briefly discussed. Nusselt correlations are formulated for each specimen and the results are compared with literature.

2. SAMPLES AND THE EXPERIMENTAL SET-UP
2.1. Fabrication And Characterization Of The Porous Specimens

Lattice structures with three different unit-cells, rotated cube (RC), tetrakaidecahedron (TK), and Weaire-Phelan (WPh), were designed in a cylindrical form with diameter of 20 mm and length of 100 mm. We used an in-house tool (developed in MATLAB R2014a) that generated lattices with a given unit-cell inside a cylindrical enclosure and cropped the cells on the boundary surfaces. The polymeric templates of the designed lattices were fabricated using a commercial 3D-printer (3DLPrinter-HD 2.0, Robot Factory S.r.l., It), with a resolution of 0.01 and 0.05 mm in vertical and horizontal axes, respectively. The printed polymeric structures were then used as the template to manufacture near-net-shape SiSiC lattices by EngiCer SA (Balerna, CH), via the replica technique followed by reactive melt infiltration of silicon. The detailed manufacturing process of the samples is described by Ortona et al. [3]. In addition, SiSiC foams were manufactured by
the same procedure utilizing commercial 5 PPI polyurethane foams as the template. A photo of the final SiSiC pieces is shown in Fig. 1. The relative density was calculated as the weight of each sample divided by the mass of the bulk material of the same volume. The total porosity, $\phi$, was determined based on the calculated relative density. The cell size, $d_c$, was defined as the unit-cell length for the lattices and the nominal cell diameter for the random foam. The strut diameter, $d_{st}$, was obtained based on 10 random measurements in the middle of the struts (center location between two strut junctions) on each of the specimens. The lattices were designed with a 5 mm cell length and a 5 PPI (pores per inch, i.e. ~5 mm nominal pore diameter) random foam was chosen for comparison. The manufacturing procedure resulted in lattices with 5% shrinkage. An overview of the microstructural properties of the investigated specimens is given in Table 1. The specific surface area, defined as the total surface of the struts per unit volume, was calculated by creating CAD models of the lattices with cell and strut diameters, measured from each manufactured lattice. The specific surface area of the random foam was obtained from a correlation proposed by Inayat et al. [42] for foams with circular struts:

$$S_v = 3 \left(1 - \frac{\phi}{d_{st}}\right)$$  \hspace{1cm} (1)

2.2. Experimental Set-Up

A scheme of the test rig used for the measurements of the pressure drop and the volumetric convective heat transfer is shown in Fig. 2. The air stream was continuously
supplied by a compressor, and passed through an air preparation unit, comprising a pressure regulator, filter and dehydrator sections, in order to decrease the supplied pressure and remove the moisture and contaminants such as oil and dust particles. A commercial mass flow controller (GSC red-y, Vögtlin, CH) was used with an accuracy of ±0.24%. The mass flow controller works based on the thermal mass flow measurement principle [43], and uses a sealed valve with very low leakage (10⁻⁶ mbar l/s He) and fast response time (300 ms) to control the flow rate. The air then entered an expansion chamber (in order to damp the possible pressure waves) and an entry pipe of 50 cm length before entering the furnace. A tubular electric furnace (XTUBE series 1600, Xerion, Freiberg, De) was used to heat up the samples. The tube is heated using two U-shaped heating elements (Fig. 2), each with a diameter of 0.8 cm, that are placed in the middle of the furnace, 6 cm apart from each other. An alumina tube was horizontally fitted in the middle of the furnace, passing through the heating elements and connected to the inlet tube by a water-cooled flange.

The samples were placed in the middle of the alumina tube, wrapped into a thin layer of expanded graphite foil to avoid air bypassing between the sample and the tube. A differential pressure transmitter (DPGM409, Omega Engineering, Inc., US) with working pressure range of 0–2500 Pa and accuracy of 0.08% BSL (best straight line) was used to measure the pressure drops. The temperatures were monitored by several K-type thermocouples, placed on the alumina tube, on the specimens and in the air stream at the inlet and the outlet. All thermocouples were calibrated to have an accuracy of ±0.4°C. For the measurements of the tube wall temperature, $T_{\text{wall}}$ in Fig. 2, seven
thermocouples were fixed on the alumina tube in different locations along the tube length, glued to the surface using a high-temperature silicate adhesive (Coltogum 1500, Blattener AG, Zürich, CH) and covered by an alumina felt to decrease the radiation effects. The set-up had a quasi-axis-symmetric configuration and, therefore, the temperature was assumed to be uniform around the tube (in azimuthal direction) for each axial position. The assumption was confirmed by a preliminary test, measuring the tube wall temperature difference at the upper and lower side of the tube (azimuth of 0° and 180°) at three positions along the axial direction (close to the inlet, middle and outlet of the specimen). The temperature along the azimuth (for same axial and radial position) differed in maximum by 0.5°C only. To determine the lattice temperature, a thermocouple with exposed tips was tightened to a strut, glued and isolated with the silicate adhesive. The inlet temperature was measured by a thermocouple placed in the middle of the duct just before the air enters the alumina tube. To measure the outlet air temperature three thermocouples were fixed on a mounting structure and inserted into the alumina tube. The thermocouples were placed 10 mm behind the porous sample in order to measure the outlet air temperature \( T_{f,\text{out},i} \) \( (i \) is the thermocouple number shown in Fig. 2). One of the three thermocouples was at the center \( (i=2) \), while the other two were fixed at off-center locations, 5 mm up \( (i=3) \) and down \( (i=1) \) from the center. Assuming the thermocouple’s bead as a perfect sphere with a diameter of 1 mm and emissivity of 0.9, the fluid temperature measurements were corrected for radiation losses using the bare-bead model [44]. Whitaker’s correlation [45] was used to obtain the convective heat transfer coefficient for the flow passing over the sphere. All data
were recorded using NI data acquisition modules (NI 9219, NI 9213 and NI 9203, National Instruments, Austin, TX, USA).

