Plasma refuelling at the SOL simulated with the GBS code

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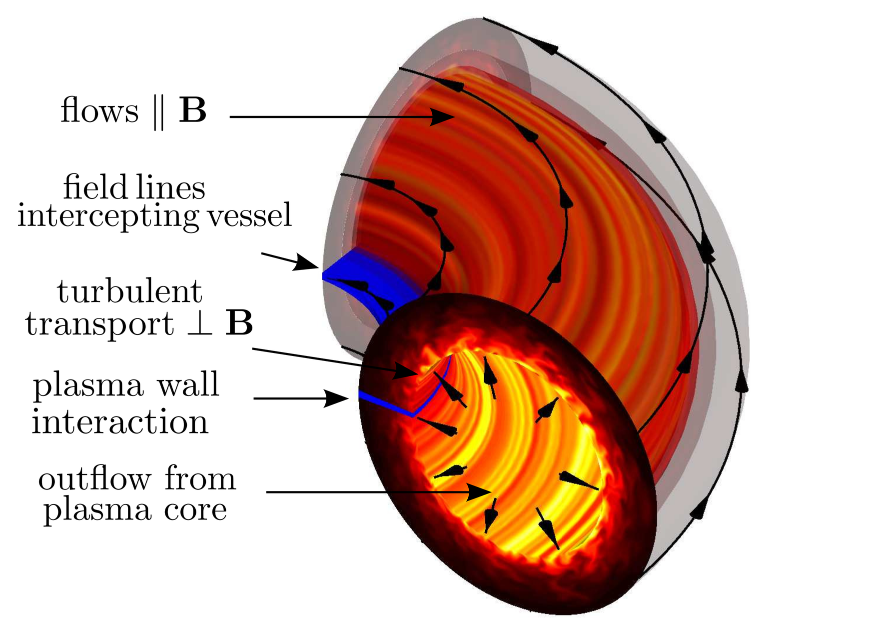
Introduction



- In tokamaks Scrape-Off Layer (SOL), **magnetic field lines intersect the walls** of the fusion device
- **Heat and particles** flow along magnetic field lines and are **exhausted to the vessel**
- **Turbulence** amplitude and size **comparable** to **steady-state** values
- **Neutral** particles interact with the plasma
- SOL plays a key role on determining the **refuelling** of the plasma

The **Global Braginskii Solver (GBS) code:**
a 3D, flux-driven, global turbulence code in limited geometry
used to study **plasma turbulence in the SOL**

- GBS is a simulation code to evolve plasma turbulence in the edge of fusion devices. [Halpern *et al.*, JCP 2016], [Ricci *et al.*, PPCF 2012]
- GBS solves 3D **fluid equations for electrons and ions**, Poisson's and Ampere's equations, and a **kinetic equation for neutral atoms**.

The Global Braginskii Solver (GBS) code

Two fluid drift-reduced Braginskii equations, $k_\perp^2 \gg k_\parallel^2$, $d/dt \ll \omega_{ci}$

$$\frac{\partial n}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, n] + \frac{2}{B}[C(p_e) - nC(\phi)] - \nabla\cdot(n v_{\parallel e}\mathbf{b}) + \mathcal{D}_n(n) + S_n + n_n\nu_{iz} - n\nu_{rec} \quad (1)$$

$$\frac{\partial \Omega}{\partial t} = -\frac{\rho_*^{-1}}{B}\nabla_\perp\cdot[\phi,\omega] - \nabla_\perp\cdot\left[\nabla_\parallel\left(v_{\parallel i}\omega\right)\right] + B^2\nabla\cdot(j_\parallel\mathbf{b}) + 2BC(p) + \frac{B}{3}C(G_i) + \mathcal{D}_\Omega(\Omega) - \frac{n_n}{n}\nu_{cx}\Omega \quad (2)$$

$$\frac{\partial U_{\parallel e}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left[\frac{\nu j_\parallel}{n} + \nabla_\parallel\phi - \frac{\nabla_\parallel p_e}{n} - 0.71\nabla_\parallel T_e - \frac{2}{3n}\nabla_\parallel G_e\right] + \mathcal{D}_{v_{\parallel e}}(v_{\parallel e}) + \frac{n_n}{n}(\nu_{en} + 2\nu_{iz})(v_{\parallel n} - v_{\parallel e}) \quad (3)$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{\nabla_\parallel p}{n} - \frac{2}{3n}\nabla_\parallel G_i + \mathcal{D}_{v_{\parallel i}}(v_{\parallel i}) + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})(v_{\parallel n} - v_{\parallel i}) \quad (4)$$

