A saddle coil system for TCV and RMP spectrum optimisation

J. Rossel, J.-M. Moret, Y. Martin and the TCV Team [4]

Ecole Polytechnique Fédérale de Lausanne (EPFL)

Centre de Recherches en Physique des Plasmas

Association Euratom Confédération Suisse

CH-1015 Lausanne, Switzerland

Introduction

Edge localized modes (**ELM**), related to the high confinement regime (H-mode), lead to a degradation of the plasma confinement and a release of energetic particles towards the vessel walls. Scaling the current experimental data to ITER predicts that the power flux related to ELMs will cause an intolerable erosion and heat load on the plasma facing components. Experiments on DIII-D [1, 3] and JET [7] have demonstrated that the application of resonant magnetic perturbation (**RMP**) is able to mitigate or suppress ELMs while keeping sufficient confinement properties. The description of the mechanism responsible for this phenomenon is still incomplete. The limits of the process, in terms of operation domain, are not yet accurately known. Experiments in different Tokamaks reveal opposite results for similar conditions. With that respect, TCV (Tokamak à Configuration Variable) unique plasma shaping and positioning capability could extend the range of accessible magnetic perturbation modes for a given RMP coil system geometry.

A multi-purpose saddle coil system (**SCS**) is therefore proposed as part of a future upgrade of TCV. This system consists of 3 rows of 8 coils, each coil having independent power supplies, and provides simultaneously error field correction (**EFC**) [5], RMP [1] and fast vertical control (**VC**). Other applications, like resistive wall mode control and controlled plasma rotation, are also considered.

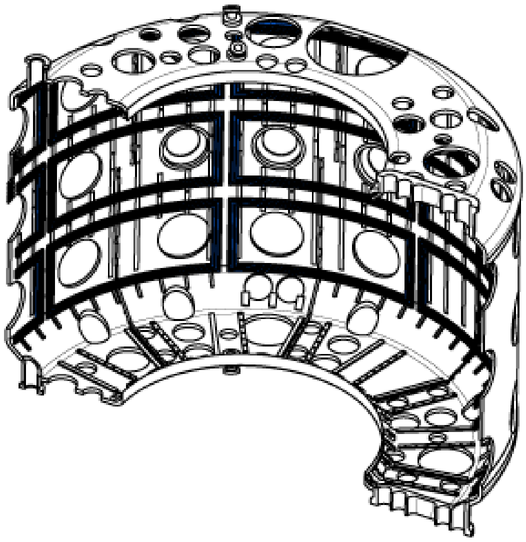


Figure 1: Proposed saddle coil system: 3 rows of 8 internal coils (10 turns). Powered with 4 kAt for RMP, 3.2 kAt for EFC and 5 kAt for VC.

Feeding the coil system with independent power supplies allows an optimisation of the coil current distribution, both toroidally and poloidally. A 3D multi-mode geometry-independent Lagrange method has been developed and appears to be an efficient way to minimize parasitic modes and current requirements while imposing the amplitude and phase of a number of target modes. For example, the relative amplitude of edge modes can be increased at the cost of lower absolute amplitudes, demonstrating a degree of controllability on the localisation of the magnetic perturbation. In order to apply the Lagrange method, the characterization of the system in terms of spectral degeneracy is crucial to determine the set of simultaneously achievable target modes.

Current distribution optimization method

The optimization method relies on the calculation of the spatial spectrum of b [2, 5, 6], the locally normalized perpendicular magnetic perturbation due to the SCS, expressed in straight field line coordinates.

$$b(\rho,\theta^*,\phi) = \frac{B_\perp R \mid \nabla\rho \mid}{B_{0,\phi}} \qquad \tilde{b}(\rho,m,n) = \frac{1}{(2\pi)^2}\iint_0^{2\pi} \mathrm{d}\phi \mathrm{d}\theta^* b(\rho,\theta^*,\phi) e^{i(-m\theta^*-n\phi)} \quad (1)$$

The cost function f minimized by the Lagrange method is defined as a linear combination of the norm of the coil current vector and the amplitude of a selection of modes:

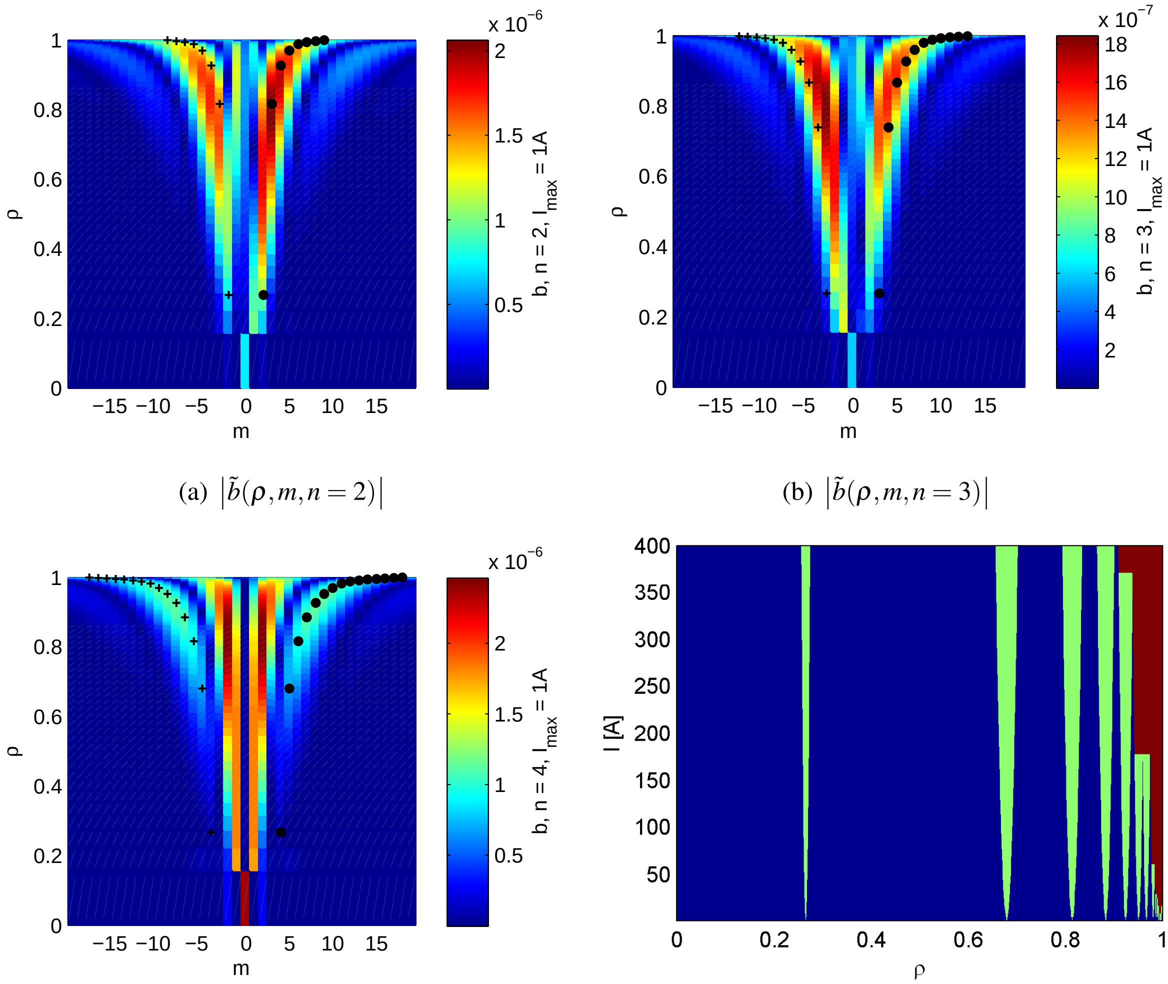
$$f(\{I_c\}) = w_{cur}\sum_c I_c^2 + \sum_k w_k \sum_{p\in S_k}\left[\left(\sum_c \tilde{b}_{cp,r} I_c\right)^2 + \left(\sum_c \tilde{b}_{cp,i} I_c\right)^2\right] \quad (2)$$

with c the coil index, w_k relative weights and normalization factors, p the index on triplets (ρ,m,n), k the index of the set of optimized modes S_k, r and i indices standing for real and imaginary parts of the modes. $w_k > 0$ (resp. $w_k < 0$) leads to a minimization (resp. maximization) of the set of modes S_k. The Lagrange optimization is obtained by solving $\nabla_{\{I_c\},\{\lambda_t\}} h = 0$ with h defined by:

$$h(\{I_c\},\{\lambda_t\}) = f(\{I_c\}) + \sum_t \left[\lambda_{t,r}\left(\sum_c \tilde{b}_{ct,r} I_c - \Re(A_t e^{i\alpha_t})\right) + \lambda_{t,i}\left(\sum_c \tilde{b}_{ct,i} I_c - \Im(A_t e^{i\alpha_t})\right)\right] \quad (3)$$

with t the index on the target modes, A_t and α_t their respective amplitude and phase, and λ_t the Lagrange multipliers. $\nabla_{\{I_c\},\{\lambda_t\}} h = 0$ yields a full rank system of linear equations, inverted with standard methods. If α_t are free parameters, the phase combination leading to the lowest cost is selected. The type of optimum returned by the method is checked by exploring the affine sub-space of current vectors fulfilling the constraints around the optimum.