3. CALCULATION PROCEDURE
3.1 Pressure Drops Coefficients

The pressure drop was experimentally measured and used to calculate permeability, $K$, and form coefficient, $C$ (also called Dupuit-Forchheimer coefficient). For each porous structure, two specimens were placed in the middle of the alumina tube, one after the other to form a 200 mm long sample. The desired air mass flow rate was selected and the pressure drop, $\Delta p$, was measured. The experiments were conducted in ambient temperature for different mass flow rates between 2.5 and 50 NL/min, corresponding to superficial velocities ranging between 0.15-2.97 m/s. For an isothermal flow with constant inlet velocity, the Darcy-Forchheimer equation can be written for 1-D domain in a non-dimensional form as [38]:

$$\frac{-\Delta p}{\mu_\ell \nu_\ell} \frac{d_c^2}{\Pi} = \frac{d_c^2}{K} + \frac{d_c}{b} \frac{\rho_\ell \nu_\ell}{\mu_\ell} \frac{d_c}{\Re},$$  \hspace{1cm} (2)

where $L$ is the length of the porous sample, $\rho_\ell$ and $\mu_\ell$ are the density and dynamic viscosity of the air at ambient temperature, and $\nu_\ell$ is the air superficial velocity in the porous medium. Knowing the non-dimensional pressure drop, $\Pi$, for different Reynolds numbers, the intercept and slope ($a$ and $b$ in Eq. 2) and consequently $K$ and $C$ were calculated by a least-square method.
3.2 Volumetric Heat Transfer Coefficients

To determine the volumetric convective heat transfer coefficients of the porous samples, a steady state approach was employed. The lattices were cut to obtain samples of 25 mm length and then placed in the middle of the alumina tube. The furnace temperature was fixed to 100°C and the air at ambient temperature was introduced at the inlet of the alumina tube. The experiments were done for five mass flow rates between 5 and 15 NL/min at ambient temperature, corresponding to the superficial velocities ranging between 0.30-0.89 m/s. For each test, the desired mass flow was set and the temperatures were recorded at steady state condition, reached after about 20 min. The pipe flow Reynolds number, $Re_{pipe}$, at the inlet varied between 380-1160, and thus a laminar flow was ensured in the duct. In order to evaluate the convective heat transfer coefficients, an inverse method was used, consisting of comparing the steady state data of the experiments with the numerical predictions and adjusting the volumetric convective heat transfer coefficient (in the simulations) until both agree. The numerical method we used to calculate the heat transfer coefficients is presented in the next section.

3.2.1. Computational Heat Transfer And Fluid Flow Model

The following assumptions and simplifications were made for the computational model: (1) the inlet flow is laminar with a uniform temperature, (2) the porous domain is modeled by two homogenous and isotropic media (solid and fluid), (3) the solid and fluid phases are in local thermal non-equilibrium (LTNE), (4) the solid phase temperature
is considered constant given by the high conductivity of the material and the short
sample length, and (5) the buoyancy forces in the fluid are not negligible due to high
radial temperature gradients and low fluid velocities.

Brinkman equations with Forchheimer corrections were solved for the porous media,
considering a formulation for low Mach number, steady state compressible flow [46]:

\[
\nabla \cdot (\rho_t \mathbf{v}_t) = 0
\]

(3)

\[
\frac{\rho_f}{\phi^2} (\mathbf{v}_t, \nabla) \mathbf{v}_t = -\nabla p + \nabla \left[ \mu_t (\nabla \mathbf{v}_t + (\nabla \mathbf{v}_t)^\top) - \frac{2}{3} \frac{\mu_t}{\phi} (\nabla \cdot \mathbf{v}_t) \mathbf{I} \right] - \left[ \frac{\mu_t}{K} + \rho_t C |\mathbf{v}_t| \right] \mathbf{v}_t + \rho_t \mathbf{g},
\]

(4)

\(K\) and \(C\) are independent of the fluid velocity and were calculated directly from the
pressure-drop experimental test results (see section 3.1).

The LTNE formulation of the energy equation was solved for the fluid phase in the
porous media domain [47]:

\[
\rho_t c_{p,f} \mathbf{v}_t \nabla T_f - \nabla \left( \phi \lambda_f \nabla T_f \right) = h_v (T_s - T_f),
\]

(5)

where \(c_{p,f}\) and \(\lambda_f\) are the specific heat capacity and thermal conductivity of air, \(T_s\) and \(T_f\)
stand for the intrinsic temperature average of the solid and fluid phases, and \(h_v\) is the
volumetric convective heat transfer coefficient \((h_v = S_v h_{st})\). For the free fluid flow
inside the inlet tube, the standard Navier-Stokes and energy equations for a single-
phase flow model were used. In all the simulations, the fluid was assumed as ideal gas
and its properties were calculated as a function of the local temperature.
3.2.2. Numerical Procedure

For every specimen and each mass flow rate, the convective heat transfer coefficient was estimated by solving the numerical model several times until it matched the experimental steady state fluid outlet temperature data. As this iterative procedure is highly time-consuming, a submodeling technique was implemented so as to reduce the calculation domain in the iterative procedure. A numerical solution for the global model was acquired initially using the entire domain, in which the porous medium was modelled with an initial guess value for the convective heat transfer coefficient. A submodel (indicated by the red rectangle in Fig. 3) was used for further iterations, which didn’t need to again solve the equations for the entry region.

The coupled continuity, momentum and energy equations, formulated in a 3D domain, were solved in Comsol Multiphysics package® (ver. 5.2) interlinked with Matlab. Fig. 3 displays the computational domain, which consisted of a cylinder with the porous specimen inside. Due to the symmetry only half of the cylinder was investigated. As we accounted for gravity, axis-symmetry was not given.

