On the modulational stability of magnetic structures in electron drift turbulence

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The generation of large-scale magnetic fields in magnetic electron drift mode turbulence is investigated. In particular, the mechanism of modulational instability caused by three-wave interactions is elucidated and the explicit increment is calculated. Also, a stability criterion similar to the known Lighthill criterion is found. © *2007 American Institute of Physics*.
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I. INTRODUCTION

In electrostatic drift wave theory, the generation of large-scale structures with additional symmetry, so-called zonal flows and streamers, is a well-known and active field of investigation.[1] These structures are spontaneously generated and sustained by small-scale drift-type fluctuations. Generation of such flows is commonly attributed to the effect of Reynolds stress generated by small-scale fluctuations,[2] using the free energy stored in density and temperature gradients.[2] The mechanism behind can be attributed to the well-known inverse cascade guaranteed in two-dimensional (and quasi-two-dimensional) fluids by the conservation of energy and enstrophy.[3] Moreover, studying the long-term dynamics has shown that flow generation by drift turbulence results in a modification of the drift wave spectra itself and finally in the saturation of the growing flows. This nonlinear "feedback" phenomenon is usually investigated in the frame of the so-called "predator-prey" model by employing the quasilinear closure (see, e.g., Refs. 4 and 5).

On the other hand, transport and amplification properties of large-scale magnetic fields are widely investigated, most of all because of their importance in different physical phenomena. One impressive effect of large, strong magnetic fields is the release of high-energy bursts in solar flares.[6] These bursts are believed to occur as a result of the reconnection of magnetic field lines, which one attempts to understand through turbulent magnetic field diffusivity, relating directly to the question of transport of large-scale magnetic fields in a turbulent environment.[7–10] Also, since the end of the 1970s, experiments have shown that strong quasisteady magnetic fields are created in laser-produced plasma.[11] This was an important result as it had often been assumed that the absence of magnetic field effects, which greatly affect heat transport, was a desirable feature of laser-produced plasmas.[12] These measurements showed clearly that strong magnetic fields can be generated even in unmagnetized plasmas.[13] Closer investigations revealed that these magnetic fields oscillate with a typical frequency in between the ion and the electron plasma frequency, and are fed by density and temperature gradients through the first-order baroclinic vector.[14]

In order to study the generation of strong magnetic fields within the mentioned frequency range, the nonlinear theory of magnetic electron drift wave turbulence in an unmagnetized inhomogeneous plasma has been developed recently.[15,16] This theory is a two-field theory, describing the magnetic field and temperature evolution, in contrast to the theory of electrostatic drift wave turbulence. It could be shown that structures very similar to large-scale flows in electrostatic drift wave turbulence can be found, the so-called zonal magnetic fields and magnetic streamers.[17–22] Furthermore, two regimes of large-scale magnetic field generation have recently been investigated in Ref. 23 and yielded a general criterion concerning the local form of the wave spectrum in the case of the kinetic regime (where resonance was assumed) and an explicit result for the hydrodynamic regime, where a monochromatic wave spectrum allowed the integration over all modes. Also, a sufficient criterion for large-scale structure generation was found in the form of Nyquist's criterion in Ref. 24.

The present work investigates large-scale magnetic field generation via modulational instability, which arises in the presence of a small-scale pump wave and its sidebands. We will find the conditions for such large-scale field generation, which occurs via triad interactions, i.e., with $\mathbf{k}+\mathbf{k}'+\mathbf{q}\sim 0$, and is an intrinsically nonlocal interaction in k space, since $|\mathbf{k}|\sim|\mathbf{k}'|\gg|\mathbf{q}|$. Here $\mathbf{k}$ and $\mathbf{k}'$ denote the small-scale and $\mathbf{q}$ the large-scale wave vectors. Thus, if one assumes the presence of a pump wave with a wave vector $\mathbf{k}$, two sidebands with wave vectors $\mathbf{k}_\pm=\mathbf{k}\pm\mathbf{q}$ will interact with the original pump wave and, as we will show, these interactions can lead to a growth of large-scale fields. Note that the long-term dynamics as well as the different saturation mechanisms of growing large-scale magnetic fields are not considered in this paper.

The rest of the paper is organized as follows: In Sec. II, we will give a short reminder of the basic equations for mag-

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netic electron drift modes as derived in Ref. 15. In Sec. III, the modulational instability arising from the interaction of a pump wave with its sidebands will be developed. Finally, the paper will be concluded in Sec. IV.

