Two-Dimensional Vlasov Simulation of Driven, Nonlinear Electron Plasma Waves

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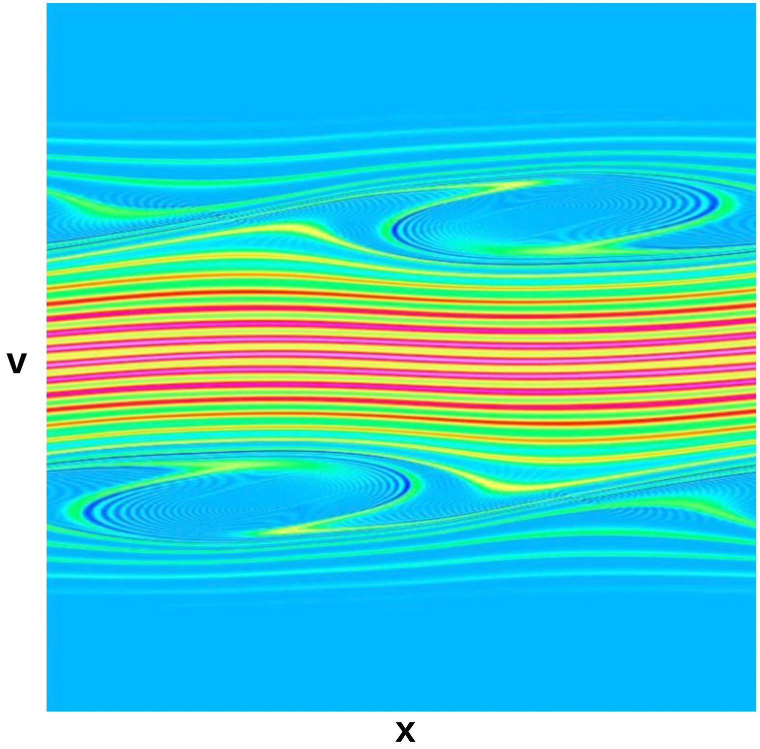
Why two-dimensional Vlasov simulation?

- Wave-particle interaction and particle trapping are important physical effects that influence:
  - Laser-plasma interactions
  - RF heating and current drive
  - Micro-instability and associated turbulence
- Simulation of wave-particle interactions requires a kinetic description
  - *Challenging for accurate and efficient representation*

1D Landau Damping (2048x2048)

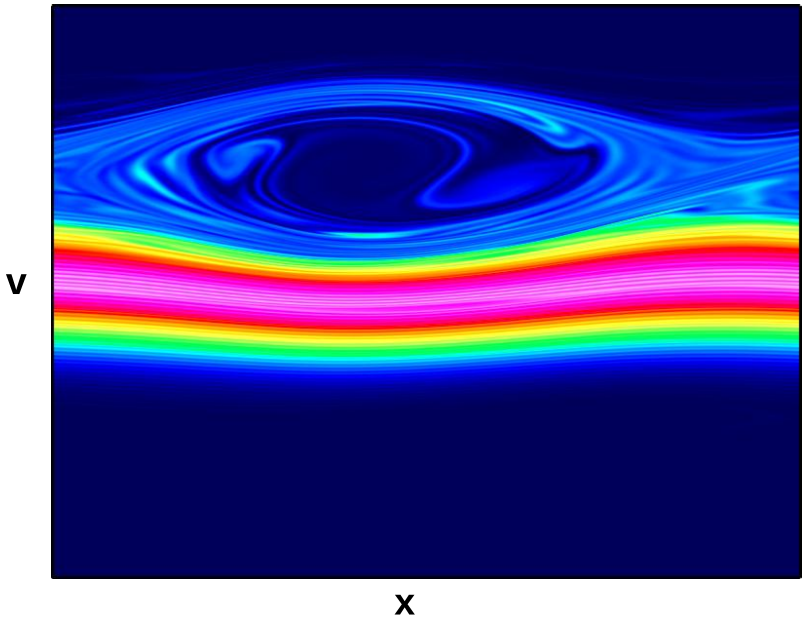


- Two-dimensional Vlasov Simulation is **expensive**, but still valuable
  - Regimes where PIC fluctuations can mask or alter physical effects
  - Benchmark comparison for PIC results