For the global mode, no slip conditions with fixed temperature on the lateral walls and constant solid temperature for the porous medium were given as boundary conditions. Furthermore, a laminar flow with uniform temperature and a velocity profile, calculated as a function of the mass flow rate and the entrance length, was considered at the inlet. No conductive heat transfer was also applied at the outlet. The outlet pressure was specified as:
\[ p_{\text{out}} = p_{\text{atm}} - \rho_f g z \] 

(6)

The exit tube length (after the porous domain) was chosen long enough to avoid backflow in the outlet. For the submodel, all the boundary conditions are the same as the global model except for the inlet. The solution of the global model was prescribed on the cutting interface as the inlet boundary conditions for the submodel.

A first order discretization scheme was implemented and the generalized minimal residual (GMRES) iterative solver with a relative tolerance of $10^{-3}$ was set for the convergence of the solutions. Convergence with this relative tolerance value was ensured by solving for a value of $10^{-4}$ and comparing temperatures, velocities and pressures. They varied by less than 0.23%. A mesh made of variable non-uniform mesh element sizes was used in the tube’s axial direction in order to have finer elements near the inlet, outlet and the porous medium regions. Grid convergence was obtained at about $9.6 \times 10^5$ and $4.6 \times 10^5$ cells for the global and the submodels. Mesh convergence was confirmed by doubling the element numbers and comparing temperatures, velocities and pressures, which varied by less than 0.5%.

The numerical procedure used to determine the volumetric convective heat transfer coefficients is as follows:

1- The global model is solved using a guess value $h_{\nu} = h_0$

2- Two values $h_0$ and $h_1 = 1.2h_0$ are used as the initial guess for $h_{\nu}$ in the submodel
3- Two solutions are obtained solving the submodel for $h_0$ and $h_1$ and the outlet air temperatures at the center location, $(T_{f,\text{out2}})_{\text{num},h0}$ and $(T_{f,\text{out2}})_{\text{num},h1}$, just after the porous medium.

4- A new $h_\nu$ is estimated as:

$$h_\nu = \frac{[ (T_{f,\text{out2}})_{\text{exp}} - (T_{f,\text{out2}})_{\text{num},h0} ]}{[ (T_{f,\text{out2}})_{\text{num},h1} - (T_{f,\text{out2}})_{\text{num},h0} ]} \cdot (h_1 - h_0) + h_0$$  \hspace{1cm} (7)

5- The new value $h_\nu$ is used as $h_2$ and step 3 and 4 are repeated for $h_1$ and $h_2$ instead of $h_0$ and $h_1$ obtaining a new $h_\nu$. The same process continues for new values $h_2$ and $h_3$ until the last iteration with $h_{n-1}$ and $h_n$ where the following criteria is fulfilled and the solution is converged:

$$| (T_{f,\text{out2}})_{\text{num},n} - (T_{f,\text{out2}})_{\text{exp}} | < 0.04 \degree C$$  \hspace{1cm} (8)

The threshold value of $0.04 \degree C$ in Eq. 8 was chosen as 10% of the uncertainty of the thermocouples. Depending on the initial guess, the numerical procedure converged after 6-10 iterations. The deviation of the numerical solution from the experimental temperature values at the three outlet positions was calculated from Eq. 9. The deviation was smaller than 0.05 for all the cases.

$$D = \sqrt{\frac{1}{3} \sum_{i=1}^{3} \left[ \frac{(T_{f,\text{out}i})_{\text{num}} - (T_{f,\text{out}i})_{\text{exp}}}{(T_{f,\text{out}i})_{\text{exp}}} \right]^2}$$  \hspace{1cm} (9)
3.3 Uncertainty Analysis

The uncertainty of the calculated $K$ and $C$ is a function of the uncertainties of the air properties, the specimen length, and the measured velocity and pressure values. In all our calculations, the uncertainty of the air properties was not considered and the uncertainty of $d_c$ was considered negligible. As the experimental values and their uncertainties ($\sigma_{\Delta p} = 2 \text{ Pa}$, $\sigma_L = 1 \text{ mm}$, $\sigma_{v_t} = 0.0024 \text{ m/s}$) were known, the uncertainty of $II$ (as defined in Eq. 2) was calculated by applying an error propagation analysis:

$$\sigma_{II} = \sqrt{\left(\frac{\partial II}{\partial \Delta p} \sigma_{\Delta p}\right)^2 + \left(\frac{\partial II}{\partial L} \sigma_L\right)^2 + \left(\frac{\partial II}{\partial v_t} \sigma_{v_t}\right)^2}.$$  \hspace{1cm} (10)

The uncertainty of the Reynolds number was calculated as:

$$\sigma_{Re} = \sqrt{\left(\frac{\partial Re}{\partial v_t} \sigma_{v_t}\right)^2}.$$  \hspace{1cm} (11)

Knowing the values and uncertainties of $II$ and $Re$ for each test, the parameters $a$ and $b$ (and consequently $K$ and $C$) and their standard errors were obtained using the iterative least squares solutions developed by York et al. [48].

The estimation of the uncertainty of the predicted volumetric convective heat transfer coefficient was not straightforward as the values were obtained through a combined experimental and numerical analysis. A sensitivity analysis of the numerical model was conducted using the uncertainty of the main input parameters. The effect of the measured tube’s surface temperature, absolute pressure of the air, and the geometry parameters was assumed negligible. The uncertainty of volumetric convective heat transfer coefficient was calculated by varying three experimental parameters - the
outlet air temperature \((T_{\text{out}})_{\text{exp}}\), the porous solid temperature, and the air inlet velocity - within their experimental uncertainties \((\sigma_T = 0.4^\circ \text{C}, \sigma_{v_f} = 0.0024v_f \text{ m/s})\).