II. BASIC EQUATIONS

As already mentioned, we consider a nonuniform unmagnetized plasma. The inhomogeneity of the plasma is due to a density and a temperature gradient, which serve as an energy source for the magnetic electron drift wave turbulence investigated here.[16] The studied drift modes are low-frequency motions with a typical time scale in between the inverse ion and electron plasma frequencies, and hence we consider plasma consisting of an unpolarized electron fluid and immobile ions, which play a passive role as a neutralizing background, and the dominant role in dynamics is played by the electrons. Therefore, density perturbations can be neglected, i.e., the electron density n equals its equilibrium value n_0.

For deriving our model equations, the momentum equation together with Maxwell's equations and the energy equation are used,[25] together with the standard assumptions of a quasi-two-dimensional case in the x-y plane. Then, all quantities are independent of z, along which the perturbed magnetic field is directed. The length scales of the fluctuations are supposed to be much smaller than those of the equilibrium quantities. The temperature will be considered the sum of an equilibrium and a perturbed part T_0+T, and the perturbed magnetic field is denoted by B. As a last assumption, we consider the equilibrium density and temperature gradients ∇n_0 and ∇T_0 along the x axis only. Taking the curl of the momentum equation, one can show, with all the above assumptions, that the basic system of equations describing both linear and nonlinear properties of magnetic electron drift modes becomes[25]

$$\frac{\partial}{\partial t}(B-\lambda^2\nabla^2 B)+\beta\frac{\partial T}{\partial y}=\frac{e\lambda^4}{m}\{B,\nabla^2 B\}, \tag{1a}$$

$$\frac{\partial T}{\partial t}+\alpha\frac{\partial B}{\partial y}=-\frac{e\lambda^2}{m}\{B,T\}, \tag{1b}$$

with $\alpha=\lambda^2(eT_0/m)(2/3\kappa_n-\kappa_T)$ and $\beta=\kappa_n/e$. We also define the inverse length scales of the density and the temperature inhomogeneities, $\kappa_n=|\nabla \ln n|$ and $\kappa_T=|\nabla \ln T_0|$, and the skin depth $\lambda=c/\omega_p$.

Note that the evolution equation for the magnetic field is nonlinear in B. This is intrinsically due to the convective derivative in the electron momentum equation. The order of perturbation of the right-hand side (RHS) in (1b) shows that the perturbed temperature should not be neglected.

Linearizing the evolution equations (1) for small perturbations $B,T\propto\exp(-i\omega t+i\mathbf{k}\mathbf{r})$, the dispersion relation for magnetic electron drift modes is obtained with the linear wave eigenfrequency[26,27]

$$\omega_k=k_y\sqrt{\frac{\alpha\beta}{1+k^2\lambda^2}}. \tag{2}$$

From the definitions of α and β, one can easily see that there is a purely growing solution for $\kappa_T>(2/3)\kappa_n$ and the growth rate vanishes for modes with $k_y=0$. However, linear theory can only predict strong magnetic fields (exponential growth) and is not capable of describing the wave-wave interactions needed for the generation of large-scale magnetic fields. In fact, we will consider ω_k to be real in this article, i.e., $\alpha\beta>0$, such that we can concentrate on the nonlinear interactions. The basic equations (1) and (2) constitute the basis of our analysis of modulational instability of magnetic electron drift waves in what follows.

III. MODULATIONAL INSTABILITY

Since zonal fields and small-scale turbulence interact via nonlocal triad interactions $\mathbf{q}+\mathbf{k}+\mathbf{k}'\sim 0$, some sidebands to the pump wave have to be involved in the interaction as well, satisfying $\mathbf{k}'=\mathbf{k}\pm\mathbf{q}$. The modelization of the four interacting waves is done via Fourier expansion, i.e., $(\bar{B},\bar{T})(\mathbf{r},t)=(B_q,T_q)(t)e^{i(\mathbf{qr}-\Omega t)}$ for the large-scale fields, and the small-scale turbulence is modeled as the sum of the pump wave and its two sidebands,

$$\begin{pmatrix}\tilde{B}\\ \tilde{T}\end{pmatrix}=\begin{pmatrix}B_0\\ T_0\end{pmatrix}+\begin{pmatrix}B_+\\ T_+\end{pmatrix}+\begin{pmatrix}B_-\\ T_-\end{pmatrix}, \tag{3}$$