To make 2D Vlasov simulation more practical, we have developed new algorithms

- Traditional Vlasov simulation typically semi-Lagrangian
- We have been developing new, high-order finite volume discretizations[†]
  - Conservative
  - Oscillation suppression
  - Inherently local (scalable)
  - Well suited for mesh adaptivity
- Our current (2+2)D Vlasov code:
  - Parallel
  - Single grid
  - Single species
  - Electrostatic and electromagnetic*

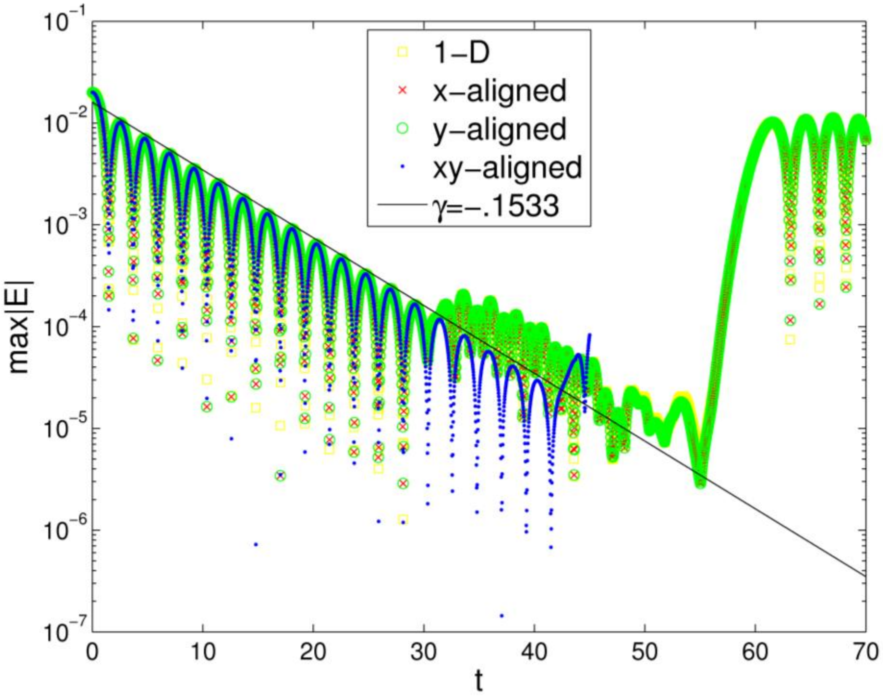
1D Bump-On-Tail Instability (512x2048)



[†] Banks and Hittinger, IEEE Trans. Plasma Sci., to appear

*general boundary conditions not yet implemented for electromagnetics

We have verified our code using a variety of physically-motivated test problems



Example: Weak Landau Damping

$$f = f_0\left[1 + 0.01\cos(k_x x + k_y y)\right]$$

$$(x, y) \in [-L_x, L_x] \times [-L_y, L_y]$$

$$(v_x, v_y) \in [-2\pi, 2\pi] \times [-2\pi, 2\pi]$$

$$N_{v_x} = N_{v_y} = 64$$

(k_x, k_y)	(L_x, L_y)	$N_x \times N_y$
$(1/2, 0)$	$(2\pi, 2\pi)$	64×64
$(1/2, 0)$	$(2\pi, 2\pi)$	64×64
$(1/2, 1/2)$	$(2\sqrt{2}\pi, 2\sqrt{2}\pi)$	128×128

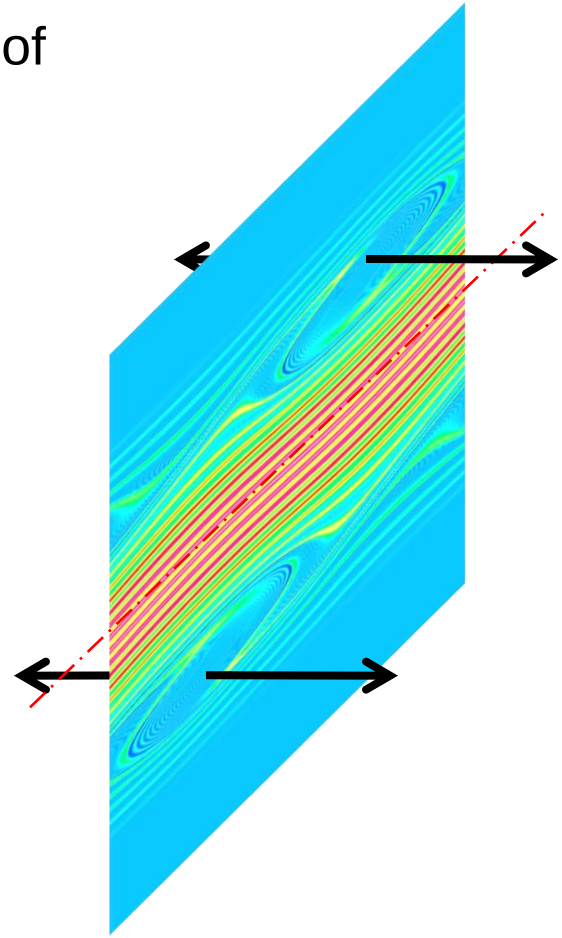
- In 2D, we performed simulations of 1D Landau damping in directions aligned with and diagonal to the grid
- We recovered the correct linear damping rate and frequency