The sensitivity of the calculations to these three parameters, \(P_i\), was calculated by:

\[
S_{hv} = \sqrt{\frac{\sum_{i=1}^{3} (\frac{\sigma_{P_i} - (h_{hv})_{P_i} - \sigma_{P_i}}{2 (h_{hv})_{P_i}})^2}{\sum_{i=1}^{3} (\frac{\sigma_{P_i} - (h_{hv})_{P_i} - \sigma_{P_i}}{2 (h_{hv})_{P_i}})^2}}
\]  

\[(12)\]

4. RESULTS AND DISCUSSION

4.1. Measured Pressure Drop

Experimental results expressed as the pressure drop per sample length for various flow rates for the four porous structures are shown in Fig. 4. The pressure drop increases exponentially when raising the mass flow rate. At low Reynolds numbers (of the order unity), the pressure drop is a linear function of the velocity (Darcy’s law). At higher velocities, before entering the turbulence regime, a second-order polynomial model fits the data best, resulting from the dominance of the inertia effects. To determine permeability and form coefficients, we only used the data at \(Re < 250\) to ensure that the flow didn’t enter the turbulent regime [49]. Table 2 shows the calculated permeability and form coefficients together with their uncertainties. The high values of the coefficient of determination, \(R^2\), from the linear fitting of Eq. 2 suggest that the Darcy-Forchheimer model fits the data well. The uncertainties of \(C\) were lower compared to the ones of \(K\). This was because of the high uncertainty of the pressure drop measurements at lower velocities, which has a higher contribution to the
uncertainty of $K$ than $C$. Fig. 5 shows the dimensionless pressure gradient with experimental uncertainties as a function of Reynolds number. For the sake of clarity, only the data from WPh lattices and the foam are shown.

Pressure gradients in WPh and TK lattices were 2-3 times higher as in the RC lattice and random foams. As shown in Table 1, designed TK and WPh lattices have higher specific surface area than the other two specimens, which causes higher viscous drag and therefor higher pressure drops. Besides, the morphology of the manufactured lattices is different from their ideal designed shape: the replica process resulted in the agglomeration of material in the strut joints (nodes) during the impregnation of the polymeric template in the ceramic slurry, which led to bigger nodes. Furthermore, during the manufacturing process, the high cohesion forces of the slurry kept a membrane in small windows, resulting in the clogged cell windows, a phenomenon that was mostly observed in the TK and WPh lattices. This was not the case for the RC lattice as the cubic unit-cell contained larger windows. In case of TK, all the square windows in each unit-cell were closed, which accounts for about 29% of the windows. For WPh, counting the closed windows in each periodic section resulted in a total of about 20% closed pores in the structure. A closed pore deviates the fluid flow passage, i.e. increases the tortuosity [50], and also reduces the available cross-sectional area for the fluid to pass. This is probably the main reason for the higher pressure gradients in the TK and WPh samples. The lower pressure-drop values in the random foam (compared to the RC lattice) are attributed to its 4.7% higher porosity and lower specific surface area.
4.2. Literature Comparison For Pressure Drop

Experimentally determined $K$ and $C$ of the foams with the same pore size (5 PPI) but different porosities (0.91-0.97) obtained from literature [30, 32, 34, 51, 52] were used for comparison (Table 3). Even though we chose reported data for samples with similar porosity and the same PPI value, it is not assured that their morphology is equivalent. The manufacturing process can, for example, affect the morphology by formation of clogged pores or different shapes of the struts. As illustrated by Inayat et al. [42], manufactured ceramic foams with different porosities have different strut shapes. The foams with porosities lower than 0.9 have a circular strut cross-section (similar to our samples), while increasing the porosity results in struts with triangular and concave triangular cross-sections. Also, the values reported in Table 3 are based on experiments with Reynolds numbers up to 5000, which clearly lies outside of the Forchheimer flow regime. At large Reynolds numbers a cubic model is recommended [53].

The foam used in our work has a permeability value within the range of permeability values reported ($2.30 \pm 0.65 \cdot 10^{-7} \text{ m}^2$) but its form coefficient is 44% larger than the average of the reported values ($188.5 \pm 18.6 \text{ m}^{-1}$). Accurate permeability values can be obtained by investigations at low Reynolds numbers, where the pressure changes are mainly due to the friction at the solid strut walls [2]. The higher form coefficient of our foam can be attributed to its lower porosity, which is about 9% less than the average porosity of the data in Table 3. In general, decreasing the porosity decreases the effective flow passage cross section leading to higher flow resistance.
For the comparison of the lattice structures, we used computational pressure drop predictions reported for regular structures. Cunsolo et al. [40] modelled a laminar flow in both WPh and TK structures and reported Darcy–Weisbach friction factors for different porosity and cell sizes. Wu et al. [39] also used a turbulence model to numerically investigate the pressure drop in an ideal TK lattice. Fig. 6 shows the results of these two works together with the results of our study. Results of Cunsolo et al. are 25-70% higher than our values. These higher values are due to the methodology they used to calculate the friction factor and Reynolds number. In their model, the porous structure was heated up by a constant flux and the convective heat transfer coefficients were determined from the volume-averaged values of the solid and fluid temperatures. However, the calculations of Reynolds number and friction factor were based on the inlet velocity, which was lower than the average velocity. We expect that using the average velocity, will decrease their calculated friction factor and increase the Reynolds number, and consequently decreases the gap between their and our results. Also, the triangular shape of the strut cross section in their models results in higher tortuosity and thus higher pressure drops [54]. Wu et al. used circular struts, which is closer to the shape of the struts in our study. However due to the clogged windows, the final shapes of the manufactured TK and WPh lattices are different from the ideal models considered in the two numerical works. Our samples had friction factors 1.4-1.9 times larger than Wu et al., which can be explained by the higher drag forces caused by the clogged pores in the manufactured samples.
4.3. Predicted Convective Heat Transfer Coefficients