where $(B_0,T_0)(\mathbf{r},t)=(B_k,T_k)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)}+c.c.$ is the pump wave and $(B_\pm,T_\pm)(\mathbf{r},t)=(B_{k\pm},T_{k\pm})e^{i(\mathbf{k}_\pm\cdot\mathbf{r}-\omega_{k\pm}t)}+c.c.$ are the upper/lower sidebands. The conditions defining the sidebands are

$$\omega_{k\pm}\equiv\omega_k\pm\Omega, \tag{4a}$$

$$\mathbf{k}\pm\ \equiv\mathbf{k}\pm\mathbf{q}. \tag{4b}$$

A. Zonal magnetic fields

1. General calculations

The model equations for the small-scale turbulence are identical to the equations given by (1). The ones describing zonal fields can be found by averaging the latter over the small/fast scales and using the fact that they are elongated along the y axis, so that we obtain

$$\partial_T(1-\lambda^2\nabla^2)\bar{B}=\frac{e\lambda^4}{m}\overline{\{\tilde{B},\nabla^2\tilde{B}\}}, \tag{5a}$$

$$\partial_T\bar{T}=-\frac{e\lambda^2}{m}\overline{\{\tilde{B},\tilde{T}\}}, \tag{5b}$$

where ∂_T denotes the partial derivative with respect to the slow time variable. The nonlinear terms on the RHS of Eq. (1) for the small and (5) for the large scales are determined using the resonance principle and the triad interactions. For zonal fields, the left-hand side (LHS) of (5) is proportional to $\exp(iqr)$, so that there are four possibilities for the RHS in order to be in resonance:

$\exp(ik_+r)\times\exp(-ikr)$, $\exp(-ikr)\times\exp(ik_+r)$, $\exp(-ik_-r)\times\exp(ikr)$ and $\exp(ikr)\times\exp(-ik_-r)$, since these are the four possibilities to decompose q: $q=k_+-k=-k+k_+=-k_-+k=k-k_-$. Note that, e.g., k_+-k and $-k+k_+$ give different contributions since the Laplacian is applied on the second exponential only on the RHS of Eq. (5a). With these considerations, the equations for the zonal field Fourier amplitudes follow directly from (5) and are given by

$$-i\Omega B_q=P_{kq}\frac{1}{1+q^2\lambda^2}[\lambda^2(k^2-k_+^2)B_{k+}B_k^* + \lambda^2(k_-^2-k^2)B_{k-}^*B_k], \tag{6a}$$

$$-i\Omega T_q=P_{kq}(T_{k+}B_k^*-B_{k+}T_k^*+B_{k-}^*T_k-T_{k-}^*B_k), \tag{6b}$$

where we defined $P_{kq}\equiv\lambda^2 e/m(\mathbf{k}\times\mathbf{q})\cdot\hat{z}$ and a star $(*)$ denotes complex conjugate. From this equation, we see that for determining the evolution of the zonal fields, one has to find the expressions of the amplitudes of the sidebands B_{k+}, T_{k+}, B_{k-}^*, and T_{k-}^*. These can be found from Eqs. (1) with the same resonance arguments as before. However, in contrast to large-scale fields, the spatial derivative with respect to the y coordinate is nonzero, which makes it necessary to diagonalize (i.e., uncouple) the equations for solving them. First, let us find the expressions for B_{k+} and T_{k+}. From resonance arguments, one obtains

$$-\omega_{k+}B_{k+}+\frac{\beta k_y}{1+k_+^2\lambda^2}T_{k+}=-iP_{kq}\frac{1}{1+k_+^2\lambda^2}\times[\lambda^2(q^2-k^2)B_qB_k], \tag{7a}$$

$$\alpha k_yB_{k+}\quad-\omega_{k+}T_{k+}=iP_{kq}(B_qT_k-B_kT_q). \tag{7b}$$

For diagonalization of the LHS of the latter system of equations, the eigenvalues can be found reading

$$\delta\omega_+^\pm\equiv-\omega_{k+}\pm k_y\sqrt{\frac{\alpha\beta}{1+k_+^2\lambda^2}}. \tag{8}$$