Consider trapping effects in a finite-amplitude Electron Plasma Wave (EPW)

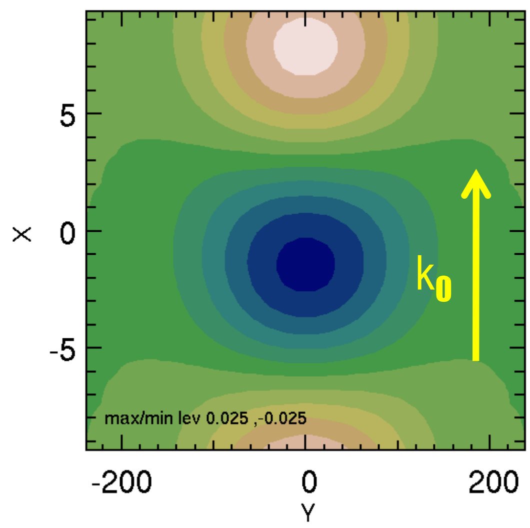
- SRS-driven waves are localized in laser speckles of transverse size $f\lambda_0 \sim 3 \times 10^{-4}$ cm
  - Electrons trapped are lost in a time:

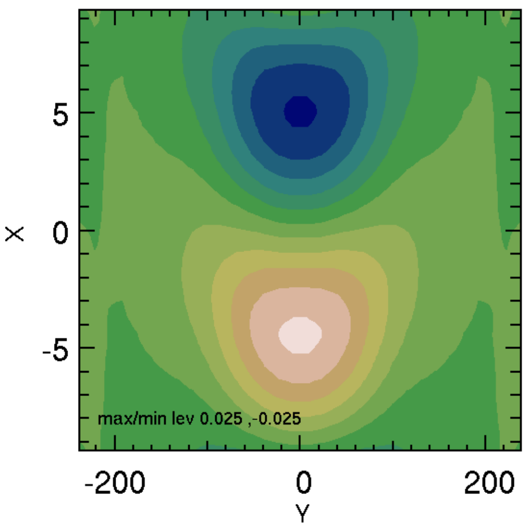
$$t_r \sim f\lambda_0 / v_{th} = 10^{-13}\ \mathrm{s}$$

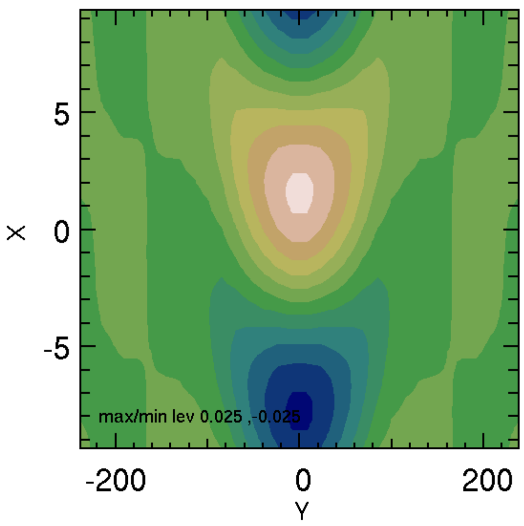
  - *Commensurate with the SRS growth time*
- Large-amplitude EPWs are more nonlinear in 2D
  - Self-focusing dependent on the transverse variation of the nonlinear frequency shift
- Two-dimensional Vlasov simulation can:
  - Test carefully theoretical models of the dependence of the trapped electron frequency shift and the damping rate on t_r
  - Eliminate doubts about the influence of non-physically large fluctuation levels