The samples were tested at five different inlet velocities (between 0.30 and 0.89 m/s). A typical result of the experimental and numerical air temperatures at the same sample cross section is presented in Fig. 7. We noted that the temperature profile was asymmetric due to the effect of buoyancy. The air properties and velocity used in the Reynolds and Nusselt formulations are based on the calculated volume averaged temperature and velocity of the fluid in the porous media. Fig. 8 shows the predicted mean Nusselt number as a function of Reynolds number. TK and WPh lattices showed similar behavior and slightly higher values than RC lattice and the random foam. RC lattice had the lowest heat transfer rates, 30% lower than WPh at higher Reynolds numbers. As mentioned by Fuller et al. [24], there are two main causes for the enhanced heat exchange in porous media: higher specific surface area and tortuosity. In porous structures with high thermal conductivity (such as SiSiC specimens in this study), higher specific surface area means extended surfaces for heat transfer from the solid walls to the fluid and, thus, is directly proportional to the volumetric convective heat transfer coefficient. Ideal structures of WPh and TK lattices have larger specific surface area than the foam and RC structures (Table 1). The clogged pores in both manufactured WPh and TK lattices can change the flow characteristics, leading to a more tortuous fluid path [31] but also an increased probability of the flow separation from the solid walls. On one side, higher tortuosity increases the thermal dispersion within the porous medium and on the other side the separation effect can decrease the heat transfer from the solid walls to the fluid [40]. Furthermore, the lattices have slightly lower porosity (on average...
about 4% lower) than the random foam. Since porosity is inversely related to the fluid pore velocity, the lower porosity results in higher pore velocity, which can result in higher local heat transfer between the solid and the fluid.

Empirical volumetric convective heat transfer coefficients are often reported in the form of Nusselt correlations as a function of porosity, Prandtl number and Reynolds number [2, 25, 26]. Nusselt number should asymptotically reach a constant value for the limit of very low Reynolds numbers [55]. However for the investigated range (70<Re<250), a simpler power correlation is postulated:

$$\text{Nu}_v = a_1 \text{Re}^{a_2}$$  \hspace{1cm} (13)

The coefficients $a_1$ and $a_2$ were obtained for the lattices and the foam by least square fitting. $a_1$ is typically a function of Prandtl number and porosity. As we didn’t test different fluids, no Prandtl dependence could be estimated. Theory predicts a $Pr^{1/3}$ dependence for the flows over a cylinder [56]. The porosity dependence for the random foams are also reported [21, 27, 57]. Table 4 lists the correlation coefficients with the $R^2$ values of the fittings.

**4.4. Literature Comparison For Heat Transfer**

Most of the data reported in literature has been obtained by analyses of random foams with small pore diameters (i.e. PPI≥10). Table 5 collects empirical correlations based on random foams of different materials, cell density, and porosities.
Fig. 9 (a) displays a comparison between the heat transfer results of the foam in this study and the values predicted by the correlations from literature. We observed a high variation between the proposed correlations, reaching a difference up to one order of magnitude. At low velocities, our foams agree best with the correlation of Xia et al. [27]. At high velocities the correlation of Wu et al. [57] has a better agreement with our experimental data.

Similar to the pressure drop results, WPh and TK lattices are compared in Fig. 9 (b) with the numerically obtained heat transfer results of Cunsolo et al. [40] and Wu et al. [57]. The overall trend of our results agrees with their predictions. At high Reynolds numbers, the correlation of Wu et al. [39] predicts similar Nusselt numbers as our results, however at lower Reynolds numbers we predict values 65% lower than theirs.

The discrepancy between the experimental works, shown in Fig 9 (a), is attributed to the different measurement techniques and their corresponding errors (steady state or transient methods), and due to the different ranges of measurement conditions (e.g. skeleton’s material or flow velocity). We applied the correlation to our material, porosity and flow conditions. Additionally, the traditional manufacturing processes result in foams that are geometrically different from sample to sample [56]. Various irregularities are present in the pore microstructure of commercial foams, which results in different cell distributions, presence of closed pores, local agglomeration of material, and varying surface roughness of each sample. These irregularities, which influence the flow path and the heat transfer rates, are not accounted for in any of the correlations shown in Table 5. Direct additive manufacturing techniques can resolve some of these irregularities.
problems, as they benefit from reproducible computer-controlled processes for the creation of complex structures.

5. CONCLUSIONS

We present pressure drop and convective heat transfer measurements of porous media with different morphologies. RC, TK and WPh lattices (obtained by additive manufacturing) and commercial random foams with similar structural parameters (porosity and cell density) are investigated. The volumetric convective heat transfer coefficients of the specimens were estimated using the steady state experiments combined with numerical simulations. The numerical model consisted of solving the Brinkman equations with Forchheimer corrections coupled with two-homogeneous phase local non-thermal equilibrium formulations.

The production of the specimens using replica technique caused undesirable cell window clogging, which was observed specifically in smaller cell windows in TK and WPh samples and affected their transport properties (compared to ideal, unclogged TK and WPh structures). The pressure drop and heat transfer analysis showed that the manufactured RC lattice and the random foam have 2-3 times lower pressure drops and slightly lower heat transfer rates compared to WPh and TK lattices. The higher pressure drop values in TK and WPh samples were mainly attributed to the clogged pores, leading to higher flow resistance. However, for the same porosity and cell density, the lower specific surface area of the RC lattice and the foam explained their lower heat transfer rates. Hence, the overall transport properties of the porous structures are not only a
function of the cell size and porosity but also depend on the geometrical anomalies, which can deviate the flow path. Finally, correlations are proposed for each of the morphologies to obtain volumetric Nusselt number as a function of Reynolds number, valid for the range of $70<\text{Re}<250$. The results were then compared to the literature showing an agreement for the foam values and general support for the values obtained for the TK and WPh lattices.