$$\frac{\partial T_e}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4T_e}{3B}\left[\frac{C(p_e)}{n} + \frac{5}{2}C(T_e) - C(\phi)\right] + \frac{2T_e}{3n}\left[0.71\nabla\cdot(j_\parallel\mathbf{b}) - n\nabla\cdot(v_{\parallel e}\mathbf{b})\right] + \mathcal{D}_{T_e}(T_e) + \mathcal{D}^\parallel_{T_e}(T_e) + S_{T_e} + \frac{n_n}{n}\nu_{iz}\left[-\frac{2}{3}E_{iz} - T_e + \frac{m_e}{m_i}v_{\parallel e}\left(v_{\parallel e} - \frac{4}{3}v_{\parallel n}\right)\right] - \frac{n_n}{n}\nu_{en}\frac{m_e}{m_i}\frac{2}{3}v_{\parallel e}(v_{\parallel n} - v_{\parallel e}) \quad (5)$$

$$\frac{\partial T_i}{\partial t} = -\frac{\rho_*^{-1}}{B}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4T_i}{3B}\left[\frac{C(p_e)}{n} - \frac{5}{2}\tau C(T_i) - C(\phi)\right] + \frac{2T_i}{3n}\left[\nabla\cdot(j_\parallel\mathbf{b}) - n\nabla\cdot(v_{\parallel i}\mathbf{b})\right] + \mathcal{D}_{T_i}(T_i) + \mathcal{D}^\parallel_{T_i}(T_i) + S_{T_i} + \frac{n_n}{n}(\nu_{iz} + \nu_{cx})\left[\tau^{-1}T_n - T_i + \frac{1}{3\tau}(v_{\parallel n} - v_{\parallel i})^2\right] \quad (6)$$

$$\rho_\star = \rho_s/R_0, \quad \mathbf{b} = \frac{\mathbf{B}}{B}, \quad [A,B] = b\cdot(\nabla A\times\nabla B), \quad C(A) = \frac{B}{2}[\nabla\times(\frac{b)}{B}]\cdot\nabla A, \quad \nabla_\parallel f = \mathbf{b}_0\cdot\nabla f + \frac{\beta_{e0}}{2}\frac{\rho_*^{-1}}{B}[\psi, f]$$

$$p = n(T_e + \tau T_i), \quad U_{\parallel e} = v_{\parallel e} + \frac{\beta_{e0}}{2}\frac{m_i}{m_e}\psi, \quad \Omega = \nabla\cdot\omega = \nabla\cdot(n\nabla_\perp\phi + \tau\nabla_\perp p_i)$$

- Equations implemented in GBS, a **flux-driven** plasma turbulence code with limited geometry to study SOL heat and particle transport
- System completed with **first-principles boundary conditions** applicable at the magnetic pre-sheath entrance where the magnetic field lines intersect the limiter [Loizu *et al.*, PoP 2012]
- Parallelized using domain decomposition, **excellent parallel scalability** up to $\sim$ 10000 cores
- Gradients and curvature discretized using **finite differences**, Poisson Brackets using Arakawa scheme, integration in time using **Runge Kutta method**
- Code **fully verified** using method of manufactured solutions [Riva *et al.*, PoP 2014]
- Note: $L_\perp \to \rho_s$, $L_\parallel \to R_0$, $t \to R_0/c_s$, $\nu = ne^2R_0/(m_i\sigma_\parallel c_s)$ normalization

The Poisson and Ampere equations

- **Generalized Poisson equation**, $\nabla\cdot(n\nabla_\perp\phi) = \Omega - \tau\nabla_\perp^2 p_i$
- **Ampere's equation** from Ohm's law, $\left(\nabla_\perp^2 - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}n\right)v_{\parallel e} = \nabla_\perp^2 U_{\parallel e} - \frac{\beta_{e0}}{2}\frac{m_i}{m_e}nv_{\parallel i}$
- Stencil based **parallel multigrid** implemented in GBS
- Elliptic equations separable in parallel direction allow for **independent 2D solutions** for each x-y plane

The kinetic neutral atoms equation

$$\frac{\partial f_n}{\partial t} + \vec{v}\cdot\frac{\partial f_n}{\partial\vec{x}} = -\nu_{iz}f_n - \nu_{cx}n_n\left(\frac{f_n}{n_n} - \frac{f_i}{n_i}\right) + \nu_{rec}f_i \quad (7)$$