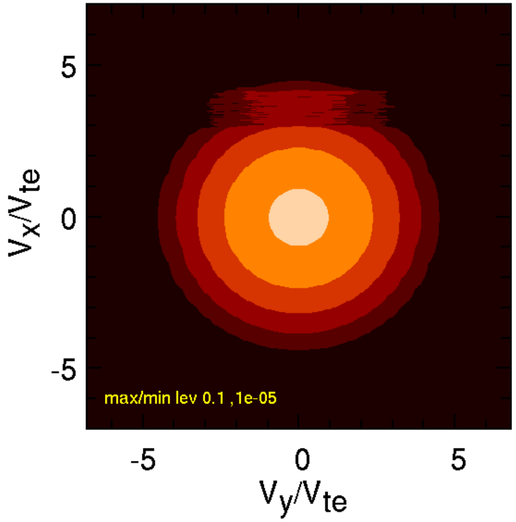


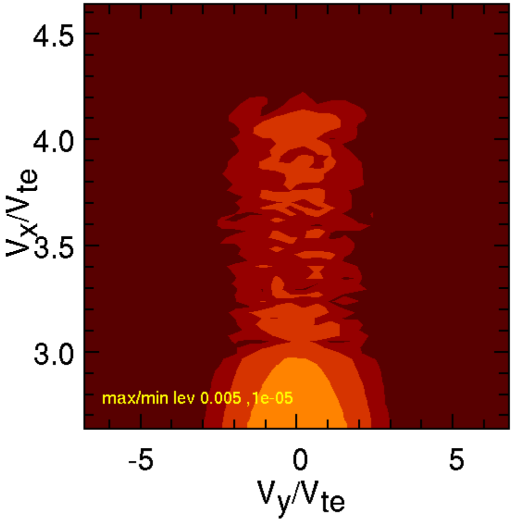
The field appears to focus and maintain its amplitude on axis even as electrons de-trap

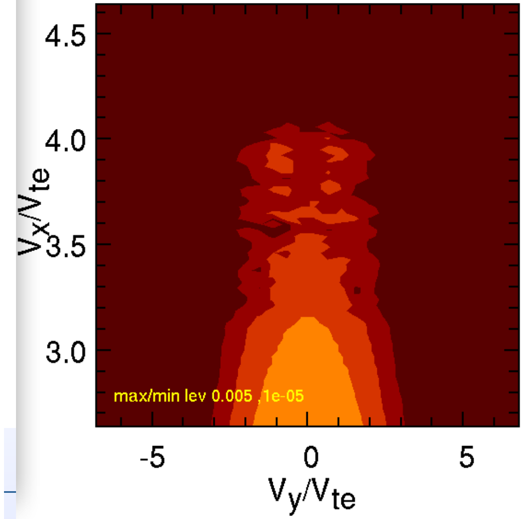

E_x vs x,y at t = 80


E_x vs x,y at t = 230


E_x vs x,y at t = 380


distribution at t= 360and y= 7.6 at ϕ_{max}


distribution at t= 360and y= 7.6 at ϕ_{max}


distribution at t= 360and y= 237.3 at ϕ_{max}

Simulation of a driven nonlinear traveling EPW of finite width in one-wavelength-long system (L_x=λ)

$$L_x = \lambda = 2\pi/k$$

$$L_y \gg L_x$$

Periodic in x
In/Outflow in y

- The wave is driven by a traveling wave potential:

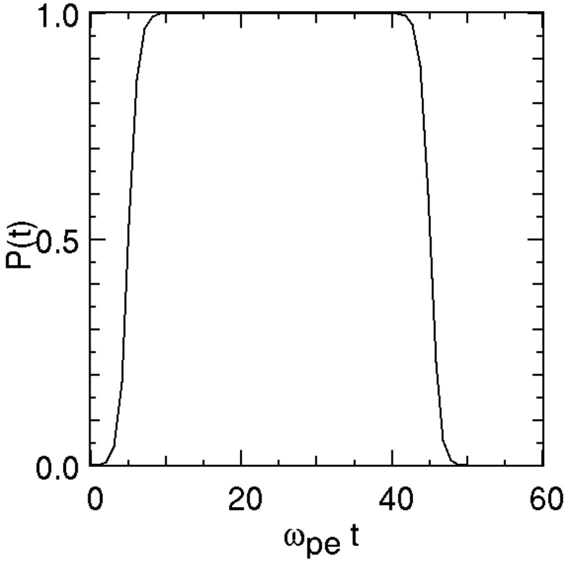
$$\mathrm{E_{ext}} = \mathrm{A(Y)\ E_0 \cos(kx - \omega t)\ P(t)}$$

with ω and *k* chosen to satisfy the linear dispersion:

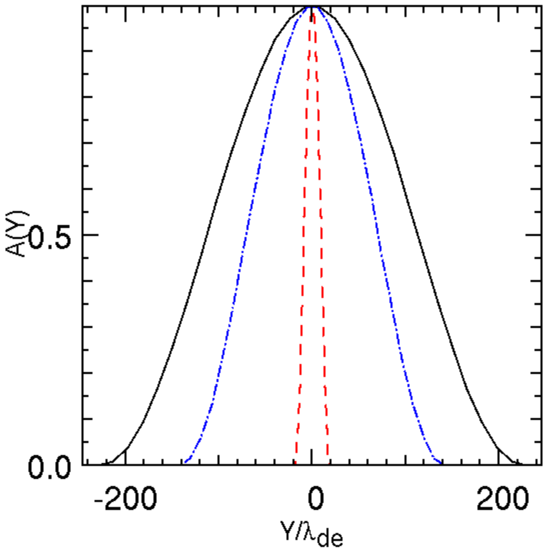
$$k\lambda_{de} = 1/3 \Rightarrow \omega/\omega_{pe} = 1.201$$

- Varied the driver width as shown: FWHM = 20,144, 220 $\lambda_{\rm de}$

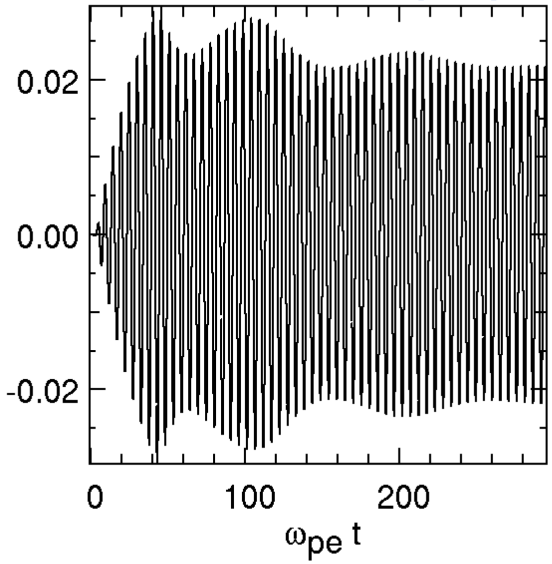
Driver Pulse



Driver shapes

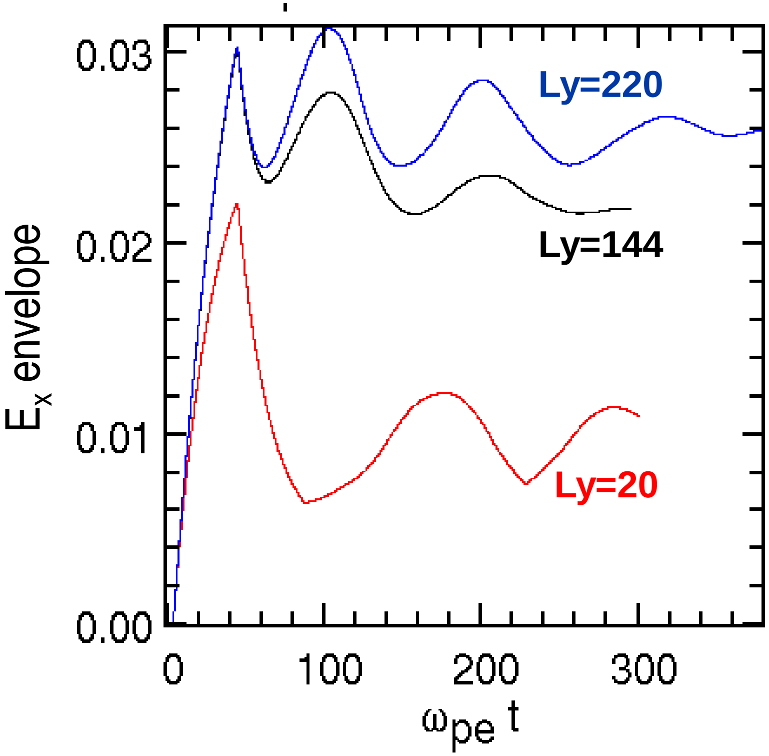


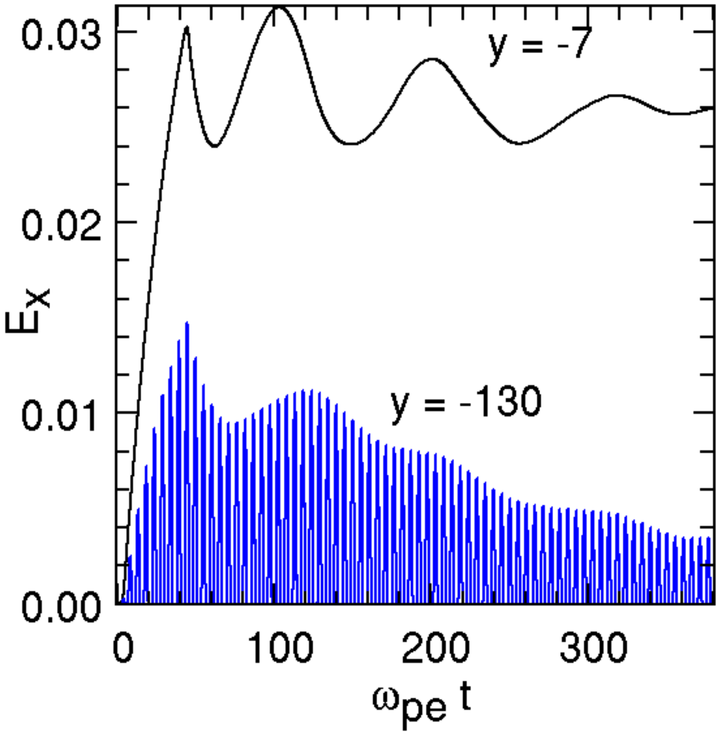
Plasma Wave (Y=0)



The final amplitude of the plasma wave depends on the width of the driving potential