For B_{k-}^* and T_{k-}^*, one has to take into account the complex conjugate. Otherwise, it is exactly the same procedure and one can show that the corresponding equations are

$$-\omega_{k-}B_{k-}^*+\frac{\beta k_y}{1+k_-^2\lambda^2}T_{k+}=iP_{kq}\frac{1}{1+k_-^2\lambda^2}[\lambda^2(k^2-q^2)B_qB_k^*], \tag{9a}$$

$$\alpha k_yB_{k-}^*\quad-\omega_{k-}T_{k-}^*=-iP_{kq}(T_qB_k^*-B_qT_k^*), \tag{9b}$$

with the eigenvalues

$$\delta\omega_-^\pm\equiv-\omega_{k-}\pm k_y\sqrt{\frac{\alpha\beta}{1+k_-^2\lambda^2}}. \tag{10}$$

Together with the corresponding eigenvectors to the above eigenvalues, we can diagonalize the systems of equations (7) and (9) with the matrix of change of basis,

$$S=\begin{pmatrix}1/\delta_\pm & -1/\delta_\pm\\ 1 & 1\end{pmatrix}, \tag{11}$$

where we defined the coefficient $\delta_\pm\equiv\sqrt{\alpha/\beta}\sqrt{1+k_\pm^2\lambda^2}$. These matrices finally diagonalize the equations describing the evolution of the pump wave's sidebands in the following way:

$$\delta\omega_{k_\pm}^+\bar{B}_\pm^+=\delta_\pm\mathrm{NL}_B+\mathrm{NL}_T, \tag{12a}$$

$$\delta\omega_{k_\pm}^-\bar{B}_\pm^-=-\delta_\pm\mathrm{NL}_B+\mathrm{NL}_T, \tag{12b}$$

with $\bar{B}_\pm^+=\delta_\pm B_{k_\pm}^{(*)}+T_{k_\pm}^{(*)}$ and $\bar{B}_\pm^-=-\delta_\pm B_{k_\pm}^{(*)}+T_{k_\pm}^{(*)}$. In (12), we defined NL_B as the nonlinear term (i.e., the RHS) of (7a) in the case of a "+," the RHS of (9a) for the case of a "−," and analogously NL_T the RHS of (7b) or (9b), respectively. We can now find explicit expressions for the sideband amplitudes B_{k+}, T_{k+}, B_{k-}^*, and T_{k-}^* as functions of known quantities describing the pump wave and the zonal magnetic field. In order to do so, we use Eq. (12) and all the necessary definitions described before, to get after some algebra

$$B_{k+}=\frac{i}{2}P_{kq}\left\{\frac{\lambda^2(k^2-q^2)}{1+k_+^2\lambda^2}\left(\frac{1}{\delta\omega_+^+}+\frac{1}{\delta\omega_+^-}\right)B_qB_k + \frac{1}{\delta_+}\left(\frac{1}{\delta\omega_+^+}-\frac{1}{\delta\omega_+^-}\right)(B_qT_k-T_qB_k)\right\}, \tag{13a}$$

$$T_{k+}=\frac{i}{2}P_{kq}\left\{\delta_+\frac{\lambda^2(k^2-q^2)}{1+k_+^2\lambda^2}\left(\frac{1}{\delta\omega_+^+}-\frac{1}{\delta\omega_+^-}\right)B_qB_k + \left(\frac{1}{\delta\omega_+^+}+\frac{1}{\delta\omega_+^-}\right)(B_qT_k-T_qB_k)\right\}, \tag{13b}$$

$$B_{k-}^*=\frac{i}{2}P_{kq}\left\{\frac{\lambda^2(k^2-q^2)}{1+k_-^2\lambda^2}\left(\frac{1}{\delta\omega_-^+}+\frac{1}{\delta\omega_-^-}\right)B_qB_k^* + \frac{1}{\delta_-}\left(\frac{1}{\delta\omega_-^+}-\frac{1}{\delta\omega_-^-}\right)(B_qT_k^*-T_qB_k^*)\right\}, \tag{13c}$$

$$T_{k-}^*=\frac{i}{2}P_{kq}\left\{\delta_-\frac{\lambda^2(k^2-q^2)}{1+k_-^2\lambda^2}\left(\frac{1}{\delta\omega_-^+}-\frac{1}{\delta\omega_-^-}\right)B_qB_k^* + \left(\frac{1}{\delta\omega_-^+}+\frac{1}{\delta\omega_-^-}\right)(B_qT_k^*-T_qB_k^*)\right\}. \tag{13d}$$

These equations together with the large-scale equations (6) theoretically solve the problem and lead to the dispersion relation of the zonal magnetic fields as a function of the pump wave amplitude and the small-scale eigenvalues. They build, however, a very complicated system and we did not yet use the assumptions about the wave vector q being small compared to k and the frequency Ω also being small with respect to ω_k. For simplification of the problem, we go back to the definition of the eigenvalues (8) and (10) and expand them around $q=0$.