ACKNOWLEDGMENT

This material is based upon work performed in cooperation with CTI Swiss Competence Centers for Energy Research (SCCER Heat and Electricity Storage).
Latin

\( c_p \)  
heat capacity at constant pressure

\( C \)  
form coefficient

\( d \)  
diameter

\( g \)  
gravity constant

\( h_{sf} \)  
interfacial heat transfer coefficient

\( h_v \)  
volumetric convective heat transfer coefficient

\( K \)  
permeability coefficient

\( L \)  
sample length

\( p \)  
pressure

\( q \)  
heat flux vector

\( S \)  
sensitivity

\( S_v \)  
specific surface area

\( T \)  
temperature

\( v \)  
superficial velocity

\( \mathbf{x}, \mathbf{y}, \mathbf{z} \)  
coordinate vectors
Greek

\( \lambda \)  \hspace{1cm} \text{thermal conductivity}

\( \mu \)  \hspace{1cm} \text{dynamic viscosity}

\( \rho \)  \hspace{1cm} \text{density}

\( \sigma \)  \hspace{1cm} \text{uncertainty}

\( \phi \)  \hspace{1cm} \text{porosity}

Subscripts

\( c \)  \hspace{1cm} \text{cell}

\( h \)  \hspace{1cm} \text{hydraulic}

\( \text{exp} \)  \hspace{1cm} \text{experimental}

\( f \)  \hspace{1cm} \text{fluid}

\( \text{in} \)  \hspace{1cm} \text{inlet}

\( \text{out} \)  \hspace{1cm} \text{outlet}

\( \text{num} \)  \hspace{1cm} \text{numerical}

\( p \)  \hspace{1cm} \text{pore}

\( s \)  \hspace{1cm} \text{solid}
Abbreviations

Pr  Prandtl number
RC  rotated-cube
TK  tetrakaidecahedron
WPh Weaire-Phelan

Definitions

\[ f_{DW} = \frac{\Delta p}{L} \cdot \frac{2d_c}{\rho_f v_t^2} \]  \quad \text{Darcy–Weisbach friction factor}

\[ \text{Nu}_v = \frac{h_v d_c^2}{\lambda_f} \]  \quad \text{Mean Nusselt number}

\[ \Pi = \frac{\Delta p}{L} \cdot \frac{d_c^2}{\mu_f v_t} \]  \quad \text{Dimensionless pressure gradient}

\[ \text{Re} = \frac{\rho_f v_t d_c}{\mu_f} \]  \quad \text{Reynolds number}
Re_{pipe} = \frac{\rho_f v_f d_{pipe}}{\mu_f} \quad \text{Reynolds number in the pipe}
REFERENCES


equations for the slope, intercept, and standard errors of the best straight line,”

Darcy to Turbulent,” in *Fundamentals of Transport Phenomena in Porous Media*, J.
256.

2004.

[51] V. V. Calmidi and R. L. Mahajan, “Forced convection in high porosity metal foams,”

Aluminum Foams,” *J. Heat Transf.*, vol. 133, no. 6, pp. 060904-060904–9, Mar.
2011.

[53] J. L. Lage, B. V. Antohe, and D. A. Nield, “Two Types of Nonlinear Pressure-Drop
Versus Flow-Rate Relation Observed for Saturated Porous Media,” *J. Fluids Eng.,*

[54] A. Inayat, M. Klumpp, M. Lämmermann, H. Freund, and W. Schwieger,
“Development of a new pressure drop correlation for open-cell foams based
completely on theoretical grounds: Taking into account strut shape and geometric


from gases and liquids to a circular cylinder in crossflow,” *J. Heat Transf.*, vol. 99,

transfer between air flow and ceramic foams to optimise volumetric solar air
receiver performances,” *Int. J. Heat Mass Transf.*, vol. 54, no. 7–8, pp. 1527–1537,

Figure Captions List

Fig. 1  Front and top view of cylindrically shaped SiSiC structures (diameter of 20 mm and length of 100 mm). From left to right: RC, TK, WPh lattices, and random foam.

Fig. 2  Scheme of the test set-up for measurements of pressure drop and convective heat transfer. The cylindrical porous samples are embedded in a cylindrical tube, which is heated in the temperature-controlled electric furnace. The pressure drop sensors (purple lines), the thermocouples placed on the solid parts (blue lines), and the thermocouples to measure the air stream inlet and outlet temperatures (red lines) are shown. The magnified region shows the two U-shaped heating elements that are placed around the tube and the thermocouples used to measure the temperature of the air stream (red thermocouples) and the porous specimens (blue thermocouple), which are placed inside the tube.

Fig. 3  Schematic of the numerical domain used when solving the coupled Eqs. 3-5. The red rectangle shows the submodel used for the determination of the volumetric convective heat transfer coefficients. The inlet boundary condition of the submodel is obtained from the global model solution on the cutting interface (red surface). Submodeling is
used assuming that changing $h_v$ in the porous medium doesn’t change the results on the cutting interface.

Fig. 4  Measured pressure drops per length for the four specimens for 10 different velocities, corresponding to Reynolds numbers in the range of 40 to 940. The regression lines are obtained only using the data points with Re < 250.

Fig. 5  Measured dimensionless pressure gradient as a function of Reynolds number. The error bar indicates the calculated uncertainty of each data point. The black and red dotted-lines, respectively, represent the best line fits for mean and their standard errors.

Fig. 6  Comparison of our experimental friction factor of WPh and TK lattices with the numerical results in the literature. The values of Wu et al. were obtained by extrapolation of their results for a TK lattice with $d_c = 4.74$ mm.