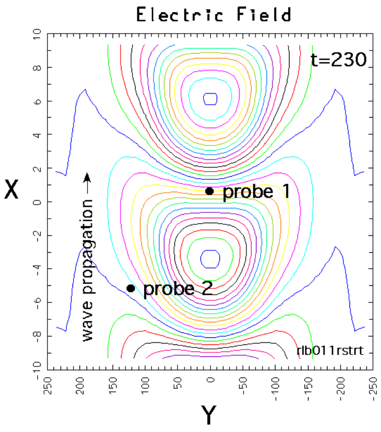
- Narrow drivers produce lower amplitude EPWs
  - For laser speckle: $f\lambda_0/\lambda_{de}$ = 225 [f/8, λ_0=351nm, Te=2.5, N_e/N_c=0.1]
- EPW amplitude in wings of driver decay after driver is off

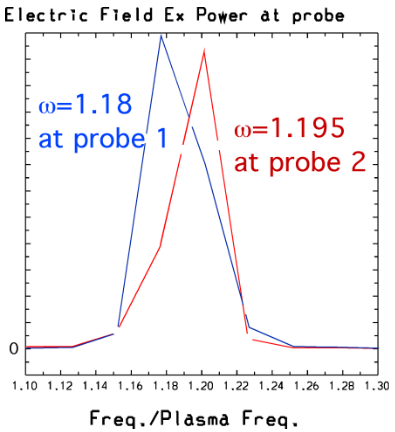


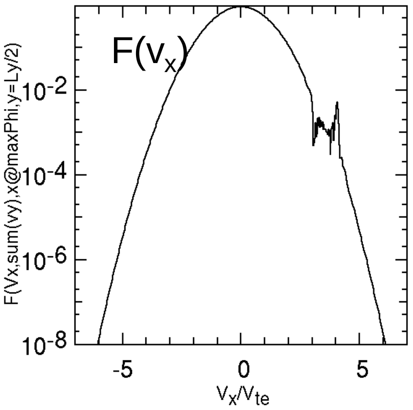


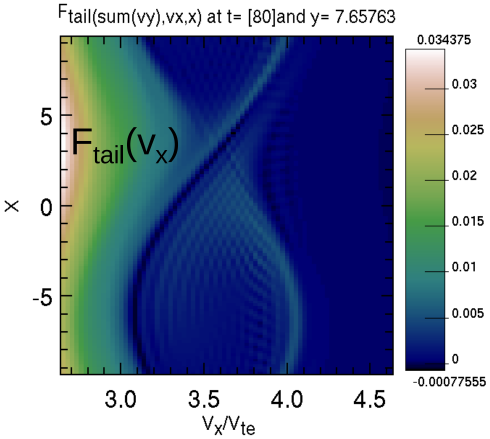
Trapping effects in a 2D finite-amplitude EPW induce wave-front bending