The linear method presented above minimizes $\sum_c (I_c)^2$, a simplified version of the non linear real constraint $\max_c(|I_c|)$. A non linear algorithm has been created to minimize f_{nl}, the non linear cost given by using $\max_c(I_c^2)$ instead of $\sum_c (I_c)^2$ in f. It is based on the exploration of the affine space of currents fulfilling the constraints, starting from the solution provided by the linear approach. If the α_t are free parameters, an effective approach consists in using the linear solution for each phase combination, but using the non linear cost to select the optimal phase combination. This method has been used to obtain the results displayed in figure 2.



(a) $\left|\tilde{b}(\rho,m,n=2)\right|$

(b) $\left|\tilde{b}(\rho,m,n=3)\right|$

(c) $\left|\tilde{b}(\rho,m,n=4)\right|$

(d) Ergodization map, $n=4$

Figure 2: Optimal spectra given by the Lagrange method for different target values of n. Equilibrium used is a standard H-mode plasma with magnetic axis located at $Z=0$. The $\bullet$ locate the resonant flux surfaces while the $+$ locate their symmetrical non resonant counterparts. Figure 2(d) shows the island width (green) and the ergodic regions (red) as a function of the maximal current fed in the SCS.

Mode degeneracy

In the problem of spectrum optimization, the questions of the number of simultaneously controllable modes, mode degeneracy and availability of optimization must be addressed. The most general approach consists in grouping coils in sets s of equivalent coils (e.g. identical coils on the same row, with arbitrary toroidal spacing) and using the first coil of the set to obtain the spectrum for the whole set:

$$\tilde{b}(\rho,m,n)=\sum_s\sum_c \tilde{b}_c^s(\rho,m,n)I_c^s=\sum_s \tilde{b}_0^s(\rho,m,n)\sum_c I_c^s e^{-in\phi_c^s}=\sum_s \tilde{b}_0^s(\rho,m,n)\hat{I}^s(n) \qquad (4)$$

with ϕ_c^s the toroidal shift between the coils of the set s and $\hat{I}^s$ the **generalised Fourier transform** of I_c^s in the current space (ϕ_c^s does not necessarily describes a regular grid). From equation (4), it follows that only 1 target mode can be given per set of equivalent coils per value of n. In the case of evenly spaced coils, $\phi_c^s = \frac{2\pi}{N_s}c$, $c \in \{0, N_s-1\}$ without necessarily being complete, degeneracy of modes occur:

$$\hat{I}^s(n+pN_s) = \hat{I}^s(n) \quad \forall p \in \mathbb{N} \qquad \text{and} \qquad \hat{I}^s(N_s-n) = \hat{I}^{s*}(n) \tag{5}$$

which now limits the previous statement to non degenerated target modes only. In addition, $\Im\left(\hat{I}^s(n \in \{\frac{N_s}{2}, N_s\})\right) = 0$, therefore limiting arbitrary phase setting of target modes in these cases. Finally, in the case of complete evenly spaced coil sets, $\hat{I}^s$ is equal to the discrete Fourier transform of I_c^s, so that modes in different values of n can be orthogonally activated by using Fourier modes for the currents in each coil row. The feature of orthogonal activation in even geometries implies the necessity to use a cross-mode cost function in the implementation of the Lagrange method, such as the cost based on the current amplitude. In addition, the norm of the current vector being related to $\sum_{s,n} |\hat{I}^s(n)|^2$, it is independant of the phase of the current modes. By using the non linear cost, based on the maximum of the current, an optimisation of this phase is achieved, generally leading to a localisation of the extrema of the current mode in the middle of 2 consecutive coils. Such an optimisation can lead to an increase of 40% of the mode amplitude for $n = 2$ in the TCV SCS case.

The considerations above allow a characterization of the features of the TCV upgrade SCS: 5 orthogonal classes of n ($n \in \{0,1,2,3,4\}$), with main degenerate pairs $\{0,8\}$, $\{1,7\}$, $\{2,6\}$, $\{3,5\}$. For classes 1, 2 and 3, the 3 coil rows allow a maximum of 3 simultaneous targets per class (without simultaneous spectrum optimisation), while for classes 0 and 4, only 1 target per class is allowed (with simultaneous spectrum optimisation).

The loss of 1 coil in an evenly spaced coil system is of interest in terms of impact on the mode control. From the theory above, the main impact of such a loss is the lost of the orthogonal activation of modes. In terms of control, this loss can be compensated by adding a cost function on the side-bands that must be avoided. The latter cannot be avoided for the classes $n = 0$ and $n = N_s/2$ however.

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