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A review of finite-element modelling in snow mechanics

The finite-element method (FEM) is one of the main numerical analysis methods in continuum mechanics and mechanics of solids (Huebner and others, 2001). Through mesh discretization of a given continuous domain into a finite number of sub-domains, or elements, the method finds approximate solutions to sets of simultaneous partial differential equations, which express the behavior of the elements and the entire system. For decades this methodology has played an accelerated role in mechanical engineering, structural analysis and, in particular, snow mechanics. To the best of our knowledge, the application of finite-element analysis in snow mechanics has never been summarized. Therefore, in this correspondence we provide a table with a detailed review of the main FEM studies on snow mechanics performed from 1971 to 2012 (40 papers), for facilitating comparison between different mechanical approaches, outlining numerical recipes and for future reference. We believe that this kind of compact review in a tabulated form will produce a snapshot of the state of the art, and thus become an appropriate, timely and beneficial reference for any relevant follow-up research, including, for example, not only snow avalanche questions, but also modeling of snow microstructure and tire–snow interaction. To that end, this correspondence is organized according to the following structure. Table 1 includes all essential information about previously published FEM studies originally developed to investigate stresses in snow with all corresponding mechanical and numerical parameters. Columns in Table 1 provide references to particular studies, placed in chronological order. Rows correspond to the main model parameters and other details of each considered case.

In order to give an overview of the studies covered by this review, we briefly summarize them below. Previously considered physical and engineering problems in snow mechanics can be roughly separated into several major categories, namely:

state of strain and stress in snowpack on slope, snow creep and compaction (Smith and others, 1971; Smith, 1972; Curtis and Smith, 1974; Smith and Curtis, 1975; Lang and Sommerfeld, 1977; Johnson, 1998; Bartelt and Christen, 1999; Bartelt and others, 2000; Teufelsbauer, 2009, 2011);

influence of snow weak layer and subcritical weak spots on the mechanical state of snowpack and slab avalanche release (McClung, 1979; Singh, 1980; Bader and Salm, 1990; Bartelt and Christen, 1999; Stoffel and Bartelt, 2003; Stoffel, 2005; Gaume and others, 2011, 2012);

skier loadings on inclined snowpack (Schweizer, 1993; Wilson and others, 1999; Jones and others, 2006; Habermann and others, 2008; Mahajan and others, 2010);

shock loading and explosive loading on snowpack (Johnson and others, 1993; Miller and others, 2011);

reproduction of mechanical experiments for studying fundamental rheological properties of snow (Mohamed and others, 1993; Meschke and others, 1996; Jamieson and Johnston, 2001; Haehnel and Shoop, 2004; Cresseri and Jommi, 2005; Cresseri and others, 2010);

forces exerted by snow cover on avalanche defense structures (Bartelt and Christen, 1999; Bartelt and others, 2000; Stoffel, 2005; Teufelsbauer, 2011);

fracture properties of snow and snow slabs, crack propagation (Bažant and others, 2003; Mahajan and Senthil, 2004; Stoffel, 2005; Sigrist and others, 2006; Sigrist and Schweizer, 2007; Heierli and others, 2008; Mahajan and Joshi, 2008);

tire/wheel–snow interaction (Haehnel and Shoop, 2004; Lee, 2009);

microstructure studies of snow volume obtained from X-ray microtomography (Pieritz and others, 2004; Schneebeli, 2004; Srivastava and others, 2010; Theile, 2010; Hagenmuller, 2011).

We have omitted some studies from Table 1 because sufficient detail was not available to us (e.g. Navarre and Desrues, 1980; Singh, 1980). Others are omitted because they did not focus purely on mechanics; for example, some studies using FEM for snow or firn studies were mainly dedicated to heat transfer (at the microstructural level or at the snow–permafrost boundary), air ventilation within pore space, water infiltration or metamorphism (Christon and others, 1994; Tseng and others, 1994; Meussen and others, 1999; Phillips and others, 2000; Pielmeier and others, 2001; Albert, 2002; Bartelt and others, 2004; Kaempfer and others, 2005). Still others focused on the transition from solid to fluid (Daudon and Dufour, 2011) or the phase-tracking snow microstructure model (Slaughter and Zabaras, 2012). Finally, FEM papers on tire–snow interaction may be found in references within Haehnel and Shoop (2004) and Lee (2009).