- Electrons are trapped in the wave
  - more are trapped ***along the axis*** of the wave where ***amplitude is stronger***
- After the driver is off, a nonlinear shift (<0) of the normal mode frequency occurs
  - ***Shift is smaller*** at a finite *y* displacement ***away from the axis***
  - Thus, the ***phase velocity is larger*** away from the axis
- This phase velocity variation causes the wave front to bow
  - Consistent with the Raman studies of Yin, *et al.,* PRL **99**, 265004 (2007)





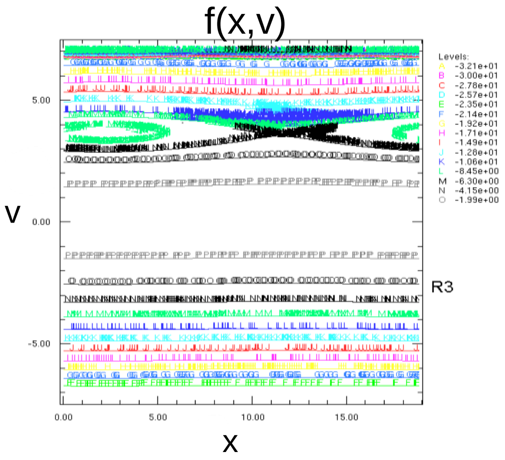


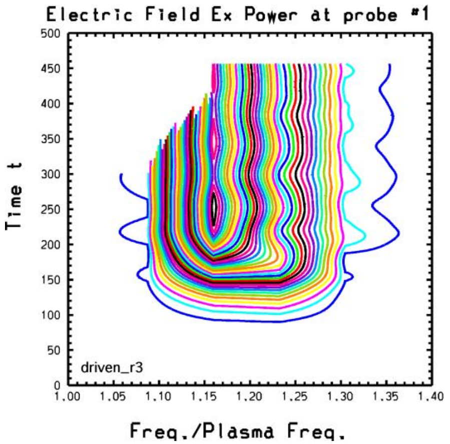


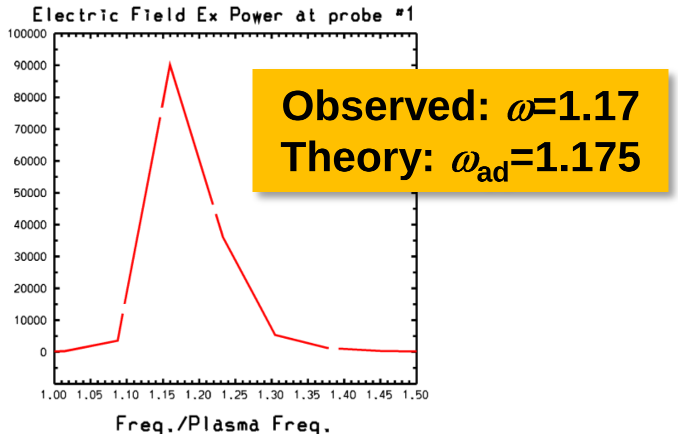
$(N_x,N_y,N_{vx},N_{vy})=(128,32,512,32)$

For sufficient resolution, frequency shift dependence on wave amplitude agrees with adiabatic theory

- Results of example 1D simulation with (Nx,Nv)=(512,1024):