Fig. 7  Comparison of experimental (points) and the numerical (lines) air temperatures at the porous sample outlet for a RC sample at three different velocities. Gravitational force is in the direction from right to left.
Fig. 8  The predicted Nusselt number as a function of Reynolds number for the porous specimens. The uncertainty of the results, calculated according to Eq. 12, is indicated with error bars.

Fig. 9  (a) Comparison of the predicted Nusselt values for the foam in the present study with correlations listed in Table 5. All correlations are plotted for a 5 PPI foam with $\phi = 0.843$. (b) Comparison between the Nusselt values of the numerical results from the literature with the values obtained in the current study for TK and WPh.
Table Caption List

Table 1  Microstructural properties of the four investigated specimens

Table 2  Permeability and form coefficients and their uncertainties for the four porous specimens

Table 3  Permeability and form coefficients reported in literature for 5 PPI foams with different porosities

Table 4  Coefficients of the proposed Nusselt correlations (Eq. 13) for each of the samples with $R^2$ values of the fitting. The correlations are valid for $70<Re<250$.

Table 5  Literature correlations based on experimental studies of foams with various porosities and pore densities
Fig. 1. Front and top view of cylindrically shaped SiSiC structures (diameter of 20 mm and length of 100 mm). From left to right: RC, TK, WPh lattices, and random foam.
Fig. 2. Scheme of the test set-up for measurements of pressure drop and convective heat transfer. The cylindrical porous samples are embedded in a cylindrical tube, which is heated in the temperature-controlled electric furnace. The pressure drop sensors (purple lines), the thermocouples placed on the solid parts (blue lines), and the thermocouples to measure the air stream inlet and outlet temperatures (red lines) are shown. The magnified region shows the two U-shaped heating elements that are placed around the tube and the thermocouples used to measure the temperature of the air stream (red thermocouples) and the porous specimens (blue thermocouple), which are placed inside the tube.
Fig. 3. Schematic of the numerical domain used when solving the coupled Eqs 3-5. The red rectangle shows the submodel used for the determination of the volumetric convective heat transfer coefficients. The inlet boundary condition of the submodel is obtained from the global model solution on the cutting interface (red surface). Submodeling is used assuming that changing $h_p$ in the porous medium doesn’t change the results on the cutting interface.
Fig. 4. Measured pressure drops per length for the four specimens for 10 different velocities, corresponding to Reynolds numbers in the range of 40 to 940. The regression lines are obtained only using the data points with $Re < 250$. 
Fig. 5. Measured dimensionless pressure gradient as a function of Reynolds number. The error bar indicates the calculated uncertainty of each data point. The black and red dotted-lines, respectively, represent the best line fits for mean and their standard errors.
Fig. 6. Comparison of our experimental friction factor of WPh and TK lattices with the numerical results in the literature. The values of Wu et al. were obtained by extrapolation of their results for a TK lattice with $d_e = 4.74$ mm.
Fig. 7. Comparison of experimental (points) and the numerical (lines) air temperatures at the porous sample outlet for a RC sample at three different velocities. Gravitational force is in the direction from right to left.
Fig. 8. The predicted Nusselt number as a function of Reynolds number for the porous specimens. The uncertainty of the results, calculated according to Eq. 12, is indicated with error bars.
Fig. 9. (a) Comparison of the predicted Nusselt values for the foam in the present study with correlations listed in Table 5. All correlations are plotted for a 5 PPI foam with $\phi = 0.843$. (b) Comparison between the Nusselt values of the numerical results from the literature with the values obtained in the current study for TK and WPh.
Table 1. Microstructural properties of the four investigated specimens.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Structure</th>
<th>Density (kg/m$^3$)</th>
<th>$\phi$ (-)</th>
<th>$d_c$ (mm)</th>
<th>$d_{st}$ (mm)</th>
<th>$S_v$ (1/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>Rotated cube</td>
<td>$578.6 \pm 5.5$</td>
<td>0.796 ± 0.008</td>
<td>4.75 ± 0.02</td>
<td>1.19 ± 0.11</td>
<td>542.85</td>
</tr>
<tr>
<td>TK</td>
<td>Tetrakaidecahedron</td>
<td>$540.3 \pm 5.4$</td>
<td>0.809 ± 0.009</td>
<td>4.75 ± 0.02</td>
<td>0.94 ± 0.14</td>
<td>833.26</td>
</tr>
<tr>
<td>Foam</td>
<td>Various polyhedra</td>
<td>$444.6 \pm 4.4$</td>
<td>0.843 ± 0.008</td>
<td>5.08</td>
<td>1.03 ± 0.14</td>
<td>456.40</td>
</tr>
<tr>
<td>WPh</td>
<td>Weaire-Phelan</td>
<td>$546.8 \pm 5.4$</td>
<td>0.807 ± 0.008</td>
<td>4.75 ± 0.02</td>
<td>1.26 ± 0.17</td>
<td>745.42</td>
</tr>
</tbody>
</table>
Table 2. Permeability and form coefficients and their uncertainties for the four porous specimens.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$K \times 10^7$ m$^2$</th>
<th>$\sigma_K$ (%)</th>
<th>$C$ m$^{-1}$</th>
<th>$\sigma_C$ (%)</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>1.80</td>
<td>34.1</td>
<td>326.1</td>
<td>15.4</td>
<td>0.963</td>
</tr>
<tr>
<td>TK</td>
<td>0.75</td>
<td>15.2</td>
<td>814.8</td>
<td>6.5</td>
<td>0.991</td>
</tr>
<tr>
<td>Foam</td>
<td>2.20</td>
<td>42.2</td>
<td>270.5</td>
<td>18.4</td>
<td>1.000</td>
</tr>
<tr>
<td>WPh</td>
<td>1.00</td>
<td>19.3</td>
<td>943.8</td>
<td>5.4</td>
<td>0.999</td>
</tr>
</tbody>
</table>
Table 3. Permeability and form coefficients reported in literature for 5 PPI foams with different porosities.