We hope that the papers collected in this review will serve to facilitate comparison between and assimilation of different mechanical approaches or numerical recipes, and that they will be useful for solving the many remaining problems.

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Table 1. Finite-element method studies on snow mechanics, 1971–2012

Parameter	Smith and others (1971)	Smith (1972)	Curtis and Smith (1974)	Smith and Curtis (1975)	Lang and Sommerfeld (1977)	McClung (1979)	Bader and Salm (1990)	Schweizer (1993)	Johnson and others (1993)
Constitutive law/ Approach	Linear elasticity	Linear elasticity	Linear elasticity	Linear elasticity and nonlinear behavior	Viscoelasticity with orthotropic snow properties	Linear elastic slab with nonlinear basal relationship (strain-softening weak layer)	Steady-state linear viscoelasticity	Linear viscosity	Linear elasticity
Young's modulus (10^6 N m^{-2})	(0.2×10^3)–(1.2×10^3)	0.2×10^3	20, 200, 2000	20, 40, 100, 150, 260 (after Nakaya, 1959)	–	3.95 (shear modulus, G), same for slab and weak layer; for $\nu=0.25$, $E=9.875$, where $E=2G(1+\nu)$	–	–	–
Poisson's ratio	0.10–0.30	0.20	0.00, 0.25, 0.45	0.25	–	0.25	–	0.10–0.40 viscous analogue of Poisson's ratio; selected arbitrarily (Mellor, 1975)	–
Viscosity (Pa s)	–	–	–	–	–	–	0, 0.5×10^8, 10^8, 5×10^8	(3.1×10^7)–(2.4×10^9) Pa s	–
Strain rates (s^{-1})	–	–	–	–	$<0.5\times10^{-7}$	–	infinitesimal, 10^{-4} to 10^{-3}	–	$>10^3$
Density or density range (kg m^{-3})	100–400	127–400	210–400	210–400	~175–418	300	430	100–300	100–520
Snow depth (slab or sample thickness) (m)	0.96	<~2.0	<~2.0	–	1.3–1.8	(1)	2 (1)	<1.0 (0.4)	(0.08)
Fracture energy of the weak layer (J m^{-2})	–	–	–	–	–	–	–	–	–
Weak-layer thickness (m)	No weak layer; single-layer homogeneous snow	not specified	not specified	not specified	0.1	0	0–0.02	0.02	–
Weak-layer length (m)	–	12 (same as model length)	~19 (same as model length)	not specified	~10 (same as model length)	~5 (same as model length)	∞ (same as model length), weak/gliding zone 0.2–80.0	10 (same as model length)	–