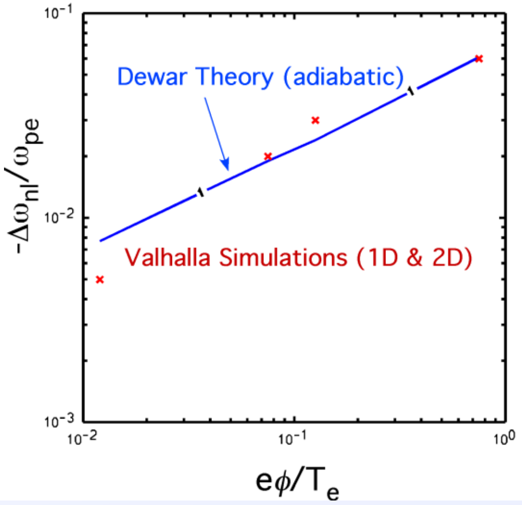






Observed: ω=1.17
Theory: ω_{ad}=1.175

- Scan in wave amplitude in 1D and 2D
  - With adequate (x,v) resolution, compares well with Dewar's theory (adiabatic drive)
- Less resolution in (x,v) space degrades the results
  - Trapped particles must be resolved
  - Particle motion links x and v resolution needs
  - Resolution scan done with uniform mesh



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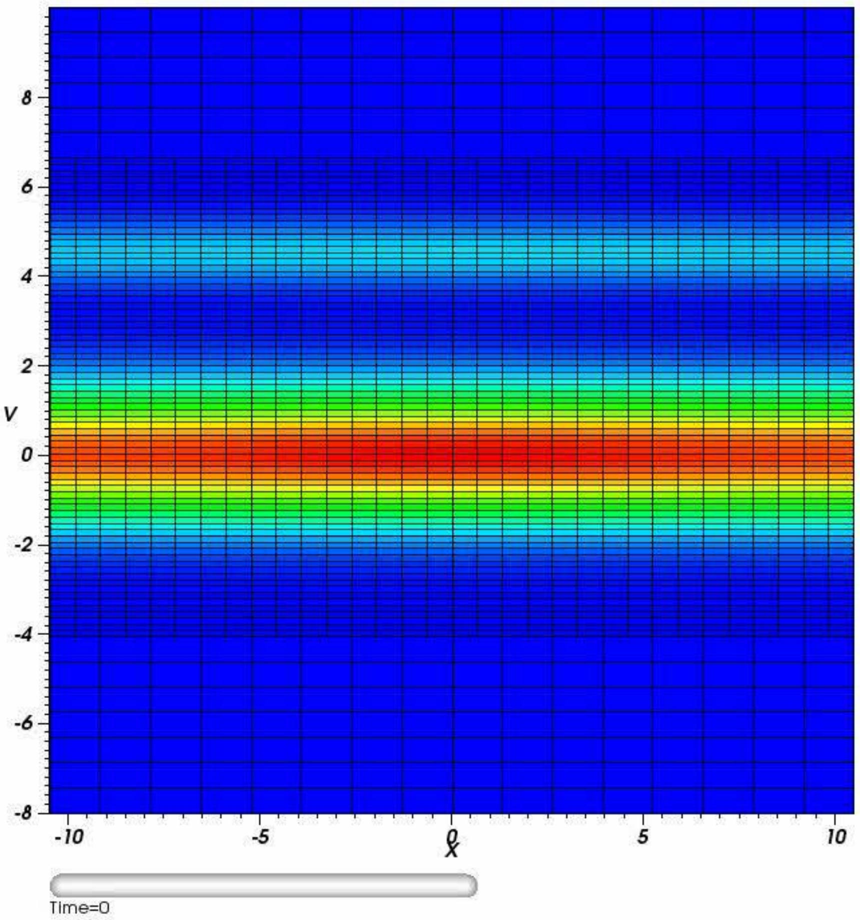
Adaptive Mesh Refinement (AMR) will further reduce the cost of 2D Vlasov simulation



- Our runs required 385 - 768 processors for ~24 hours
- AMR could further reduce the expense
  - Fewer cell
  - Larger time steps
  - Savings increase geometrically with dimension
- Current AMR code:
  - 1D Vlasov-Poisson
  - Multi-species
  - Same discretizations
  - Based on SAMRAI library

Conclusions and Future Work

- We have demonstrated two-dimensional detrapping effects on driven, nonlinear EPWs using Vlasov simulation
- We will continue to investigate Raman-relevant problems:
  - Parametric studies of driven EPWs
  - Vlasov-Maxwell simulation of SRS
- We will continue to improve our simulation capability:
  - Extend AMR implementation to 2D
  - Extend AMR implementation to electromagnetics
  - Investigate improved refinement criteria
  - Investigate improved time integration techniques
  - Investigate improved non-reflective boundary conditions