2. Simplification

In the definitions of the eigenvalues, (8) and (10), the second term is very similar to the linear frequency ω_k given in (2). Let us denote this term by $\omega(k_\pm)$ and expand it around $q=0$,

$$\omega(k_+) \simeq \omega(k+q)|_{q=0} + \left.\frac{\partial\omega(k+q)}{\partial(k+q)}\right|_{q=0}[(k+q)-k] + \frac{1}{2}\left.\frac{\partial^2\omega(k+q)}{\partial(k+q)^2}\right|_{q=0}[(k+q)-k]^2 = \omega_k + qv_g + \frac{q^2v_g'}{2}, \tag{14a}$$

$$\omega(k_-) \simeq \omega(k-q)|_{q=0} + \left.\frac{\partial\omega(k-q)}{\partial(k-q)}\right|_{q=0}[(k-q)-k] + \frac{1}{2}\left.\frac{\partial^2\omega(k-q)}{\partial(k-q)^2}\right|_{q=0}[(k-q)-k]^2 = \omega_k - qv_g + \frac{q^2v_g'}{2}, \tag{14b}$$

where the group velocity and its derivative are defined as $v_g=\partial\omega_k/\partial k_x$ and $v_g'=\partial^2\omega_k/\partial k_x^2$. The four eigenvalues have now the explicit form

$$\delta\omega_+^+ = -\left(\Omega - qv_g - \frac{q^2v_g'}{2}\right), \tag{15a}$$

$$\delta\omega_-^+ = \Omega - qv_g + \frac{q^2v_g'}{2}, \tag{15b}$$

$$\delta\omega_+^- = -\left(\Omega + 2\omega_k + qv_g + \frac{q^2v_g'}{2}\right), \tag{15c}$$

$$\delta\omega_-^- = \Omega - 2\omega_k + qv_g - \frac{q^2v_g'}{2}. \tag{15d}$$

But q/k is not the only small parameter involved. The second one is Ω/ω, and thus it can now be seen that all terms proportional to $1/\delta\omega_{k_\pm}^+$ are much larger than terms in $1/\delta\omega_{k_\pm}^-$. We will therefore neglect the latter and note that in this case, (12) gives us a relation between the magnetic field and the temperature Fourier amplitudes $T_{k_\pm}^{(*)}=\delta_\pm B_{k_\pm}^{(*)}$. The complete set of equations after this simplification is given by Eqs. (6) and

$$B_{k+} = \frac{i}{2}P_{kq}\left\{\frac{\lambda^2(k^2-q^2)}{1+k_+^2\lambda^2}\frac{1}{\delta\omega_+^+}B_qB_k + \frac{1}{\delta_+}\frac{1}{\delta\omega_+^+}(B_qT_k - T_qB_k)\right\}, \tag{16a}$$

$$B_{k-}^* = \frac{i}{2}P_{kq}\left\{\frac{\lambda^2(k^2-q^2)}{1+k_-^2\lambda^2}\frac{1}{\delta\omega_-^+}B_qB_k^* + \frac{1}{\delta_-}\frac{1}{\delta\omega_-^+}(B_qT_k^* - T_qB_k^*)\right\}, \tag{16b}$$

$$T_{k+} = \delta_+B_{k+}, \tag{16c}$$

$$T_{k-}^* = \delta_-B_{k-}^*. \tag{16d}$$

3. Dispersion relation

With (16) and (6), we finally find the evolution equation for the large-scale Fourier magnetic field amplitude in the above-mentioned limits. We will furthermore assume $q\lambda \ll 1$ and that $1+k_\pm^2\lambda^2 \approx 1+k^2\lambda^2$ (i.e., $\delta_\pm \approx \delta$). This yields then

$$-i\Omega B_q = \frac{i}{2}P_{kq}^2\left\{\frac{1}{1+k^2\lambda^2}\left[\lambda^2(k^2-k_+^2)\lambda^2(k^2-q^2)\times\frac{1}{\delta\omega_+^+}B_q|B_0|^2 + \lambda^2(k_-^2-k^2)\lambda^2(k^2-q^2)\times\frac{1}{\delta\omega_-^+}B_q|B_0|^2\right] + \frac{1}{\delta}\left[\lambda^2(k^2-k_+^2)\frac{1}{\delta\omega_+^+}B_qT_kB_k^* - \lambda^2(k^2-k_+^2)\frac{1}{\delta\omega_+^+}T_q|B_0|^2 + \lambda^2(k_-^2-k^2)\times\frac{1}{\delta\omega_-^+}B_qT_k^*B_k - \lambda^2(k_-^2-k^2)\frac{1}{\delta\omega_-^+}T_q|B_0|^2\right]\right\}. \tag{17}$$