Weak-layer properties	–	Bottom layer with lower Young's modulus (0.2×10^8 N m^{-2})	Bottom layer with lower Young's modulus (0.2×10^8 N m^{-2})	Sub-layer with nonlinear stress–strain curve	Basal layer with densities 245–255 and 257–348 kg m^{-3}	1-D line spring elements at the base of the slab allowing strain softening	Thin middle layer with low viscosity (0.5×10^8 or 10^8 Pa s) containing zone with zero viscosity (ideal gliding), which was replaced by corres-ponding interface conditions for the top and bottom layers	Soft layer 50 kg m^{-3}, with low viscosity (3.1×10^7 Pa s)	–
Temperature or temperature range (°C)	–	–	–	–	–	–	–	–5.15	–23.0 to –2.0
Calculation times	–	–	–	–	3 or 5 days	–	0–60 min	–	<200 µs
1-D, 2-D or 3-D mesh	2-D	2-D	2-D	2-D	2-D	2-D	2-D	2-D	1-D and 2-D
Element shape	Triangular	Triangular	Triangular	Triangular	4-node quadrilateral	not specified	4-node element	4-node quadrilateral	–
FEM code or software used	Program by Wilson and Clough (1963)	Program by Wilson and Clough (1963)	Program by Wilson and Clough (1963)	Program by Wilson and Clough (1963)	FE code by Goudreau and others (1967)	–	TPS10	TPS10	PRONTO 2-D
Issues addressed	State of stress in homogeneous snow on slope	State of stress in layered snow on slope, effect of new layer and weak sub-layer	State of stress in layered snow on slope, refinement of bottom weak sub-layer	State of stress in layered snow on slope, with nonlinear stress analysis	Study of snow deformation on avalanche slope with FE five- or seven-layer model corresponding to local stratigraphy in the experimental area, compared to measured deformation rates	Calculation of snow slab stress conditions prior to shear band propagation	Influence of weak layer on stress and strain distributions in a layered snowpack with consideration of snow creep; conditions and speed of fracture propagation	Skier-induced stress and snow layering	Shock loading of snow, pressure–density relationships, edge wave superposition on primary shock wave, impedance mismatch between stress gauges and snow
Laboratory or field studies used as references for validation	–	Mellor (1966, 1968)	Mellor (1966)	–	Experimental measured deformation of a sloping snowpack as part of the same study	–	–	–	–
Sites investigated for validation	Lift Gully, Berthoud Pass, CO, USA	Lift Gully, Berthoud Pass, CO, USA	Lift Gully, Berthoud Pass, CO, USA (1967/68)	Lift Gully, Berthoud Pass, CO, USA (1965/66)	Avalanche slope at Berthoud Pass, CO, USA	–	–	–	–

Table 1. (continued)

Parameter	Mohamed and others (1991)	Meschke and others (1996)	Johnson (1998)	Wilson and others (1999)	Bartelt and Christen (1999)	Bartelt and others (2000)	Jamieson and Johnston (2001)	Stoffel and Bartelt (2003)	Bažant and others (2003)
Constitutive law/ Approach	Linear-plastic behavior with nonlinear material	Modified nonlinear elastic Cam-clay and Drucker–Prager models	Dynamic, with roughly spherical elastic–plastic particles	Linear elasticity	Viscoelasticity with microplane *N*-directional 1-D material laws ($N=10$ was found to be optimal)	Viscoelasticity with nonlinear and instationary material with macroscopic and microscopic parameters used in constitutive laws (Lagrangian coordinate system)	Linear elasticity	Temperature-dependent viscoelasticity	Linear elasticity with equivalent linear clastic fracture mechanics
Young's modulus (10^6 N m^{-2})	–	7.5 (shear modulus) assuming $\nu=0.25$, $E=18.75$	–	0.25–10.0 (after Mellor, 1975; Shapiro, 1997)	–	–	0.2–1.0	$E_0=f(\rho)$, after von Moos and others (2003), which agrees well with Mellor (1975); Voytkovskiy (1977)	0.9875 (after McClung, 1977, 1979)
Poisson's ratio	0.00	–	–	0.25	0.25 ($300 \leq p \leq 500$ kg m^{-3}) (after Mellor, 1975)	–	0.25	0.00	0.25 (after McClung, 1977, 1979)
Viscosity (Pa s)	–	–	–	–	$\eta=f(\rho, T)$ Empirical laws after many authors; see original text	$\eta=f(\rho, T)$ Empirical laws after many authors; see original text	–	–	–
Strain rates (s^{-1})	3×10^{-4} (for compression tests)	–	0.4–20 m s^{-1}	$>10^{-4}$	$<10^{-3}$	–	$>2.5\times10^{-3}$	10^{-9}–10^{-4}	–
Density or density range (kg m^{-3})	350–600 (initial and final)	400, 520	–	not specified	115, 138–237 (initial)	213–242, 300 (initial)	160–200	180–350	–
Snow depth (slab or sample thickness) (m)	0.178, 0.79	(0.076, 0.04, 0.08)	–	1.0 (0.3, 0.5)	0.9 or 1.0 (initial)	0.24, 3.0	(~0.04)	<2.0	<3.5
Fracture energy of the weak layer (J m^{-2})	–	–	–	–	–	–	–	–	2.51, 2.73, 2.97
Weak-layer thickness (m)	–	–	–	0.005	–	–	0.002	0.0	–
Weak-layer length (m)	–	–	–	6 (same as model length)	–	–	0.156/0.208 (below shear frame/ including substratum)	0–20	∞ (same as model length)
Weak-layer properties	–	–	–	E_0 lower than neighboring snow layers (0.25 MPa)	–	–	2 mm soft layer (160 kg m^{-3}) with lower E_0 (0.2 MPa)	Springs transferring normal stress with shear resistance = 0	Crack propagates at or near underlying rigid–elastic interface