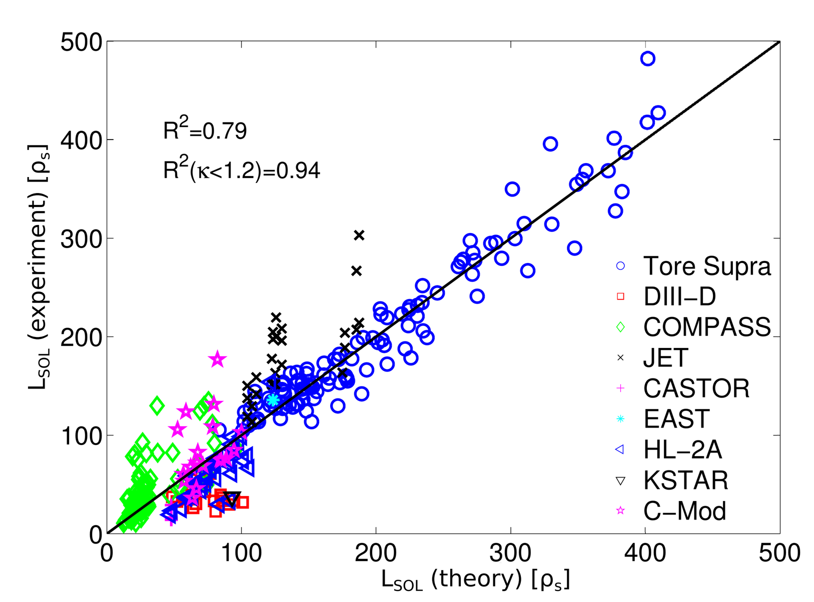
- **Method of characteristics** to obtain the formal solution of f_n [Wersal *et al.*, NF 2015]
- **Two assumptions**, $\tau_{\text{neutral losses}} < \tau_{\text{turbulence}}$ and $\lambda_{\text{mfp, neutrals}} \ll L_{\parallel,\text{plasma}}$, leading to a 2D steady state system for each x-y plane
- **Linear integral equation** for neutral density obtained by integrating f_n over $\vec{v}$
- **Spatial discretization** leading to a linear system of equations

$$\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} = \begin{bmatrix} K_{p\to p} & K_{b\to p} \\ K_{p\to b} & K_{b\to b}\end{bmatrix}\cdot\begin{bmatrix} n_n \\ \Gamma_{out}\end{bmatrix} + \begin{bmatrix} n_{n,rec} \\ \Gamma_{out,rec} + \Gamma_{out,i}\end{bmatrix} \quad (8)$$

- This system is solved for neutral density, n_n, and neutral particle flux at the boundaries, Γ_{out}, with the threaded LAPACK or MUMPS (serial or parallel) solvers.

Past achievements of GBS

- Characterization of **non-linear turbulent regimes** in the SOL
- **SOL width scaling** as a function of dimensionless / engineering plasma parameters
- Origin and nature of **intrinsic toroidal plasma rotation** in the SOL
- Mechanisms regulating the SOL **equilibrium electrostatic potential**



Moving towards a density-conserving model

- Current version of the GBS code does not conserve charged particle density since:
  - the inverse aspect ratio $\epsilon = \frac{r}{R_0}$ is taken constant over the simulation domain, $\epsilon_0 = \frac{a_0}{R_0}$
  - parallel gradient components of Poisson brackets and curvature operators neglected
- Studying the plasma refuellling requires a density-conserving model to be implemented in GBS
- GBS must conserve the total sum of the ion+neutral density over the whole simulation domain
- This is important to address refuelling and Greenwald density limit physics
- Continuity equation must compute the exact variation of the ion density
- To make the model density-conserving, we implemented in GBS:
  - **Radially variable inverse aspect ratio** $\epsilon = \frac{r}{R_0}$ to take into account **curvilinear geometry**
  - **Parallel gradient terms** included in Poisson brackets and curvature operators

$$[\phi, A] = P_{yx}[\phi, A]_{yx} + \mathbf{P}_{\mathbf{x}\parallel}[\phi, \mathbf{A}]_{\mathbf{x}\parallel} + \mathbf{P}_{\parallel\mathbf{y}}[\phi, \mathbf{A}]_{\parallel\mathbf{y}}, \quad C(A) = C^x\frac{dA}{dx} + C^y\frac{dA}{dy} + \mathbf{C}^\parallel\nabla_\parallel\mathbf{A}$$