Given the relation between the small-scale Fourier temperature and magnetic field amplitudes in (16), we can replace the temperature in the mixed terms (that is, in those terms containing neither $|B_0|^2$ nor $|T_0|^2$) in the latter equation. As to the equation for the temperature, (6b), it vanishes when we use the same relations. Note that this confirms the result obtained previously in Refs. 16 and 15, where the model equations (1) were directly applied to the limit for zonal fields, neglecting the noise emitted into the flows by incoherent coupling of the drift wave turbulence. With these last points, the large-scale amplitude equations become

$$-\Omega B_q = \frac{1}{2}P_{kq}^2\left\{\left(\frac{\lambda^2(k^2-q^2)}{1+k^2\lambda^2}+1\right)\left(\frac{\lambda^2(k^2-k_+^2)}{\delta\omega_+^+} + \frac{\lambda^2(k_-^2-k^2)}{\delta\omega_-^+}\right)B_q|B_0|^2 - \frac{1}{\delta}\left(\frac{\lambda^2(k^2-k_+^2)}{\delta\omega_+^+} + \frac{\lambda^2(k_-^2-k^2)}{\delta\omega_-^+}\right)T_q|B_0|^2\right\}, \tag{18a}$$

$$-\Omega T_q = 0. \tag{18b}$$

Remember that the equation for the temperature contains the nonlinear term due to interaction between the temperature and the magnetic field, whereas in the equation for the magnetic field, the nonlinear term is due to self-interaction of the magnetic field. Equations (18) tell us then that the coupling between the temperature and the magnetic field is much weaker than the self-interaction of the magnetic field. Furthermore, the Poisson brackets of the form $\{\cdot,\nabla^2\cdot\}$ in (1a) are known to cascade energy toward the large scales, whereas the brackets of the form $\{\cdot,\cdot\}$ are responsible for the direct cascade into the small scales.[28] We see then from (18) that the direct cascade is much less important in our case than the inverse cascade described in (18a). This confirms, of course, the generation of large-scale structures by small-scale turbulence.

The common term in the brackets of (18a) has to be calculated explicitly,

$$\frac{\lambda^2(k^2-k_+^2)}{\delta\omega_+^+}+\frac{\lambda^2(k_-^2-k^2)}{\delta\omega_-^+} = \frac{2k_xq^3v_g'+2q^2(\Omega-qv_g)}{(\Omega-qv_g)^2-(q^2v_g'/2)^2} \approx \frac{2k_xq^3v_g'}{(\Omega-qv_g)^2-(q^2v_g'/2)^2}, \tag{19}$$

since $\Omega-qv_g\sim q^2v_g'/2\ll k_xqv_g'$. The final form of (18a) is then

$$\Omega[(\Omega-qv_g)^2-(q^2v_g'/2)^2]B_q = -P_{kq}^2\left(\frac{\lambda^2(k^2-q^2)}{1+k^2\lambda^2}+1\right)k_xq^3v_g'\lambda^2|B_0|^2B_q. \tag{20}$$

This equation will give us the dispersion relation we are looking for. In order to find the expression for the frequency Ω, let us discuss its form to be expected. First of all, we expect a resonance between the large-scale flows and the perturbation, and secondly, the neglecting of the second term in (19) already anticipated a frequency of the form $\Omega=qv_g+i\gamma$. The imaginary part has, of course, to be added for instability to be possible. Plugging this form of the frequency into (20), the imaginary part of the equation yields the first three solutions, which are $\gamma=0$ and $\gamma=\pm iq^2v_g'/2$. These solutions are real corrections to the form of the frequency we have chosen and confirm our assumption previously taken. The real part of the equation, however, yields a complex frequency with imaginary part,

$$\gamma=\pm\sqrt{\frac{v_g'}{v_g}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\hat{\mathbf{z}}\right)^2\left(\frac{\lambda^2(k^2-q^2)}{1+k^2\lambda^2}+1\right)|B_0|^2k_xq^2\lambda^2-(q^2v_g'/2)^2}. \tag{21}$$