Temperature or temperature range (°C)	–	–	–	–15.0 to –5.0	–10.0	–10 to –2	–	–12.0 to 0.0	–
Calculation times	–	–	–	–	40 days	3.5 and 20 days	–	24–48 hours	–
1-D, 2-D or 3-D mesh	2-D	2-D	3-D	2-D	2-D	2-D	2-D	2-D	1-D, 2-D
Element shape	Triangular and 1-D joint elements	Single finite and bilinear plane-strain elements	3-D quadrilateral or smoothed particle hydro-dynamic elements	8-node quadrilateral	Triangular	3-node triangular	8-node quadrilateral	4-node quadrilateral	–
FEM code or software used	FE program by Hanna (1975)	MARC	PRONTO 3-D (Taylor and Flanagan, 1989)	PATRAN, ABAQUS	–	–	–	–	–
Issues addressed	Modeling plate penetration test (with snow properties obtained by confined compression and direct shear tests)	Modified Cam-clay plasticity model and Drucker–Prager plasticity model validated on hydrostatic and shear-box tests for snow and used for simulations of interactions between a single rubber thread block and snow-covered surface	Constant-speed uniaxial strain compaction of snow at different rates to study stages of compaction	Skier-induced shear stress and warming effects	Instationary viscoelastic creep in snow (including large volumetric and shear strains, temperature-dependent behavior with phase change, fracture or progressive creep damage of weak layers, forces exerted on defense structures)	Development of FE program *Haefeli* solving instationary heat transfer and creep (viscoelastic deformation), its validation for temperature drop imposed on a snow block and forces exerted on avalanche defense structures to compare with Swiss Guideline calculation procedure	Determination of shear stress distribution in the weak layer caused by shear frame (inserted into weak layer or above it)	Study of effects of weak-layer length, deformation rate, temperature variation	Formulation of size effect law for fracture triggering in dry snow slabs for studying material fracture parameters (fracture toughness and snow slab thickness)
Laboratory or field studies used as references for validation	Part of the same study	Part of the same study and Meschke and others (1993)	–	–	De Quervain (1945); Kojima (1974)	Cold laboratory experiments on snow metamorphism by M. Schneebeli and B. Brown, in 1998	Part of the same study	–	Perla (1977)
Sites investigated for validation	–	–	Compared to Byrd Station polar snow pressure-density profile (Gow, 1968)	–	–	–	–	–	–

Table 1. (continued)

