Calibration of Parallel Kinematics Machine-Tools Using Small Displacement Torsors

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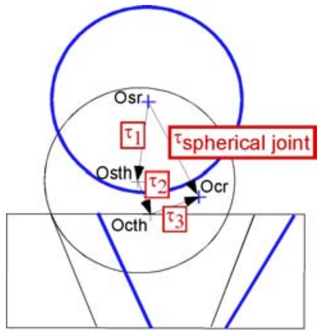
1. Introduction.

The accuracy of a machine-tool depends on manufacturing and assembly errors, backlash in the structure and the links, thermal variations, vibrations, and others. The method presented in this paper deals with manufacturing and assembly errors but it can be adapted to other sources of errors.

This method is based on the use of Small Displacement Torsors ([Bourdet 87] and [Ballot 95]) to model the defects of the surfaces that constitute the joints. This concept is used to the calibration of parallel kinematics machine-tools.

2. Principle of the Small Displacement Torsors (SDT).

Considering a joint, for example a spherical joint composed of a sphere and a cone. The transformation needed to obtain the position and orientation of the real surfaces from the theoretical one is assumed to be a composition of translations and rotations (both theoretical and real surfaces have the same typology). This transformation is represented by a torsor [Techniques de l'ingénieur] called SDT because we make the hypothesis that the amplitude of the defects are small compared to the nominal dimensions of the parts. In other words, the displacement that is applied to transform the theoretical sphere into the real one is small. The same hypothesis is taken for all the surfaces of a joint.



If we explicit the case of the spherical joint, we obtain the following torsor $[\tau_{\text{spherical joint}}]_{Osth}$ (between Osr and Ocr), calculated at point Osth (Osth is the theoretical centre of the sphere, Osr is the real centre of the sphere, idem for the points Octh and Ocr characterising the cone):

$[\tau_{sphere \to th\_sphere}]_{Osth}=[R_{sphere \to th\_sphere}\ ,\ D_{sphere \to th\_sphere,Osth}]$ in {Rsphere} frame (represented by $[\tau_1]$ in the picture);

$R_{sphere \to th\_sphere} = (0,0,0)_{\{Rsphere\}}$ is the resultant of the torsor expressed in Osth (null because a sphere has a symmetry of revolution along all axis);

$D_{sphere \to th\_sphere,Osth} = (u_{sph},v_{sph},w_{sph})_{\{Rsphere\}}$ is the torque of the torsor, it represents the vector Osr-Osth.

$[\tau_{cone \to th\_cone}]_{Octh}=[R_{cone \to th\_cone}\ ,\ D_{cone \to th\_cone,Octh}]$ in {Rcone} frame (represented by $-[\tau_3]$ in the picture);

$R_{cone \to th\_cone} = (\alpha_{cone},\beta_{cone},0)_{\{Rcone\}}$ is the resultant of the torsor expressed in Octh (the cone has a symmetry of revolution along its axis, Z in this case);

$D_{cone \to th\_cone,Octh} = (u_{cone},v_{cone},w_{cone})_{\{Rcone\}}$ is the torque of the torsor, it represents the vector Ocr-Octh.

If we consider that the spherical joint is preloaded, it means that Osth and Octh are overlapped, then the SDT representing the backlash between both surfaces is null: $[\tau_2]_{Osth} = 0$. So the SDT of the spherical joint is:

$$[\tau_{\text{spherical joint}}]_{Osth} = [\tau_1]_{Osth} + [\tau_2]_{Osth} + [\tau_3]_{Osth} = [\tau_{sphere \to th\_sphere}]_{Osth} - [\tau_{cone \to th\_cone}]_{Osth}$$

u_{sph}, v_{sph}, w_{sph}, α_{cone}, β_{cone}, u_{cone}, v_{cone}, w_{cone} are the 8 parameters needed to model a preloaded spherical joint.

3. Small Displacement Torsors used to calibrate parallel kinematics structures.

The calibration method consists in:

- modelling each link of the structure by a SDT expressed in the local frame of the link. n parameters characterize all the defects of the structure;
- writing each torsor in the frame of the structure where measurements are done (this frame is fixed compared to the end-effector) and at the tool centre point of the machine. So the components of the torsors depend on the pose (position and orientation) of the end-effector;
- calculating the relations between the SDT of the joints and the pose of the end-effector using the kinematical chains of the structure. If the structure has N kinematical chains then N equations of torsors exist;
- measuring the error in the pose of the end-effector in m points. These points must be optimally located. If measures are taken along axis X, Y and Z for the errors of orientation and position then s=6 informations are available for each measuring point. If only the errors of position along the axis X, Y and Z are measured then

$s=3$ informations are obtained for each measuring point. So $N*s$ equations between the defects of the structure and the errors of position and/or orientation are obtained. Then m must verify the relation: $m*N*s \geq n$;

- solving the equation $E = M \cdot D$ where E is the vector containing the errors of pose in the measuring points (size $m*N*s \times 1$); M is the matrix that represents the relations between parameters and errors, as expressed in the N equations between the torsors and the pose of the end-effector (size $m*N*s \times n$); D is the unknown vector containing the parameters of the torsors (size $n \times 1$). Note that this equation is inverted to calculate vector D, so matrix M must be invertible. This system of equations is linear;
- verifying the hypothesis of small displacements when vector D is calculated, it means when the torsors of each joints are calculated;
- finally the error of pose of the end-effector can be calculated in all the working volume using one of the N equations. If Err is the pose error at point P then improving the accuracy of the structure will consist in programming a movement in $(P - Err)$ in order to reach the desired point P.

4. Main difficulties and advantages of this method.

One of the main difficulties is to find a parameterisation of the structure such that the matrix M becomes invertible. In our simulations and with the model of the joints we took, the columns of the matrix M are not linearly independent because some parameters have the same effect on the error of pose either because they are redundant or because the points of measure are not well located. So a systematic method must be found to determine which are the independent parameters, considering measuring points in optimal positions.

Simulations showed that the errors of pose calculated at each point of the working space depend on the equation of the kinematical chain used. Then some equations should be more adapted to this calculation. Some tests are needed to determine the equation that gives most accurate results.

A preloaded spherical joint needs 8 parameters (u_{sph}, v_{sph}, w_{sph}, α_{cone}, β_{cone}, u_{cone}, v_{cone}, w_{cone}), a prismatic joint needs 10 parameters and a rotative joint needs 8 parameters so the number of parameters of a structure is often high.

The local frames of the joints are associated to the theoretical structure so the relations between local frames and the frame used to make measures are known as soon as the direct geometrical model of the theoretical structure is known. The geometrical models of the theoretical structure can be known in a numerical way for the identification phase of the parameters of the torsors.

An other advantage is that with this method, only the theoretical geometrical models are needed. Moreover manipulating torsors is well known and quite easy.

5. Conclusion.

A new method to calibrate parallel kinematics structures is presented in this paper. The following hypothesis is made: the defects of the structure can be modelled by a combination of small displacements at the contact surfaces of the joints. One of the advantages of this method is that the system of equations to solve is linear and only the geometrical models of the perfect structure are needed. The main problem of the method is the choice of the parameters in order to obtain a linearly independent system of equations.

The Hita-STT machine-tool , which was presented at the 3rd Chemnitzer Parallelkinematik Seminar [Thurneysen 02], is going to be calibrated using the Small Displacement Torsors method.

6. References.

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