Calculating the group velocity, we can replace the term $v_g'/v_gk_x\lambda^2$ and obtain

$$\gamma=\pm\sqrt{-\frac{v_g'}{\omega_k}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\hat{\mathbf{z}}\right)^2\left(\frac{\lambda^2(k^2-q^2)}{1+k^2\lambda^2}+1\right)(1+k^2\lambda^2)^{3/2}|B_0|^2q^2-(q^2v_g'/2)^2}. \tag{22}$$

In order to have modulational instability, the system must satisfy

$$\frac{v_g'}{\omega_k}<0, \tag{23}$$

which is merely the well-known Lighthill criterion. Explicitly, we can calculate v_g' from the definition of ω_k in (2), which yields

$$v_g=\frac{\partial\omega_k}{\partial k_x}=-\omega_k\frac{k_x\lambda}{(1+k^2\lambda^2)^{3/2}}, \quad \Rightarrow v_g'=\frac{\partial v_g}{\partial k_x}=-\omega_k\lambda^2\frac{1-2k_x^2\lambda^2+k_y^2\lambda^2}{(1+k^2\lambda^2)^2}. \tag{24}$$

One can easily see that there is a stabilizing effect for $1-2k_x^2\lambda^2+k_y^2\lambda^2<0$. The latter result is exactly the same as was found for the case of a monochromatic wave spectrum in Ref. 23. In fact, if we compare the increments directly with the corresponding definitions of the coefficients used, we find that in the limit $k\lambda\ll1$, the increment of the modulational instability is four times as high as the one for the hydrodynamic regime. Otherwise, the expressions are the same if we neglect the second-order term within the increment for the modulational instability (22). This is not surprising since the mechanism considered is the same, only that the sidebands have been neglected in the case of a monochromatic wave spectrum through an integration over a delta function $\delta(k-k_0)$.

B. Magnetic streamers

All the above results are valid for zonal magnetic fields. In the case of magnetic streamers, however, the large-scale equations are more complicated than for zonal fields, Eqs. (5), and have to be diagonalized as well as the small-scale equations before. This is because the wave vector for magnetic streamers is along the y axis, $\mathbf{q}=(0,q,0)$, and therefore the derivatives with respect to y in the basic model equations (1) do not disappear. Specifically, the new large-scale equations take the form

$$-i\Omega B_q+i\frac{\beta q}{1+q^2\lambda^2}T_q=\lambda^4\frac{e}{m}\overline{\{\tilde{B},\nabla^2\tilde{B}\}}, \tag{25a}$$

$$-i\Omega T_q+i\alpha qB_q=-\lambda^2\frac{e}{m}\overline{\{\tilde{B},\tilde{T}\}}. \tag{25b}$$

The rest of the derivation is using the same approach as the case described in Sec. III A, and we thus give directly the new dispersion relation [corresponding to Eq. (20)] of magnetic streamers

$$\frac{(\Omega^2-q^2\alpha\beta)^2}{\Omega q\sqrt{\alpha\beta}}[(\Omega-qv_g)^2-(q^2v_g'/2)^2]^2-\Gamma\left(\frac{\chi}{\Omega}-\frac{K^2}{q\sqrt{\alpha\beta}}\right)\times(\Omega^2-q^2\alpha\beta)[(\Omega-qv_g)^2-(q^2v_g'/2)^2]=0, \quad (26)$$

where we defined the coefficients $\Gamma=P_{kq}^2kq^3v_g'\lambda^2|B_0|^2$, $K^2=1+\lambda^2(k^2-q^2)/(1+k^2\lambda^2)$, and $\chi=1/\sqrt{1+k^2\lambda^2}$. The first four solutions are easily found to be

$$\Omega_{1,2}=qv_g\pm\frac{q^2v_g'}{2}, \quad (27)$$

$$\Omega_{3,4}=\pm q\sqrt{\alpha\beta}, \quad (28)$$

which are all real and represent thus stable solutions. Excluding those, the dispersion relation takes the form

$$\frac{(\Omega^2-q\alpha\beta)[(\Omega-qv_g)^2-(q^2v_g'/2)^2]}{\Omega q\sqrt{\alpha\beta}}=\Gamma\left(\frac{\chi}{\Omega}-\frac{K^2}{q\sqrt{\alpha\beta}}\right). \quad (29)$$