Parameter	Mahajan and Senthil (2004)	Haehnel and Shoop (2004)	Schneebeli (2004)	Pieritz and others (2004)	Stoffel (2005)	Jones and others (2006)	Sigrist and others (2006)	Sigrist and Schweizer (2007)	Mahajan and Joshi (2008)
Constitutive law/ Approach	Static viscoelasticity with phenomenological constitutive law for cohesive surface elements (Needleman, 1990), which create a new surface	Capped Drucker–Prager constitutive law for low-density snow	Linear elasticity (snow fabric obtained from X-ray microtomography)	Linear elasticity under isothermal conditions (for raw snow volume data obtained from X-ray microtomography)	Temperature-dependent viscoelasticity/ Tensorial constitutive law with linear fracture mechanics (2-D); *N*-directional approach with damage mechanics (3-D) (used for the first time on a geophysical material)	Linear elasticity for compressible material	Linear elasticity	Linear elasticity	Static nonlinear plane strain; dynamic linear elasticity with elastic constitutive law for cohesive surface elements (Xu and Needleman, 1994), which create a new surface
Young's modulus (10^6 N m^{-2})	3, 10	1.379–13.79 (after Shapiro and others, 1997)	$E_i = 9.5\times10^3$ 61.8–226 (simulated)	$E_i = 9.0\times10^3$	$E_0 = f(\rho) = 0.1873e^{0.0149\rho}$ ($r^2 = 0.928$), for fine-grained well-bonded snow with ρ=180–450 kg m^3, temp. = –20°C to –2°C (after von Moos and others, 2003)	0.3, 1.5, 7.5 (after Mellor, 1975; Shapiro and others, 1997)	Obtained from snow micro-penetrometer (SMP) penetration resistance signal	5.0–10.0 (determined via penetration resistance profile made with SMP)	0.1–10.0
Poisson's ratio	0.23	0.30	$\upsilon_i = 0.30$	$\upsilon_i = 0.33$	0.00	0.25, 0.49	$\upsilon = \upsilon_0 + c(\rho - \rho_0)$, where $\upsilon_0 = 0.2$, $\rho_0 = 300$ kg m^{-3}, $c = 5\times10^{-4}$ m^3 kg^{-1} (after Mellor, 1975)	$\upsilon = \upsilon_0 + c(\rho - \rho_0)$, where $\upsilon_0 = 0.2$, $\rho_0 = 300$ kg m^{-3}, $c = 5\times10^{-4}$ m^3 kg^{-1} (after Mellor, 1975)	0.23
Viscosity (Pa s)	10^8, 5×10^{10}	–	–	–	–	–	–	–	–
Strain rates (s^{-1})	–	–	–	–	10^{-10}–10^{-3}	>10^{-4}	–	–	–
Density or density range (kg m^{-3})	100, 300	150–250 (average 200)	243, 308	~260	180–300	105, 185, 275	~234–348	173, 187, 202	100, 300
Snow depth (slab or sample thickness) (m)	2.0 assumed, bottom 0.105 modeled	0.392–0.514	0.0036, 0.00756	0.0015 (diameter)	<3.0	2.1–2.7 (0.1–0.7)	(<0.2)	(0.21–0.3)	<1.0
Fracture energy of the weak layer or interface (J m^{-2})	0.05	–	–	–	–	–	0.04 ± 0.02 (critical energy release rate)	0.07 (critical energy release rate; determined by the study)	$\phi_t = 0.01$–0.02 $\phi_n = 0.05$
Weak-layer thickness (m)	0.002–0.01	–	–	–	0.0	0.003	0.0	–	0.005–0.0113
Weak-layer length (m)	2 (same as model length)	–	–	–	0–20	10 (same as model length)	0.5 (same as specimen length)	0.6–1.8	2 (same as model length)