$$[\phi, A]_{uv} = \frac{d\phi}{du}\frac{dA}{dv} - \frac{d\phi}{dv}\frac{dA}{du}, \quad P_{yx} = \frac{a}{Jb^\varphi}, \quad \mathbf{P}_{\mathbf{x}\parallel} = \frac{\mathbf{b}_{\theta^*}}{\mathbf{J}\mathbf{b}^\varphi}, \quad \mathbf{P}_{\parallel\mathbf{y}} = \frac{\mathbf{a}\mathbf{b}_\mathbf{r}}{\mathbf{J}\mathbf{b}^\varphi}$$

$$C^x = -\frac{2B}{J}\frac{dc_\varphi}{d\theta^*}, \quad C^y = \frac{aB}{2J}\left[\frac{dc_\varphi}{dr} + \frac{1}{q}\left(\frac{dc_{\theta^*}}{dr} - \frac{dc_r}{d\theta^*}\right)\right], \quad \mathbf{C}^\parallel = \frac{\mathbf{B}}{2\mathbf{J}\mathbf{b}^\varphi}\left(\frac{\mathbf{dc_r}}{\mathbf{d}\theta^*} - \frac{\mathbf{dc}_{\theta^*}}{\mathbf{dr}}\right)$$

Field-aligned right-handed coordinates set: (θ^*, r, φ)

θ^* defined by $b^\varphi = qb^{\theta^*}$ (with q the safety factor) $\quad c_i = \frac{b_i}{B} \quad J = rR_0\frac{\left(1-\epsilon^2\right)^{3/2}}{(1-\epsilon cos(\theta^*))^2}$

Converts to (y, x, z) coordinates set with: $y = a\theta^*$, $x = r$, $z = R_0\varphi$

- Continuity equation is now density-conserving
- Gauss Theorem can be used when taking time and volume integration of the continuity equation, expressing volume-integrated density variation in terms of the fluxes across the volume's boundary.

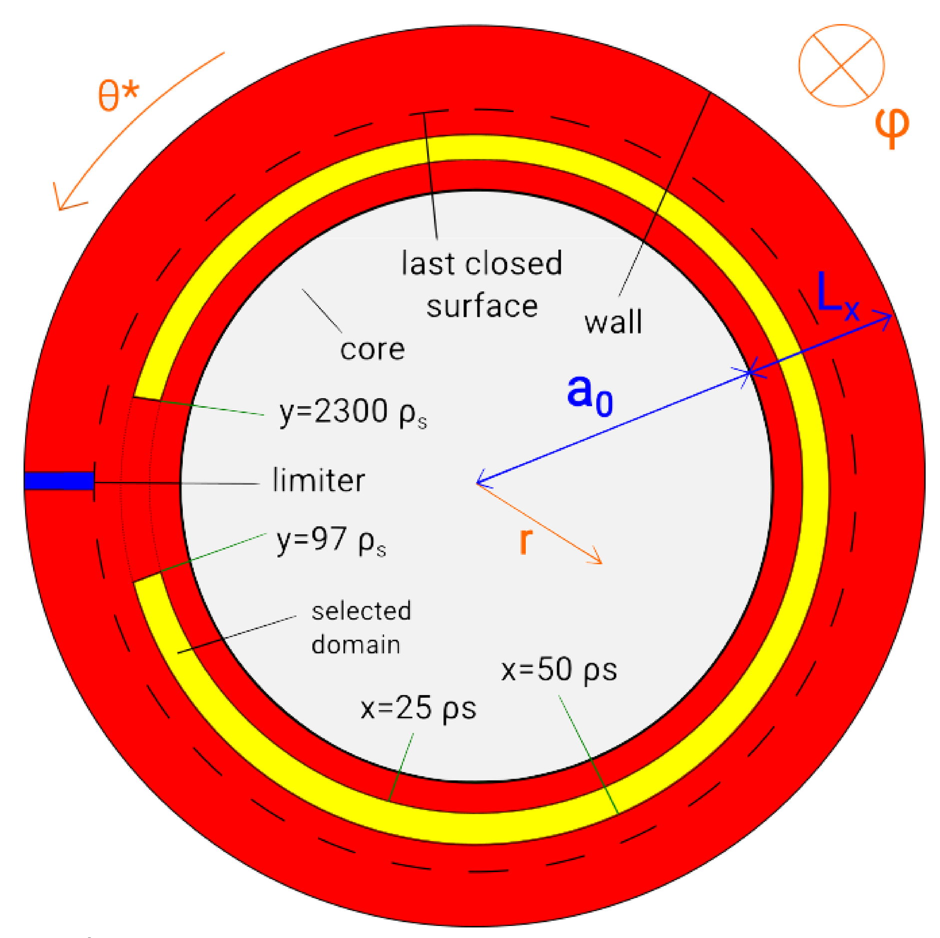
$$\int dt\int\frac{dn}{dt}dV = -\int dt\int(n\mathbf{v}_{de} + n\mathbf{v}_{\mathbf{E}\times\mathbf{B}} + nv_{\parallel e}\mathbf{b})\cdot d\mathbf{S} + \int dt\int(n_n\nu_{iz})dV \quad (9)$$