In the limit $k\lambda\ll 1$, both χ and K^2 are unity and the latter equation has the simpler form

$$(\Omega+q\sqrt{\alpha\beta})[(\Omega-qv_g)^2-(q^2v_g'/2)^2]=-\Gamma. \quad (30)$$

Here, the group velocity corresponds to the derivative of the frequency with respect to k_y, which is equal to

$$v_g=\frac{\partial\omega_k}{\partial k_y}=\sqrt{\alpha\beta}\frac{1+k_x^2\lambda^2}{(1+k^2\lambda^2)^{3/2}}\xrightarrow{k\lambda\ll 1}\sqrt{\alpha\beta}. \quad (31)$$

This means that in the dispersion relation, $q\sqrt{\alpha\beta}=qv_g$, and if we assume $\Omega=qv_g+i\gamma$, we get

$$(2qv_g+i\gamma)[-\gamma^2-(q^2v_g'/2)^2]=-\Gamma. \quad (32)$$

The imaginary part of (32) has the solutions

$$\gamma_1=0\Rightarrow\Omega=qv_g=q\sqrt{\alpha\beta}, \quad (33)$$

$$\gamma_{2,3}=\pm i\frac{q^2v_g'}{2}\Rightarrow\Omega=qv_g\pm\frac{q^2v_g'}{2}. \quad (34)$$

These solutions have already been found before and (34) represents a small real correction to the frequency. The real part of the dispersion relation yields

$$\gamma=\pm\sqrt{\frac{v_g'}{2v_g}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\hat{\mathbf{z}}\right)^2k_yq^2\lambda^2|B_0|^2-(q^2v_g'/2)^2}. \quad (35)$$

But in the case of magnetic streamers, $v_g'=-\sqrt{\alpha\beta}(1+k_x^2\lambda^2)3k_y\lambda^2/(1+k^2\lambda^2)^{5/2}<0$ and $v_g>0$, so that the term under the square root is always negative. In other words, in the limit $k\lambda\ll 1$, magnetic streamers are always stable for this kind of instability.

In the opposite limit $k\lambda\gg 1$, $K^2\gg\chi$, and (29) yields in this case

$$\frac{\Omega^2-q^2\alpha\beta}{\Omega}[(\Omega^2-qv_g)-(q^2v_g'/2)^2]=-\Gamma K^2, \quad (36)$$

and the group velocity is larger than $\sqrt{\alpha\beta}$. So, when we set again $\Omega=qv_g+i\gamma$, we can neglect $q^2\alpha\beta$ with respect to Ω^2 and the latter equation can be written as

$$\frac{q^2\alpha\beta(qv_g-i\gamma)}{q^2v_g^2-\gamma^2}[\gamma^2+(q^2v_g'/2)^2]=-\Gamma K^2. \quad (37)$$

The imaginary part yields exactly the same stable results as in the above case, whereas (if we neglect γ^2 with respect to $q^2v_g^2$ in the denominator) the real part yields

$$\gamma=\pm\sqrt{-v_g'\frac{v_g}{\alpha\beta}\left(\lambda^2\frac{e}{m}(\mathbf{k}\times\mathbf{q})\cdot\hat{\mathbf{z}}\right)^2k_yq^2\lambda^2|B_0|^2-(q^2v_g'/2)^2}. \quad (38)$$

Since $v_g>0$ and $v_g'<0$, unstable solutions are possible for magnetic streamers in the case $k\lambda\gg 1$, verifying the Lighthill criterion for modulational instability.

IV. CONCLUSIONS

In this paper, the properties of magnetic electron drift mode turbulence is studied and a mechanism of generation of large-scale fields is investigated. It is shown that in the presence of a small-scale pump wave with a wave vector $\mathbf{k}$, an upper and a lower sideband will interact with the pump wave due to nonlocal triad interactions with $\mathbf{k}+\mathbf{k}'+\mathbf{q}\sim 0$, where the wave vectors of the sidebands are $\mathbf{k}'=\mathbf{k}\pm\mathbf{q}$, and the condition $|\mathbf{q}|\ll|\mathbf{k}|$ is satisfied. These interactions are elucidated, an expression for the resulting increments of both zonal fields and magnetic streamers is calculated, and a condition similar to the Lighthill criterion for instability is found. It is also noted that the resulting criterion is very similar to the one previously found in Ref. 23 for the case of a monochromatic wave spectrum. However, because of its more general nature, it shows that the monochromatic case underestimates the increment by a factor of 4 and it also yields a higher-order correction to this increment.

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