Weak-layer properties	Soft layer (100 kg m^{-3}) with lower E_0 and viscosity (3 MPa; 10^8 Pa s) with normal and shear strength of interface as 0.00184 and 0.002–0.008 MPa	–	–	–	Springs transferring normal stress with shear resistance = 0	Low-density layer (105 kg m^{-3}) with E_0 = 0.3 MPa	Interface, E_0 = 4–20 MPa (average = 11.6 MPa)	Assumed as the same material as the adjacent upper slab (observed: faceted crystals and depth hoar overlain by a 2 mm crust)	Soft layer (100 kg m^{-3}) with lower E_0 (0.05, 0.1, 0.75, 1.0 MPa) with normal and shear strength of interface as 0.00184 and 0.00085–0.0017 MPa
Temperature or temperature range (°C)	–	–10 to –1	(Temperature gradient 100°C m^{-1})	–5 (for E_i and υ_i)	–32.5 to –1.0	–	–	–	–
Calculation times	10 hours	–	–	–	–	–	–	–	<30 ms
1-D, 2-D or 3-D mesh	2-D	1-D, 2-D, 3-D	3-D	3-D	2-D, 3-D	2-D	2-D	2-D	2-D
Element shape	4-node quadrilateral	Single- and multi-element (no details)	8-node brick element	4-node tetrahedral	Quadrilateral (2-D) Tetrahedron (3-D)	4-node quadrilateral	not specified	–	4-node quadrilateral, triangular
FEM code or software used	ABAQUS	ABAQUS	FE program by Van Rietbergen and others (1996)	Package (RCFEA) developed as objective of the study (C++ with OOP)	–	PATRAN, ABAQUS	ANSYS	–	ABAQUS
Issues addressed	Interface crack propagation in layered snow (length of crack, slope angle, interface properties, weak-layer thickness)	Simulating radially confined uniaxial compression test, plate-sinkage test, wheel rolling through fresh snow (for homogeneous and multilayer snow)	Determination of elastic modulus and its dependence on microstructure, stress distribution in snow volume obtained from X-ray microtomography	Development of code for micromechanics modeling with 3-D geometry obtained from X-ray microtomography; simulation of uniaxial compression text	Snow creep, snow glide, stress on defense structure, avalanche formation (stress intensity factor at crack, weak-layer length, fresh snow loading, temperature rise)	Layered snowpack loaded by skier (slab thickness, relative stiffness of layers and stress concentrations)	Evaluate energy release rate in mode II fracture in layered snow	Determination of critical energy release rate for fracture propagation in a weak layer	Interface crack propagation in layered snow (rapid crack growth, crack velocity under skier loads)
Laboratory or field studies used as references for validation	–	Abele and Gow (1975); Alger and Osborne (1989); Shoop and Alger (1998); Blaisdell and others (1990); Green and Blaisdell (1991); Richmond (1995)	Part of the same study	–	De Quervain (1945); Kirchner, (2002a,b); Baggi (2003)	–	Part of the same study	Part of the same study	Part of the same study
Sites investigated for validation	–	–	–	–	Vallée de la Sionne, 'Matte'-Frauenkirch (Switzerland)	–	–	–	–

Table 1. (continued)

Parameter	Heierli and others (2008)	Habermann and others (2008)	Teufelsbauer (2009)	Lee (2009)	Mahajan and others (2010)	Srivastava and others (2010)	Cresseri and others (2010); Cresseri and Jommi (2005)	Theile (2010)
Constitutive law/ Approach	Linear elasticity	Static linear elasticity	Viscoelasticity	Drucker–Prager cap-hardening model for fresh snow	Static nonlinear plane strain; dynamic linear elasticity with elastic constitutive law for cohesive surface elements (Xu and Needleman, 1994), which create a new surface	Linear elasticity (snow fabric obtained from X-ray microtomography)	Elasto-viscoplastic law with analytical expression for sintering	Linear elasticity (snow fabric obtained from X-ray microtomography) with maximal principal stress failure criterion; Glen's creep laws for mono- or polycrystalline models
Young's modulus (10^6 N m^{-2})	7.5 ± 2.5 1.5 ± 0.8 5.0 (Shapiro, 1997)	0.15–7.5 Also use $(E_0(\rho) = A(\rho/\rho_0)^{2.94}$ where $A = 968$ MPa, $\rho_0 = 917$ kg m^{-3} after Sigrist (2006)	–	13.79	0.1, 1.0	$E_i = 9.5 \times 10^3$ (Sanderson, 1988) ~460 to 1007 (calculated primary Young's moduli, E_1)	2.1–12.4 (shear modulus) (various sources) assuming $\nu = 0.25$, $E = 5.25$–31.0	$E_i = 10 \times 10^3$
Poisson's ratio	0.20	0.25	–	0.30	0.23	$\upsilon_i = 0.30$ (after Sanderson, 1988)	–	0.3
Viscosity (Pa s)	–	–	After Gubler (1994)	–	–	–	–	–
Strain rates (s^{-1})	–	–	–	–	–	–	10^{-7} to 10^{-4}	10^{-8} to 10^{-6}, $>10^{-4}$
Density or density range (kg m^{-3})	187, 134, 200	100–270	Measured for initialization (~40–360)	200	100, 300	~368–428	200–400	$\rho_i = 920$, 180–520
Snow depth (slab or sample thickness) (m)	(0.26, 0.11)	<2.2 (0.2–1.2)	Lidar data	0.05–0.60	0.605 (0.1)	(0.00257)	–	(<0.004)
Fracture energy of the weak layer or interface (J m^{-2})	0.07 ± 0.02 0.03 (specific fracture energy per unit of crack surface)	–	–	–	$\phi_t = 0.01, 0.02$ $\phi_{n,wl} = 0.05$	–	–	–
Weak-layer thickness (m)	0.01	0.001–0.011	–	–	0.005	–	–	–
Weak-layer length (m)	∞	10 (same as model length)	–	1.46 (domain length)	8 (same as model length)	–	–	–