$$\mathbf{v}_{de} = \frac{1}{B^2}\nabla p_e\times\mathbf{B}, \quad \mathbf{v}_{\mathbf{E}\times\mathbf{B}} = -\frac{n}{B^2}\nabla\phi\times\mathbf{B}$$

- Diffusion $\mathcal{D}_n(n)$ is neglected at this stage, as well as source terms S_n and $n\nu_{rec}$

Numerical results

- **GBS Simulations** were run for 10 time steps taking the following parameters:
  - $\epsilon_0 = 0.2546$; $R_0 = 1500\rho_s$; circular centered magnetic flux surfaces
  - Simulation of an annular domain with $L_y = 2\pi a_0 = 2400\rho_s$ and $L_x = 150\rho_s$ (while $L_z = 2\pi R_0$)
  - Limited region at $x = 75 - 150\rho_s$
  - CG (coarse grid) with $N_y = 495$, $N_x = 191$, $N_z = 64$ and time step $\Delta t = 3.75\times10^{-6}$s
  - FG (fine grid) with $N_y = 990$, $N_x = 382$, $N_z = 128$ and time step $\Delta t = 1.875\times10^{-6}$s
- First, each of the four terms on the right hand side of (9) was taken separately in the continuity equation in GBS; then, all terms were taken into account



- GBS results were post-processed to obtain $\int dt\int\frac{dn}{dt}dV$ for a space domain and the integral of the right hand side of (9), the relative error between the two being computed.
- Results are presented for a domain inside the **closed flux surfaces region** defined by: $x = 25 - 50\ \rho_s$, $y = 97 - 2300\ \rho_s$, $z = 0.68 - 5.48\ R_0$

Terms considered in the equation	Relative error (%) for CG	Relative error (%) for FG
$n\,\mathbf{v}_{de}$	0.80%	0.12%
$n\,\mathbf{v}_{\mathbf{E}\times\mathbf{B}}$	0.020%	0.11%
$n\,v_{\parallel e}\,\mathbf{b}$	4.1%	6.0%
$n_n\,\nu_{iz}$	9.2×10^{-6}%	2.2×10^{-6}%
all terms	0.057%	0.010%

Discussion and conclusions

- Greatest contribution for particle transport in the closed flux surfaces region comes from perpendicular $E\times B$ transport, while the ionization contribution is also important ($\sim$10 times smaller)
- Non-converging relative error values are found for the $n\,v_{\parallel e}\,\mathbf{b}$ and $n\,\mathbf{v}_{\mathbf{E}\times\mathbf{B}}$ terms due to the numerical scheme used in GBS for the parallel gradient computation
- Since $k_\perp^2 \gg k_\parallel^2$ holds, errors arising from the parallel gradient contributions are negligible when taking the whole continuity equation
- The continuity equation in GBS is consistent with density conservation up to errors $<$ 0.1%.

Next step: plasma + neutrals conservation

- Neutral density variation can be obtained from the integral form of the neutral continuity equation:

$$\int dt\int\frac{dn}{dt}dV = -\int dt\int(n_n\mathbf{v}_n)\cdot d\mathbf{S} - \int dt\int dV(n_n\nu_{iz}) \quad (10)$$

- Ion flux obtained by taking the first order moments of f_n considering the contributions from charge-exchange in the plasma and electron-ion recycling at the limiter and walls

EUROfusion

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The views and opinions expressed herein do not necessarily reflect those of the European Commission.