<table>
<thead>
<tr>
<th>References</th>
<th>PPI</th>
<th>$\phi$ (-)</th>
<th>$K \times 10^7 \text{ m}^2$</th>
<th>$C \text{ m}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bhattacharya et al. [30]</td>
<td>5</td>
<td>0.973</td>
<td>2.7</td>
<td>186.7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.912</td>
<td>1.8</td>
<td>200.3</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.972</td>
<td>2.52</td>
<td>191.2</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.946</td>
<td>2.17</td>
<td>212.5</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.905</td>
<td>1.74</td>
<td>187.0</td>
</tr>
<tr>
<td>Liu et al. [32]</td>
<td>5</td>
<td>0.914</td>
<td>3.7</td>
<td>164.1</td>
</tr>
<tr>
<td>Mancin [34]</td>
<td>5</td>
<td>0.921</td>
<td>1.52</td>
<td>151.3</td>
</tr>
<tr>
<td>Mancin [52]</td>
<td>5</td>
<td>0.920</td>
<td>2.36</td>
<td>205.0</td>
</tr>
<tr>
<td>Calmidi and Mahajan [51]</td>
<td>5</td>
<td>0.973</td>
<td>2.7</td>
<td>186.7</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.912</td>
<td>1.8</td>
<td>200.3</td>
</tr>
</tbody>
</table>
Table 4. Coefficients of the proposed Nusselt correlations (Eq. 13) for each of the samples with $R^2$ values of the fitting. The correlations are valid for $70<Re<250$.

<table>
<thead>
<tr>
<th>Samples</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RC</td>
<td>0.80</td>
<td>0.76</td>
<td>0.989</td>
</tr>
<tr>
<td>TK</td>
<td>0.75</td>
<td>0.81</td>
<td>0.976</td>
</tr>
<tr>
<td>Foam</td>
<td>0.55</td>
<td>0.84</td>
<td>0.985</td>
</tr>
<tr>
<td>WPh</td>
<td>0.40</td>
<td>0.94</td>
<td>0.990</td>
</tr>
</tbody>
</table>
Table 5. Literature correlations based on experimental studies of foams with various porosities and pore densities.

<table>
<thead>
<tr>
<th>Reference</th>
<th>Correlation</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Younis et al. [22]</td>
<td>$\text{Nu}<em>{\text{Yo}} = \frac{h_v d_p^2}{\lambda_f} = 0.819 \left[ 1 - 7.33 \left( \frac{d_p}{L} \right) \text{Re}</em>{\text{Yo}} \right]^{0.36} \left( \frac{d_p}{\bar{\tau}} \right)^{1.55}$</td>
<td>Based on 5 specimens of alumina foams with: 0.83 &lt; $\phi$ &lt; 0.87, 10 &lt; PPI &lt; 65, 5.1 &lt; Re$<em>{\text{Yo}}$ = $\frac{\rho</em>{\text{f}} v d_p}{\mu_f}$ &lt; 564</td>
</tr>
</tbody>
</table>
| Dietrich [26]            | $\text{Nu}_{\text{Di}} = \frac{h_s d_h}{\lambda_f} = 0.31 \text{Hg}^{\frac{1}{2}} \text{Pr}^{\frac{1}{2}}$  

$Hg = 110 \text{Re}_{\text{Di}} + 1.45 \text{Re}_{\text{Di}}^2$

$d_h = \frac{4 \phi}{A_{st}}, S_v = \frac{2.87}{d_w + d_s} (1 - \phi)^{0.25}$ | Based on 18 specimens of alumina, mullite and oxidic-bonded SiC foams with: 0.75 < $\phi$ < 0.85, 10 < PPI < 45, 10$^{-1}$ < Re$_{\text{Di}}$ = $\frac{\rho_{\text{f}} v d_h}{\phi \mu_f}$ < 10$^5$ |
| Kamiuto and Yee [25]     | $\text{Nu}_{\text{Ky}} = \frac{h_s d_{st}^2}{\lambda_f} = 0.124 (\text{Re}_{\text{Ky}} \text{Pr})^{0.791}$

$d_{st} = 2d_c \left( 0.5 + \cos \left( \frac{1}{3} \cos^{-1} \left( 2\phi - 1 \right) + \frac{4\pi}{3} \right) \right) \sqrt{\pi}$ | Based on 16 specimens of cordierite–aluminite, nickel, alumina, cordierite and mullite foams with: 0.74 < $\phi$ < 0.95, 6 < PPI < 60, 1 < Re$_{\text{Ky}}$Pr = $\frac{\rho_{\text{f}} v d_{st}}{\mu_f}$Pr < 1000 |
| Wu et al. [57]           | $\text{Nu}_{\text{Wu}} = \frac{h_v d_c^2}{\lambda_f} = 32.504 \phi^{0.38} - 109.94 \phi^{1.38}$

$+ 166.65 \phi^{2.38} - 86.98 \phi^{3.38} \text{Re}_{\text{Wu}}^{0.438}$ | Based on parametric numerical study on TK lattices with: 0.66 < $\phi$ < 0.93, 1.414 < $d_c$ < 2.828, 70 < Re$_{\text{Wu}}$ = $\frac{\rho_{\text{f}} v d_c}{\lambda_f}$ < 800 |
| Xia et al. [27]          | $\text{Nu}_{\text{Xi}} = \frac{h_v d_p^2}{\lambda_f} = 0.34 \phi^{-2} \text{Re}_{\text{Xi}}^{0.61} \text{Pr}^{0.7}$ | Based on 9 specimens of Cu, Ni, SiC foams with: 0.87 < $\phi$ < 0.97, 10 < PPI < 40, 20 < Re$_{\text{Xi}}$ = $\frac{\rho_{\text{f}} v d_p}{\mu_f}$ < 1000 |