Weak-layer properties	Collapsible brittle interface layer in elastic medium	Soft layer (100 kg m^{-3}) with lower E_0 (0.15 MPa)	–	–	Soft layer (100 kg m^{-3}) with lower E_0 (0.1 MPa) with normal and shear strength of interface as 0.00180 and 0.00085–0.00170 MPa, and weak-layer strength 0.002 MPa	–	–	–
Temperature or temperature range (°C)	–	–	~–26.0 to 2.0	–	–	–12 to –2 (temperature gradient 96°C m^{-1})	–	–11 to –6 (for experiments)
Calculation times	–	–	~1632 hours	0.2 s	0.1–0.16 s	–	–	Up to 20 days for creep runs
1-D, 2-D or 3-D mesh	2-D	2-D	2-D	3-D	2-D	3-D	2-D	3-D
Element shape	not specified	4-node quadrilateral	Triangular	4-node axisymmetric (CAX4R)	4-node quadrilateral	Trilinear isotropic brick elements	–	Beam elements, tetrahedral and octahedral meshes, and spheres
FEM code or software used	ANSYS 9.0	ANSYS	MATLAB	ABAQUS Explicit 6.4	ABAQUS	Voxel-based FE program (Garboczi, 1998)	ABAQUS	ANSYS
Issues addressed	Mixed-mode anticrack, notch crack energy and critical length	Snowpack layering influence on skier-induced shear stress	SnowSim model using terrestrial laser scanning data to study temperature field (also density, stress and strain)	Pressure–sinkage relationships of tire–snow interaction studied by using rigid plate and static pneumatic tire indentation models	Interface and middle-layer failure in layered snow under skier loads including slope-normal collapse	Simulation of linear elastic properties and their dependence on microstructure (ice volume fracture, mean intercept length) in snow volume obtained from X-ray microtomography	Integration of an elastic–viscoplastic constitutive model and evolution law for sintering validated on variety of previous laboratory tests	Study of mechanical behavior of snow based on microstructure derived through computed tomography scans (snow creep, ski/snow interaction, brittle failure of snow)
Laboratory or field studies used as references for validation	Sigrist and Schweizer (2007); Gauthier and Jamieson (2008)	37 snow profiles, Columbia Mtns, Canada	–	–	–	Part of the same study	Meschke and others (1996); Scapozza and Bartelt (2003a,b); Desrues and others (1980); Von Moos and others (2003)	Part of the same study
Sites investigated for validation	–	–	Slopes near Lech am Arlberg, ~2000 m a.s.l. (Austria)	–	–	–	–	–

Table 1. (continued)

Parameter	Teufelsbauer (2011)	Miller and others (2011)	Hagenmuller (2011)	Gaume and others (2011)	Gaume and others (2012)
Constitutive law/Approach	Isotropic viscous fluid	Snow is considered as Lagrangian solid with volumetric and deviatoric constitutive relationships. Volumetric contribution is obtained from an equation of state (with shock Hugoniots). Deviatoric part is modeled as linear elastic with maximum principal stress failure criterion, $f(\rho)$	Linear elasticity (snow fabric obtained from X-ray microtomography) with maximal principal stress failure criterion	Elasto-plastic (Drucker–Prager model for a slab and interface with shear softening for the weak layer)	Linear elasticity with a quasi-brittle (strain-softening) interfacial law for weak layers
Young's modulus (10^6 N m^{-2})	–	0.2, 60 (after Mellor, 1975)	$E_i = 10 \times 10^3$	1.0	1.0
Poisson's ratio	$0.0 < \upsilon_{viscous} = f(\rho, T) < 0.49$ (see original)	–	0.3	0.30	0.20
Viscosity (Pa s)	$\eta_s = 0.05 \times \rho^{-0.0371T+4.4} \times (10^{-4} e^{0.018\rho} + 1)$, where $\eta_s = 2\eta(1-\nu)/(1-2\nu)$	–	–	–	–
Strain rates (s^{-1})	$<10^{-5}$	–	$>10^{-4}$	–	–
Density or density range (kg m^{-3})	30–57; 115; 244–449 (initial)	111, 400	$\rho_i = 916.7$, 324–621	250	250
Snow depth (slab or sample thickness) (m)	0.36–0.5; ~3.36; terrestrial laser scanning	1.51 (0.5)	(<0.004)	(0.3–2.1)	(<2.0)
Fracture energy of the weak layer or interface (J m^{-2})	–	–	–	–	–
Weak-layer thickness (m)	–	0.01	–	0	0
Weak-layer length (m)	–	10 (same as model diameter)	–	30 (same as model length)	50 (same as model length)
Weak-layer properties	–	Soft layer (111 kg m^{-3}) with lower E_0 (0.2 MPa) with maximum compressive normal stress, maximum shear stress and maximum octahedral stress = 0.03, 0.04 and 0.015 MPa, respectively	–	Elasto-plastic interface with shear softening (characteristic length = 2 mm; $\tau_{res} = \tau_{peak}/2$); Mohr–Coulomb rupture criterion with no cohesion	Elasto-plastic interface with shear softening (characteristic length = 2 mm; $\tau_{res} = \tau_{peak}/2$); Mohr–Coulomb rupture criterion with spatially varying cohesion (average = 0.006–0.015 MPa, std dev. = 0.003 MPa), and a friction angle = 30°.
Temperature or temperature range (°C)	–32.5 to –1.0	–	–20 (for experiments)	–	–
Calculation times	–	<~100 ms	–	–	–
1-D, 2-D or 3-D mesh	2-D	3-D	3-D	2-D	2-D

Element shape	triangular	2-D axisymmetric square mesh	Beam elements with circular cross section (ANSYS SOLID188); voxel (8-node brick elements – ANSYS SOLID45); tetrahedron elements (ANSYS SOLID185 and 92)	4-node quadrilateral	4-node quadrilateral
FEM code or software used	MATLAB/SnowSim	ANSYS/ AUTODYN	ANSYS	Cast3M (Laborderie and Jeanvoine, 1994)	Cast3M (Laborderie and Jeanvoine, 1994)
Issues addressed	Development and validation of snow creep model with variable temperature and density-dependent Poisson ratio (part of SnowSim model)	Study of snow's response to explosive air blasts (including nonlinear interactions of air and layered snow)	Study of snow strength (brittle failure) as a function of snow microstructure (derived from computed tomography scans); mesh type and element number influence on simulated snow strength	Studying heterogeneity of weak layer and avalanche release angle	Obtaining avalanche release depth distributions through coupling of mechanical and meteorological factors
Laboratory or field studies used as references for validation	De Quervain (1945); Hiller and Bader (1990); Gubler (1994); Abe (2001)	–	Part of the same study	–	Database of avalanche release depths from La Plagne ski resort, France
Sites investigated for validation	Avalanche test site in Lech am Arlberg, Austria	–	–	–	–

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